CEEPR Center for Energy and Environmental Policy Research

Using Futures Prices to Filter Short-term Volatility and Recover a Latent, Long-term Price Series for Oil

by

Miguel Herce, John E. Parsons and Robert C. Ready

06-005 WP April 2006

A Joint Center of the Department of Economics, Laboratory for Energy and the Environment, and Sloan School of Management

Using Futures Prices to Filter Short-term Volatility and Recover a Latent, Long-term Price Series for Oil

Miguel Herce[*], John E. Parsons[**] and Robert C. Ready[***]

Revised November 2006

Oil prices are very volatile. But much of this volatility seems to reflect short-term, transitory factors that may have little or no influence on the price in the long run. Many major investment decisions should be guided by a model of the long-term price of oil and its dynamics. Data on futures prices can be used to filter out the short-term volatility and recover a time series of the latent, long-term price of oil. We test a leading model known as the 2-factor or short-term, long-term model. While the generated latent price variable is clearly an improvement over the raw spot oil price series, we also find that (1) the generated long-term price series still contains some of the short-term volatility, and (2) a naïve use of a long-maturity futures price as a proxy for the long-term price successfully filters out a large majority of the short-term volatility and so may be convenient alternative to the more cumbersome model.

INTRODUCTION

A striking characteristic of the spot oil price series is the fact that it occasionally exhibits large swings up or down which are then followed by reversals back towards some central tendency. The most dramatic example of such a swing is the sharp price spike occasioned by the first Gulf War in late 1990. Starting from a level below \$18 per barrel in mid-July, the price peaked above \$40 per barrel in October, then falling back below \$18 per barrel by late-February 1991. A less dramatic example is the drop in prices that occurred in 1993 when conflicts within OPEC resulted in a temporary glut of supplies and prices went from over \$18 per barrel in August to below \$14 per barrel in

[*] CRA International, Boston, Massachusetts, USA.

[**] Corresponding author: MIT Center for Energy and Environmental Policy Research, E40-435, 77 Massachusetts Ave., Cambridge, MA 02139 USA, E-mail: jparsons@mit.edu

[***] Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania, USA.

December, recovering back to over $18 per barrel again in June 1994.[1] From December 1998 to November 2000 the price nearly tripled from $11 to $35 per barrel. But this dramatic increase was marked by several reversals down by $10 per barrel before the upward trend recovered. Then within the space of slightly more than a year, the price fell again down to $18 per barrel and recovered as quickly back above $30, continuing its rise marked by swings of as much as $10 per barrel. For each of the swings just mentioned the recovery occurs within less than a year, sometimes within a couple of months.

These dramatic fluctuations can be seen in Figure 1 which shows the graph of the spot price of oil from September 1989 through May 2006.[2] The huge price run-up since late 2001 dominates the figure, but even during this rise the trend has been marked by significant swings. As we are writing this, the price has fallen 20% from its high in mid-summer 2006 in the space of only a few months. One result has been newsworthy losses in a number of investment funds that had profited from the earlier run-up.

How much of the daily movements in the oil price is composed of such transitory, short-term swings? How much volatility is left after the effect of these short-term movements is filtered out? When we observe a sharp spike in the price, how much of that is likely to be reversed, and how quickly? Answering these questions is important for a large number of decisions.

[1] It was this temporary drop in the spot price that occasioned the brush with bankruptcy by the German company Metallgesellschaft due to its speculation in oil futures.

[2] Consistent with the practice in the finance literature, we refer to the price of the near month futures contract as the spot price, although it is not a price for truly immediate physical delivery as the term ‘spot’ denotes. Futures prices have a number of useful properties that make it worthwhile to work with the shortest maturity futures contract instead of with the actual spot price series.

