CEEPR Center for Energy and Environmental Policy Research

On Coase and Hotelling

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09-003 April 2009

A Joint Center of the Department of Economics,
MIT Energy Initiative, and Sloan School of Management

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April 12, 2009

Abstract

It has been long recognized that an exhaustible-resource monopsonist faces a commitment problem similar to that of a durable-good monopolist. Indeed, Hörner and Kamien (2004) demonstrate that the two problems are formally equivalent under full commitment. We show that there is no such equivalence in the absence of commitment. The existence of a choke price at which the monopsonist adopts the substitute (backstop) supply divides the surplus between the buyer and the sellers in a way that is unique to the resource model. Sellers receive a surplus share independently of their cost heterogeneity; a result in sharp contrast with the durable-good monopoly logic. The resource buyer can distort the equilibrium through delayed purchases, but the Coase conjecture arises under extreme patience (zero discount rate).

JEL Classification: D42; L12; Q30.

Keywords: durable goods, exhaustible resources, Coase conjecture

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1 Introduction

The theory on durable-good monopoly, originated with Coase's conjecture, is one the best understood models of dynamic competition in economics.[1] It would be of great value if the implications of a structure so well explored can be exported to other fields of economics. An important field where Coase's insight seem to apply is the theory of exhaustible resources, put forward by Hotelling (1931). Indeed, Hörner and Kamien (2004) establish an important equivalence between the durable-good monopoly and the exhaustible-resource monopsony: under full commitment, the two problems "differ only in the interpretations placed on symbols". Since the two problems share the same structure, one may very well conclude that the equilibrium outcomes without commitment are also equivalent, so that the conditions well understood for the Coase conjecture could be readily applied to the resource problem. In this paper, we challenge this view. The surplus-sharing in the resource model is shaped by the resource substitute, an essential element of the resource model having no natural counterpart in durable-good monopoly theory.

For durable goods, the Coase conjecture can arise only if consumer valuations decline with the stock of the good in the market. As lower valuation consumers are expected to enter the market at some future date, current buyers can gain from waiting. According to the conjecture, the monopolist sells at the lowest valuation price when buyers are patient enough. In the resource monopsony, the stock is the amount of resource already extracted, and the value changing with the stock is the sellers' extraction cost. The conjecture is then that if the low-cost sellers can wait for the high-cost sellers to enter the market, they will do so and, thereby, force the buyer to pay his choke value for the resource, i.e., the price above which its demand for the resource falls to zero. In this sense, the buyer's monopsony power vanishes 'in the twinkling of an eye", as expressed by Coase in connection with the durable-good monopoly.

Our result is that the above analog does not hold when the resource choke value is determined by the utility the buyer receives from switching to a substitute supply. In

[1] The conjecture was presented in Coase (1972). In the literature that follows, the conjecture is understood as the entire loss of monopoly power when consumers are patient enough. Early formalizations are Stokey (1981), and Bulow (1982). The monopolist may escape the conjecture, at least partially, if: marginal production costs are convex (Kahn, 1986); reputational strategies can be used (Ausubel and Deneckere, 1989); a price-quantity scheme can be used to discriminate among discrete buyers (Bagnoli et al., 1989); the good depreciates (Karp, 1996); there is entry of new consumers (Sobel, 1991); or there are capacity costs (McAfee and Wiseman, 2008).

resource economics, this is the prevailing, if not the only, interpretation of the choke price (see, e.g., Dasgupta and Heal, 1979). Substitutes that put a cap on the resource price are often called backstop technologies, after Nordhaus' (1973) work on the effect of future backstops on resource prices. A switch to the backstop or alternative supply implies that the resource stock will be consumed during a finite period of time. According to the conjecture, in this situation sellers would take a larger share of the surplus the steeper is the rise in their cost curve. When extraction costs do not depend on the evolution of the stock, the buyer should achieve its first-best and appropriate the whole surplus.

We find that the surplus-sharing in the resource market follows a logic different from that suggested by the durable-good analog, provided that the buyer switches to a substitute *at some date*. Sellers always get some surplus due to their ability to wait for the buyer's outside option; even in the absence of extraction costs. In other words, we do not require competitive agents to have varying valuations (costs) to force the buyer to give up some of its monopsony rents, which is in sharp contrast with the durable-good monopoly theory where consumers' heterogeneity is the driving force behind the Coase conjecture. Moreover, for durable goods, extreme patience means that transactions are frequent, so that the market is served quickly in real time, and this type of patience is enough for the conjecture to arise. For resources, the physical depletion of the stock may take long even when transactions are frequent and, therefore, the Coase conjecture requires extreme patience, i.e., the willingness to wait for the exhaustion date and the arrival of the substitute. In the absence of discounting, we find that the Coase conjecture arises and sellers capture the full resource surplus, even though the conjecture does not arise in the equivalent durable-good model. The result can still be called after Coase since it arises from the market's ability to wait for the buyer's outside valuation price, and thus the mechanism follows Coase's reasoning. With positive discounting but arbitrarily frequent transactions, the parties share the surplus depending on the relative sizes of the substitute utility and remaining stock.