Short-term swings in the oil price don't change the value of a major oil-related investment, so it would be useful to be able to filter them out. If we can filter them out and arrive at a cleaner measure of the long-term volatility of oil prices, this would be the right volatility parameter to use when valuing oil-related projects with large option features. For example, Gibson and Schwartz (1991), Schwartz (1998) and Schwartz and Smith (2000) all make the point that even when oil prices are driven by a more complicated process involving both short- and long-term factors, a simpler, one-factor model can be employed to value the options embedded in long-term assets. Schwartz and Smith (2000) illustrate this with an example of the right to develop an oil property where development is completed with a 3 year lag and the decline in production is slow—exponential at 5% a year. The asset value can be approximated using a single-factor model in which the long-term oil price follows a geometric Brownian motion. Since there are many useful and widely known valuation formulas derived for this price process, being able to rely upon it is convenient. The right volatility parameter to use in this single-factor model is not the raw spot price volatility: rather, it is the volatility of the long-run price only, i.e., the volatility cleaned of the effects of the short-run factors.

The large short-term volatility in the oil spot price is at the center of a current dispute over how oil companies should calculate the reserves they report in their financial statements. "Proven reserves" are those that are likely to be recovered under existing economic conditions, including price. What price should be used in making such a calculation? The Financial Accounting Standards Board (FASB) and the SEC recommend that companies use the end-of-year spot price. Many companies, however, would prefer to use a more stable forecasted price which they claim is used when they actually plan their development of reserves.

Until recently, many oil companies resisted the pressure to report reserves based on the end-of-year spot price. The recent scandal at Shell over misreporting reserves put new pressure on all companies to move to the uniform methodology advocated by the FASB and SEC. When in February 2005 Exxon Mobil made the switch and announced its 2004 results using the end-of-year pricing method, it simultaneously issued a press release to advertise the method's main flaw: the arbitrary effect of short-term volatility. Exxon Mobil was forced to remove from its reserves approximately 500 million barrels in its Cold Lake field, a heavy oil-bitumen steam project in Canada. Although bitumen prices "were strong for most of 2004,…on the day of December 31, 2004, prices were unusually low due to seasonally depressed asphalt sales and industry upgrader problems in Western Canada. Prices quickly rebounded from December 31, and through January 2005, returned to levels that have restored the reserves to the proved category."[3] The end-of-year pricing method resulted in a reserve replacement ratio for Exxon of 83%: under Exxon's former methodology it would have been 112%.

Exxon Mobil together with other oil companies, including Anadarko, BP, Chevron, ConocoPhillips, El Paso, Kerr-McGee and Marathon are lobbying for consideration of alternatives to the end-of-year pricing methodology. One alternative that has been proposed is using a futures price for oil instead of the spot price in these

[3] Exxon Mobil press release, February 18, 2005, "Exxon Mobil Corporation Announced 2004 Reserves Replacement".

estimates. Other alternatives being advocated include using modeled prices.[4] Among the many things to be considered in evaluating such proposals, is whether the futures price is free of the short-term volatility that is the alleged culprit of the current system, and whether modeled prices provide sufficient marginal advantage over the raw futures price, including whether the modeled prices have filtered out significantly more of the short-term volatility. This paper addresses these two questions and so provides helpful information on this policy issue.

2. INFORMATION FROM FUTURES PRICES

One tool for filtering out the short-term, transient volatility is data on futures prices. The price of a futures contract of a given maturity reflects expectations about the future spot price at that horizon.[5] So the dynamics of the futures price at any given maturity is unaffected by short-term factors that dissipate within that maturity horizon. Prices on longer maturity futures contracts should reflect less and less of the short-term factors and should give a cleaner picture of the remaining volatility.

Gibson and Schwartz (1990) and Schwartz (1997) develop a two factor dynamic model of the term structure of oil futures prices which exploits this feature of futures prices. Baker, Mayfield and Parsons (1998) and Schwartz and Smith (2000) show that the

[4] These companies sponsored a study of the issue by CERA which was released in February 2005 under the title *In Search of Reasonable Certainty: Oil and Gas Reserves Disclosures*.

[5] There is, of course, a large literature on the information contained in futures prices, on the expectations hypothesis and the forecast accuracy of futures prices. This goes back at least as far as Dow (1941) and Working (1942). A good review of various aspects of the literature is in Williams and Wright (1991). One point worth emphasizing is that the focus in this paper is exclusively on obtaining a good estimate of the long-term volatility of the oil price. We do not address here whether the futures price may be systematically biased.

two-factors can be represented as a short-term, transient component in the spot oil price and a long-term, lasting component. Each component is subject to shocks. Shocks to the short-term component do not have a lasting effect on the future price of oil. They dissipate gradually. In contrast, shocks to the long-term component are lasting and so cumulate. The observed volatility of the spot price is a function of the volatilities of both factors. Estimation of the model allows us to filter out the portion of the spot price volatility due to the short-term transitory factor and determine the volatility due to the long-term factor.