The market for oil has motivated much of the literature on optimal tariffs on exhaustible resources (e.g., Maskin and Newbery, 1990; Karp and Newbery, 1993). The durable-good analog would imply a large potential for extracting the sellers' resource rent when the oil production cost is largely independent of the stock level. This situation best describes the conventional, cheap oil stock which is mainly held by the OPEC group. One may argue that the conventional oil stock is the exhaustible-resource in the oil market, and the nonconventional oils are rather the substitute commodities. For this situation, our results suggests a conclusion that is quite different from that of the durable-good analog. According to our results, the cheap oil producers receive a price comparable to the cost of supplying the substitute, not their own cost. The buyer's side effort to coordinate demand reduction through tariffs or other policies can depress the price but they do so only by delaying the arrival of the substitute —and the potential for price depressing is greater the larger is the remaining resource stock.

We organize the rest of the paper as follows. In Section 2 we introduce the commitment solutions for both models using Kahn's (1986) framework for the durable-good monopoly. This model is a natural choice as it embeds common interpretations of both resource and durable-good problems. With the help of this model we show that explicit attention to the choke price is not important for the commitment solutions (which is consistent with Hörner and Kamien result). In Section 3, we study the subgame-perfect equilibrium for the resource model and establish the main result of the paper. The final section concludes and discusses ways to escape the conjecture in the resource context.

2 Commitment solutions: First look at differences

As in Kahn (1986), the durable-good monopolist is a single producer of a perfectly durable good. For exposition, we assume a perfect re-sale or rental market.[2] The monopolist sells the good by giving away the property right, but the subsequent users may also rent it. The flow valuation of the service provided by the good is a function of the total cumulative stock of the good produced up to time t. We denote the stock by S_t and the flow valuation by $P(S_t)$ which is a monotone non-increasing function defined on $[0, \bar{S}]$. There is a continuum of potential buyers. If the path $(S_z)_{z\geq t}$ is known and sales take place at t, the market clears at a price that gives the capitalized value of the marginal unit sold at t:

$$\tilde{p}_t = \int_t^\infty P(S_z)e^{-\delta(z-t)}dz \tag{1}$$

where δ is the discount rate.

The monopolist has convex cost of production $\gamma(q_t)$ that depends on the rate of production $q_t = dS_t/dt$. If the monopoly can commit to a path $(S_z)_{z\geq t}$ at $t = 0$, it will choose this path solving

$$\max_{q_t} \int_0^\infty \{\tilde{p}_t q_t - \gamma(q_t)\}e^{-\delta t}dt$$

[2]The existence of a rental market is inconsequential when there is a continuum of agents. We can assume that each consumer buys either one or zero unit of the good, and disappears after purchase. Alternatively, buyers can be intermediaries who use the goods to serve the resale or rental market.

subject to (1), $q_t = dS_t/dt$, and $S_0 \geq 0$. Using (1), this problem can be rewritten as

$$\max_{q_t} \int_0^\infty \{P(S_t)(S_t - S_0) - \gamma(q_t)\}e^{-\delta t}dt$$

for which the first-order conditions imply that positive sales satisfy

$$P(S_t) + P'(S_t)(S_t - S_0) = \delta\gamma'(q_t) - \gamma''(q_t)\frac{dq_t}{dt}. \tag{2}$$

The left-hand side of (2) is the marginal revenue from renting an additional unit, and the right-hand side is the marginal cost of producing that unit today rather than tomorrow.

For the Hotelling monopsony, we assume that there is a single importer (buyer) of an exhaustible resource. The buyer's utility depends on the rate of consumption q_t. We denote his utility by $U(q_t)$. Each supplier has one unit of the resource and a given cost of extracting and selling that unit. We assume a continuum of suppliers indexed by $S \in [0, \bar{S}]$ and that the unit cost depends on this index; the unit cost is given by a nondecreasing function $c(S_t)$. In continuous time then, $c(S_t)q_t$ is the cost of extracting at rate q_t when the stock already extracted or consumed is S_t.

For a given path $(S_z)_{z\geq t}$, market clearing requires that at all times, where sales take place, the sales price satisfies the following arbitrage condition (i.e., the Hotelling rule)

$$\frac{dp_t}{dt} = \delta(p_t - c(S_t)),$$

which, after some manipulation, can be rewritten as

$$p_t = e^{\delta t}(K - \int_0^t \delta e^{-\delta z}c(S_z)dz) \tag{3}$$

where K is a constant of integration that corresponds to p_0. Since p_t is bounded by some finite value, namely the price of the alternative supply, we can let $t \longrightarrow \infty$ and evaluate K as

$$K = \int_0^\infty \delta c(S_t)e^{-\delta t}dt. \tag{4}$$

which leads to

$$p_t = \int_t^\infty \delta c(S_z)e^{-\delta(z-t)}dz. \tag{5}$$

Note already that expression (5) looks remarkably similar to (1) but for the 'symbols".

If the resource monopsony can commit to a path $(S_z)_{z\geq t}$ at $t = 0$, it will choose this path by solving

$$\max_{q_t} \int_0^\infty \{U(q_t) - p_t q_t\}e^{-\delta t}dt$$

subject to (3), $q_t = dS_t/dt$, $S_t \leq \bar{S}$, and $S_0 = 0$. Using (5), we can rewrite the objective function to see that the optimal $(S_z)_{z\geq 0}$ also maximizes

$$\int_0^\infty \{U(q_t) - \delta c(S_t)S_t\}e^{-\delta t}dt.$$

Over the interval of time of positive sales, the optimal consumption must satisfy[3]

$$c(S_t) + c'(S_t)S_t = U'(q_t) - \frac{1}{\delta}U''(q_t)\frac{dq_t}{dt}. \tag{6}$$

The left-hand side of (6) is the marginal cost from buying an extra unit and the right-hand side is the marginal benefit of consuming that unit today rather than tomorrow.

In view of the conditions (2) and (6) and price functions (1) and (5), the equivalence noted by Hörner and Kamien (2004) is apparent: renaming $P(S)$ as $-\delta c(S)$, $\gamma(q)$ as $-U(q_t)$, and $\tilde{p}_t$ as $-p_t$ shows that the commitment solutions differ only in the interpretations placed on symbols. Note in particular that when cost $\gamma(q)$ is strictly convex, the Coase monopoly has preferences for production smoothing. This corresponds to preferences for consumption smoothing in the Hotelling model, arising when the utility function is strictly concave.

The Coasian commitment problem is known to arise because of declining consumer valuations given by strictly declining $P(S)$. Current consumers can benefit from waiting and delaying purchases only if lower valuation consumers are anticipated to enter the market in the future. Formally, the source of the inconsistency can be seen from equation (2) where the initial stock S_0 enters only if $P'(S) \neq 0$; if the monopoly could reconsider the plan $(S_z)_{z\geq t}$ chosen at $t = 0$ at some future date $t' > 0$, the initial condition would change from S_0 to $S_{t'}$, leading to a change in the solution.

The above reasoning suggests that the problem of commitment arises in the resource model only when the extraction costs $c(S_t)$ are strictly increasing. According to the conjectured analog, the low cost sellers can benefit from delaying sales only when high cost sellers are anticipated to enter the market in the future. In other words, if $c'(S_t) = 0$, the consumption dynamics solved at any future date $t > 0$ are identical to those solved at $t = 0$. In this case the buyer has no incentives to deviate from the consumption path that he announced at $t = 0$.

[3]In terms of the remaining stock, denoted by $Q_t = \bar{S} - S_t$, eq. (6) becomes (see, e.g., Karp and Newbery, 1993, p. 894)

$$c(Q_t) - c'(Q_t)(Q_0 - Q_t) = U'(q_t) - \frac{1}{\delta}U''(q_t)\frac{dq_t}{dt}$$

where $Q_0 = \bar{S}$, $dQ_t/dt = -q_t$, and $c'(Q_t) \leq 0$.

We challenge this reasoning: it does not hold if the resource buyer switches to an alternative supply at some date. Suppose the buyer's benefit from switching to the alternative supply is $W > 0$ per period. In our setting, this benefit can be easily captured as $W \equiv W(\bar{q}) = U(\bar{q}) - \bar{p}\bar{q}$, where $\bar{p} = U'(\bar{q})$ is the unit price of the alternative supply —also known as the choke price— and $\bar{q}$ is the amount of alternative supply measured in exhaustible-resource equivalents. We argue that the Coasian commitment problem follows a different logic in the resource model as long as $W(\bar{q}) > 0$; no matter how small this value is.

Before stating our result formally, let us note that the difference between the models is already visible in the commitment solutions. For example, when $c(S_t) = c$, the price path for the commitment solution is given by

$$p_t = e^{\delta t}[p_0 - c] + c$$

for $q_t > \bar{q}$. The initial price p_0 is a choice variable for the monopsonist in that its level can be chosen by the shape of the consumption path. In particular, the buyer prefers to commit not to use the substitute for a long time if the backstop utility is negligible, i.e., if $W(\bar{q})$ is close to zero. In this case, the buyer will commit to a path $(S_z)_{z \geq 0}$ that postpones the consumption of an ε-amount —more precisely, the smallest possible amount— of the resource for far enough into the future.[4] This destroys the sellers' option of waiting: p_0 collapses to cost c as the sellers race for early sales.[5] Clearly, if such commitment is not feasible in (subgame-perfect) equilibrium, sellers cannot be left with no rents.

By looking at price equations (1) and (5), it is evident that the equivalence found by Hörner and Kamien (2004) requires that prices be governed by the exact same rules. As indicated in (1), in a Coase-world prices are fully determined by the stock path chosen by the monopolist. Likewise, equation (5) says that in a Hotelling-world prices would also be fully governed by the stock path chosen by the monopsonist. But (5) totally neglects the fact that the existence of an alternative supply prevents the very last unit of the stock to be sold for anything less than $\bar{p}$ (recall that in constructing (5) we never imposed $\bar{p}$ as the terminal price).[6] As long as $\bar{p}$ represents the substitute cost, the commitment solutions are not exactly identical; although the difference is negligible when $W(\bar{q})$ is close to zero.

[4] In the durable-good equivalent, that is, with $P'(S) = 0$, the monopolist does not need leave an ε-fraction of consumers for late delivery in other to implement its first-best.

[5] The time τ at which the buyer consumes the ε-amount solves $\bar{p}e^{-\delta\tau} = c$.

[6] With or without commitment, right after buying the last unit for something less than $\bar{p}$ the buyer is ready to buy at $\bar{p}$ from the alternative supply. Such a price jump cannot occur in either case.

Yet, this slight difference in the commitment solutions explains why the subgame-perfect solutions can differ so dramatically. We turn to this now.

3 The difference between the models

In the resource model it is natural to have the stock be consumed gradually over time and the switch to the substitute supply take place at some date. These properties are ensured by the strictly concave utility function and the substitute benefit $W(\bar{q}) > 0$. For durable goods, the corresponding specification exhibits strictly convex production cost $\gamma(q)$, and $P(\bar{S}) > \gamma(0) = 0$. From Kahn (1986), we know that the durable-good seller's commitment problem arises from the changing consumer valuation, not from convex costs. One may thus conjecture that the monopoly can achieve its first-best in the subgame-perfect equilibrium when the consumer valuation is constant, i.e., $P(S) = v$ for all $S \in [0, \bar{S}]$, but costs are still convex. For the resource model, the analogous specification favoring the strategic buyer is one in which extraction costs are constant, i.e., $c(S_t) = c < \bar{p}$, while the utility is strictly concave.

To isolate the commitment problem coming from the substitute price, it helps to state the difference between the models using constant valuations for the durable good and constant costs for the exhaustible resource. We will show that the bargaining powers emerge in opposite ways in the two models. Moving to increasing stock-dependent costs does not eliminate the sellers' bargaining power coming from the substitute price; on the contrary, it is expected to reinforce it.

3.1 The durable-good benchmark

Assume thus for the durable-good model the following variant of the Kahn's (1986) framework: the consumer valuation is a constant $P(S) = v$ for all $S \in [0, \bar{S}]$, and the production cost $\gamma(q)$ is assumed to be strictly convex. Note that there is a continuum of consumers. We consider the subgame-perfect equilibrium, and for this we want to assume discrete time periods to make the extensive form of the game clear. The discrete periods extend to infinity, $t = 0, 1, 2, ...$ At each period t, the monopolist chooses q_t, and after this, the competitive market determines price p_t.

We look for a sales strategy $q_t = q_t(h_t)$ that depends on the history h_t at each t where

$$h_t = ((q_0, p_0), (q_1, p_1), ..., (q_{t-1}, p_{t-1})) \in R_+^{2t}.$$

The pricing strategy for the market is a function of the history and the seller's current choice, $p_t = p_t(h_t, q_t)$. Finding the equilibrium for this specification is a simple undertaking. We verify that the monopolist's first best is a subgame-perfect equilibrium. The result holds for any period length, implying that the commitment built into the period length is not important.

In discrete time, the commitment solution is a sequence $\{S_t\}_{t=0}^{N}$ such that (i) $S_N = \bar{S}$, (ii) the marginal profit is the same from each period in present value, and that (iii) it is not optimal to extend the sales period from N. The last two requirements imply

$$\begin{aligned} v - \gamma'(q_t) &= \beta(v - \gamma'(q_{t+1})) \text{ for all } 0 \leq t < N, \\ v - \gamma'(q_N) &\geq \beta(v - \gamma'(0)), \end{aligned}$$

where $\beta = e^{-\delta\Delta}$ is the continuous-time discount factor over the period Δ (we can set $\Delta = 1$ here). The monopolist maximizes social surplus. When the consumer valuation is constant, the seller has socially optimal incentives for production smoothing. Note that the market is served in finite time, $N < \infty$.

Denote now the socially optimal production rule by $q_t^* = q^*(S_t)$. Note that when the equilibrium horizon is finite, it is sufficient to let sales depend exclusively on the current stock; hence, we can let $h_t = S_t$ be the payoff-relevant history (see, e.g., Kahn, 1986). Consider then the strategy $q_t(h_t) = q^*(S_t)$ for the seller, and $p(S_t, \cdot) = v$ for the market.[7] Clearly, the seller cannot have profitable one-shot deviations from $q^*(S_t)$. But neither has the buyer side of the market from $p(S_t, \cdot)$; no surplus is available in any conceivable continuation game.

For intuition, note that rental value v for the good remains even when the monopolist leaves the market. The market cannot resist buying the last units at that value, which gives the bargaining power to the seller. If there is no rental market and individuals leave the market at purchase, the conclusion remains the same. The seller can achieve the first-best by leaving an ε-mass of consumers unserved in the last period N. The price will jump to v, because no surplus is expected in continuation games as it is subgame-perfect for the seller to follow the same strategy of leaving some remaining consumers unserved in all subsequent periods.[8] The welfare loss can be made arbitrarily small. And if instead of setting quantitities the monopoly seller is setting prices, the first-best remains an equilibrium both with and without the rental market: the seller prices at v for all t and buyers consume along the efficient path.

[7] We can drop the dependence on t in strategies because the strategies defined this way are stationary.

[8] If the seller brings the ε-amount to the market the remaining ε-consumers will bid it down to zero.

3.2 The resource substitute and the Coase conjecture

Let us then turn to the equivalent exhaustible-resource monopsony, where $c(S_t) = c$ for all $S \in [0, \bar{S}]$, utility $U(q)$ is assumed to be strictly concave, and there is finite choke value $\bar{p} = U'(\bar{q}) > c$. The socially optimal allocation is a sequence $\{S_t\}_{t=0}^{N}$ exhausting the stock, $S_N = \bar{S}$, equalizing present-value net marginal utilities, and keeping the latter above the choke value:

$$\begin{aligned} U'(q_t) - c &= \beta(U'(q_{t+1}) - c) \text{ for all } 0 \leq t < N, \\ U'(q_N) - c &\geq \beta(U'(\bar{q}) - c). \end{aligned}$$

This plan is also the monopsonist's first-best consumption plan if he can commit to it. When the substitute utility $W(\bar{q})$ is small, the buyer would like to commit to switch to the substitute far in the future. We have already explained how commitment can transfer the full surplus to the buyer: an ε-amount is left for later consumption enough for destroying the "boundary value" in the sellers' price path. The buyer purchases at cost and consumes according to the first-best plan.

However, the buyer's commitment plan is not subgame-perfect if $W(\bar{q}) > 0$ (or $\bar{q} > 0$), not matter how small $\bar{q}$ and $W(\bar{q})$ are. For ease of presentation, from now on we will focus on the stock still left in the ground,

$$Q_t = \bar{S} - S_t.$$

Proposition 1 *Consider a given Q_t, constant cost $c < \bar{p} = U'(\bar{q}) < U'(0)$, and period length Δ for consumption. As $\Delta \to 0$:*

1. Buyer shares the resource surplus with the sellers as long as $W(\bar{q}) > 0$ and $\delta > 0$;

2. Coase conjecture arises if $\delta = 0$ and $W(\bar{q}) > 0$;

3. Buyer receives the full surplus if $\delta > 0$ and $W(\bar{q}) = 0$.

Without loss of generality we set $c = 0$ here and in the Appendix where we present the formal proof for the continuous-time consumption rule. (The results do not depend on whether the buyer sets quantities or prices in each period; we assume quantity setting in this proof.) We start by explaining the first result where the buyer and the sellers share the resource surplus when there is discounting ($\delta > 0$) and some substitute utility ($\bar{q} > 0$). Both the Coase conjecture and the last item, corresponding to the durable-good analog, follow as limiting cases.

Let us first explain what defines the respective bargaining powers of the buyer and the sellers using discrete time periods, $\Delta = 1$. Suppose that stock Q_0 is depleted in N periods. The first N prices are then

$$p_0 = \beta p_1 = ... = \beta^{N-1} p_{N-1} = \beta^N \bar{p} \tag{7}$$

because the sellers must be indifferent between sales periods as long as there is some stock left.[9] If the buyer decides to delay the exhaustion of the stock, this can be achieved by demanding $\bar{q}$ units less during the N first stages and save these units to stage $N+1$. This one period delay of the substitute arrival implies prices

$$p_0 = \beta p_1 = ... = \beta^N p_N = \beta^{N+1} \bar{p}. \tag{8}$$

The consumption-cost difference in the scenarios (7) and (8) is

$$\beta^N \bar{p}(Q_0 + \bar{q}) - \beta^{N+1} \bar{p} Q_0 = \beta^N (1-\beta) \bar{p} Q_0 + \beta^N \bar{p} \bar{q} \tag{9}$$

In both scenarios, consumptions and prices are the same after stage $N+1$, because $\bar{q}$ is consumed with price $\bar{p}$ in each period. We can thus focus on the difference in the first $N+1$ periods. In (7), the buyer consumes $\bar{q}$ in period $N+1$, but in (8) this consumption comes from the stock, which explains the last term in (9). The interpretation of (9) is then that by reducing consumption by $\bar{q}$ over N periods, the buyer receives a price discount on the full stock, $\beta^N(1-\beta)\bar{p}Q_0$, plus the savings from the substitute cost in one period, $\beta^N \bar{p}\bar{q}$.

In subgame-perfect equilibrium, the buyer chooses consumption and, thus, how much to delay the arrival of the substitute on a period-by-period basis; not over the entire consumption plan as described above. To get an idea of the costs and benefits of today's consumption choice, consider the first period equilibrium consumption q_0, and a one-shot deviation from the equilibrium such that the full consumption q_0 is saved for later consumption. If the period length is $\Delta \neq 1$, the implied overall saving is Δq_0. Since we are considering a deviation from the equilibrium, the consumption sequence is delayed by exactly one period with no further effect on equilibrium choices. This gives the price gain per unit of consumption saved as

$$\beta^N (1-\beta) \bar{p} \frac{Q_0}{\Delta q_0} + \beta^N \bar{p}.$$

[9] The last price p_{N-1} at stage $N-1$ equals the next period discounted choke price $\beta\bar{p}$ since the buyer demands some arbitrarily small ε less than the remaining stock to allocate some sellers to sell jointly with the subsitute and, thereby, achieve the lowest possible price at stage $N-1$.

In the continuous-time limit, $\Delta \to 0$, this expression becomes

$$e^{-\delta T}\delta\bar{p}\frac{Q_0}{q_0} + e^{-\delta T}\bar{p} = p_0\delta\frac{Q_0}{q_0} + p_0,$$

where T is the equilibrium time to exhaustion, and p_0 is the current price that equals the discounted choke price. In continuous time, even small changes in current consumption will alter the overall depletion time, allowing us to express the cost of giving up current consumption as the current marginal utility loss. Therefore, at any t before exhaustion, the continuous-time equilibrium condition that balances the buyer's costs and benefits of delaying consumption is

$$U'(q_t) = \delta p_t\frac{Q_t}{q_t} + p_t. \tag{10}$$

In Appendix we derive this equilibrium consumption rule formally. Note that the first term in right hand side of (10) includes the interest earning on total present-value purchases along the equilibrium path from time t onwards. That sum is divided by q to transform it into marginal units, relevant for current consumption choice.

In view of the above, it is clear that the buyer's bargaining power arises from the ability to destroy overall surplus through delayed purchases. This incentive to delay consumption drives the wedge between the price and marginal utility given in (10). Using the price arbitrage $dp_t/dt = \delta p_t$ together with the boundary condition $p_T = \bar{p}$, the stock depletion equation $dQ_t/dt = -q_t$, and condition (10), the equilibrium path is fully determined. Figure 1 depicts how the price and the marginal utility develop over time.[10]

Let us now discuss the limiting cases. Sellers can expect a surplus share due to their ability to wait for the substitute price, and when $\delta \to 0$, the price path shifts up together with the marginal utility path. When $\delta = 0$, patience is extreme and the Coase conjecture arises. Competitive sellers capture the full resource surplus; there is no reason to accept a lower price than the buyer's outside option.

As $\bar{q} \to 0$ (and $W(\bar{q}) \to 0$), the equilibrium converges to the outcome suggested by the durable-good analog (Hörner and Kamien, 2004). We see from the figure that the marginal utility path is S-shaped. As the outside option vanishes, the buyer initially follows a marginal utility path that is growing at a rate close to the interest rate but,

[10]The figure was plotted using $U(q) = log(q)$, which leads to an explicit solution for the (remaining) stock

$$Q_t = e^{\delta t}(Q_0 - \frac{1}{2\delta\overline{p}}e^{\delta T}) + \frac{1}{2\delta\overline{p}}e^{\delta(T-t)},$$

where T is found from the boundary condition $Q_T = 0$. The descriptive features of the equilibrium follow from this solution.

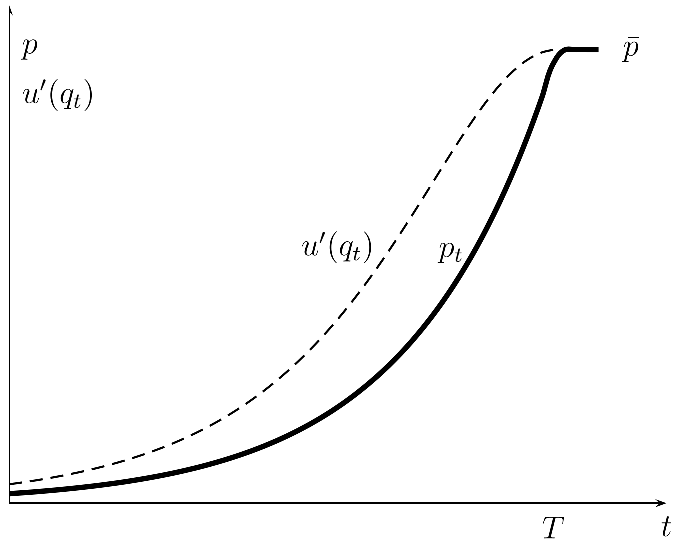

Figure 1: Equilibrium price path and marginal utility

in the end, distorts consumption choices by stretching the overall consumption period. When $\bar{q} = 0$, this stretching becomes extreme, the stock is never exactly exhausted, and the price collapses to zero.

It is natural ask how things change when extraction costs become dependent on the remaining stock, i.e., when unit cost increases with depletion, $c'(Q_t) < 0$. Using the boundary for the price path and the Hotelling rule, we can express the equilibrium price as

$$p_t = e^{-\delta(T-t)}\bar{p} + \int_t^T \delta c(Q_\tau) e^{-\delta(\tau - t)} d\tau.$$

The resource cannot be sold for anything less than $\bar{p}$ and, therefore, the equilibrium price converges to this level independently of whether the stock is economically (last units not extracted, $\bar{p} < c(0)$) or physically depleted (all units extracted, $\bar{p} > c(0)$). The equilibrium delay of consumption by the buyer lowers the price path by postponing the arrival of the substitute much the same way as explained above – the substitute price appears independently of the cost structure in the price equation. This effect is a source of surplus-share to the buyer, when the switch to the substitute takes place at some date, and discounting is positive. For these reasons, the Proposition applies under more general cost structures.

4 Concluding remarks

We conclude by discussing some features of resource markets that may help the resource buyer to escape the conjecture, and how one might restore the equivalence between the resource and durable-good models. For the latter, one might ask what is the analog of the resource substitute in the durable-good model? Recall that in the resource model, the substitute provides an outside valuation for the market (the choke price) with which the good can be ultimately sold. We have shown that in subgame-perfect equilibrium the substitute changes the economic logic of the resource model in a fundamental way. It is therefore important to understand if a similar mechanism can be imported to the durable-good model.

To import the idea of the substitute to the durable-goods, one would need to assume two types of goods, durable and non-durable, such that the monopolist first serves the pool of customers buying the highly-valued durable good, and then switches to serve the (competitive) non-durable segment of the market for some flow profit of $W > 0$. Knowing the existence of the non-durable segment, which can always procure their non-durables at some competitive price, say $\underline{p}$, the monopolist would not be able to credibly commit to never attend the non-durable segment at price $\underline{p}$, which would ultimately prevent him from pricing its last durable units above $\underline{p}$. Thus, these “outside consumers” would in principle improve the bargaining position of the durable-good buyers, forcing the monopolist to leave a fraction of the durable-good surplus with the consumers (for example, when they have a constant valuation for the durable). It is immediately clear that this ”backstop” interpretation is not at all that natural in the durable-good case —the non-durable or flow benefit from buying a durable is already part of the original model—, while the substitute is an essential part of the resource model. Hence, there are good substance-related and economic reasons to argue that the two theories are distinct.

One can still speculate if any of the strategies that alleviate the Coase conjecture in the durable-good model can work in the resource model. It seems not. Ausubel and Deneckere (1989) result would require the buyer to never adopt the substitute. Strategies aimed at slowing down production, either through capacity constraints or convex costs (McAfee and Wiseman, 2008; Kahn, 1986), are already part of the resource model. Neither the introduction of discrete agents (Bagnoli et al. 1989) or the entry of new resource suppliers (Sobel, 1991) seem to change nature of the resource problem. And Karp's (1996) depreciation result does not seem to eliminate the determinant of surplus sharing identified in this paper.

There is nevertheless a different way in which the buyer might be able to escape the conjecture, or more precisely, retain a larger share of the overall surplus. To maintain a close connection to the durable-good framework, in this paper we adopted the traditional and somewhat stark view on the backstop arrival. Once the choke price is reached, the substitute enters the market with perfectly elastic supply. Recent research has developed a multi-sector description of the resource substitution process such that the transition is gradual as sectors move substitutes at different times (e.g., Chakravorty, Roumasset, and Tse 1997). Gerlagh and Liski (2008) have shown that adjustment costs in the form of time-to-build period for the substitute, can bring about considerable bargaining power to the buyer side of the resource market. This, again, is a resource-market specific addition to the Coase conjecture discussion. We believe it is a fruitful agenda to further explore elements that may shape the strategic and dynamic relationships in exhaustible-resource markets.

5 Appendix: Buyer's continuous-time consumption rule

The buyer's equilibrium payoff in continuous time satisfies

$$V(Q_t) = \int_t^T [U(q_\tau) - p_\tau q_\tau] e^{-\delta(\tau - t)} d\tau + e^{-\delta(T-t)} \frac{W(\bar{q})}{\delta} \tag{11}$$

where the choices at time points are evaluated along the equilibrium path (recall that $W(\bar{q}) = U(\bar{q}) - \bar{p}\bar{q}$). When time is discrete and the period length is Δ, the payoff to the buyer at stock level Q_t can be expressed as

$$V(Q_t) = [U(q_t) - p_t q_t]\Delta + e^{-\delta\Delta} V(Q_t - \Delta q_t). \tag{12}$$

For a small Δ, this equation can be Taylor approximated as

$$0 = [U(q_t) - p_t q_t]\Delta - \Delta\delta e^{-\delta\Delta} V(Q_t - \Delta q_t) - \Delta e^{-\delta\Delta} q_t V_Q(Q_t - \Delta q_t).$$

In the continuous-time limit, $\Delta \to 0$,

$$\delta V(Q_t) = [U(q_t) - p_t q_t] - q_t V_Q(Q_t).$$

The equilibrium choice of q_t maximizes the right hand side of (12), and satisfies

$$[U'(q_t) - p_t - \frac{\partial p_t}{\partial q_t} q_t]\Delta - \Delta e^{-\delta\Delta} V_Q(Q_t - \Delta q_t) = 0, \tag{13}$$

or, in the limit $\Delta \to 0$,

$$[U'(q_t) - p_t - \frac{\partial p_t}{\partial q_t} q_t] - V_Q(Q_t) = 0, \tag{14}$$

To find an expression for $V_Q(Q_t)$, totally differentiate the equilibrium value function (11) to get

$$\begin{aligned} dQ_t V_Q(Q_t) &= dV = \\ & \int_t^T [U'(q_\tau) - p_\tau - \frac{\partial p_\tau}{\partial q_\tau} q_\tau] dq_\tau e^{-\delta(\tau - t)} d\tau + && (15) \\ & e^{-\delta(T-t)} [U(q_T) - p_T q_T - U(\bar{q}) + \bar{p}\bar{q}] dT - && (16) \\ & \int_t^T q_\tau dp_\tau e^{-\delta(\tau-t)} d\tau. && (17) \end{aligned}$$

By the fact that we are considering q_t along the equilibrium path, the expression on line (15) is zero: marginal perturbation of the choice variable yieds a zero improvement in the value. In addition, since the buyer switches to the substitute at T, we have $q_T = \bar{q}$ and $p_T = \bar{p}$; hence, the value of the expression on line (16) is also zero. For the last term, note that the consumption cost can be written as

$$\int_t^T q_\tau p_\tau e^{-\delta(\tau-t)} d\tau = e^{-\delta(T-t)} \bar{p} \int_t^T q_\tau d\tau = e^{-\delta(T-t)} \bar{p} Q_t,$$

because equilibrium prices grow at the rate of interest. Therefore, the differential on line (17) captures the effect coming from postponement of the choke price. Thus, the expression for $dQ_t V_Q(Q_t)$ simplifies to

$$dQ_t V_Q(Q_t) = -\delta e^{-\delta(T-t)} \bar{p} Q_t dT = -\delta p_t Q_t d\tau, \tag{18}$$

where $dT = d\tau$, because marginal increase in T equals the period length $d\tau$. Since also $dt = d\tau$, and $dQ_t = -q_t dt$, we can write (18) as

$$V_Q(Q_t) = \delta p_t \frac{Q_t}{q_t}. \tag{19}$$

Reconsider now the first-order condition (13), and use $dt = \Delta$ to rewrite it as

$$[U'(q_t) - p_t - \frac{\partial p_t}{\partial q_t} q_t] dt - e^{-\delta dt} V_Q(Q_t - dt q_t) dt = 0.$$

Evaluate (19) at Q_{t+dt} and rewrite the first-order condition further to obtain

$$[U'(q_t) - p_t - \frac{\partial p_t}{\partial q_t} q_t] - e^{-\delta dt} \delta p_{t+dt} \frac{Q_{t+dt}}{q_{t+dt}} = 0.$$

As $dt \to 0$, the equilibrium direct price effect is zero, $\partial p_t / \partial q_t = 0$. The equilibrium price path depends on Q_t only, as instantaneous consumption has no effect on what the market can expect to receive in the future. The equilibrium consumption rule then becomes

$$U'(q_t) - p_t - \delta p_t \frac{Q_t}{q_t} = 0,$$

which completes the proof.

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