Formally, the model is written as $\ln(P_t) = \xi_t + \chi_t$, where ξ is the log of a latent, long-run component of the oil price and χ is the short-run component which produces a temporary deviation of the current spot price from the long-run price. The latent, long-run price component is modeled as a simple random-walk with drift: $d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi$, where μ_ξ is the instantaneous rate of growth, σ_ξ is the volatility of the long-term component, and dz_ξ is the standard increment to a Wiener process. The short-run component is modeled as a simple mean reverting process: $d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi$, where κ is the rate of mean reversion, σ_χ is the volatility in the short-run component, and dz_χ is the standard increment to a Wiener process. The correlation between the two Wiener processes is written $dz_\xi dz_\chi = \rho_{\xi\chi} dt$. This model is a hybrid of a pure random walk model and a pure mean reverting model. The spot price is mean reverting, but the mean to which it reverts is itself evolving as a random walk.

This model of spot prices combined with a theory for the valuation of futures contracts implies a specific term structure for the set of futures prices at a moment in time. Writing $F_{T,t}$ for the price for a futures contract with time to maturity T quoted at time t, the log of the futures price is given by the equation:

$$\ln(F_{T,t}) = \left(\xi_t + (\mu_\xi + \tfrac{1}{2}\sigma_\xi^2)\,T\right) - \lambda_\xi T$$

$$+ \left(e^{-\kappa t}\chi_t + \tfrac{1}{2}\left[(1-e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} + 2(1-e^{-\kappa t})\frac{\rho_{\xi\chi}\sigma_\xi\sigma_\chi}{\kappa} \right] - (1-e^{-\kappa T})\frac{\lambda_\chi}{\kappa} \right).$$

where λ_ξ and λ_χ are the parameters for the market price of risk associated with the uncertainty in the future value of the long- and short-run factors. This formula can be understood as a combination of (i) the forecasted long-run factor, $\xi_t + (\mu_\xi + \frac{1}{2}\sigma_\xi^2)\,T$, (ii) an adjustment for the market price of risk associated with the long-run factor, $-\lambda_\xi T$, (iii) the expected contribution of the current short-run factor value to the forecasted spot price, a contribution that declines over time, $e^{-\kappa t}\chi_t$, (iv) the expected contribution of short-run volatility to the level of the forecasted spot price, a contribution that asymptotes to a fixed value in time, $\frac{1}{2}\left[(1-e^{-2\kappa t})\frac{\sigma_\chi^2}{2\kappa} + 2(1-e^{-\kappa t})\frac{\rho_{\xi\chi}\sigma_\xi\sigma_\chi}{\kappa} \right]$, and (v) , the adjustment for the market price of risk associated with the short-run factor, an adjustment that asymptotes to a fixed value in time, $-(1-e^{-\kappa T})\frac{\lambda_\chi}{\kappa}$.

This specific structure makes it possible to infer the parameters of the underlying process from observations of the term structure. The front-end of the term structure reveals information about the short-run factor and whether the spot price is above or below the long-run price. Essentially, whenever the short-run factor is very positive and therefore the spot price is significantly above the current long-run price, the term structure will initially slope sharply downward. Whenever the short-run factor is very negative so that the spot price is significantly below the current long-run price, the term structure will initially slope upwards. In both cases, the slope or curvature of the term structure will attenuate as we look at futures contracts with greater maturity. The rate of attenuation in the slope reveals the speed of mean reversion and therefore the rate of dissipation of the short-run factor. At longer maturities, the term structure approximates a flat line, and the slope reveals information about the long-term component—whether the drift of the long-term factor, net of the discount for risk, is positive or negative.

Observing a sequence of term structures allows us to infer what portion of the movement in the oil price is due to movements in the short-run factor and what portion is due to movements in the long-run factor, and also to infer the volatility of the short-run factor and the long-run factor as well as their correlation. We do this estimation using the Kalman filter methodology. Our implementation of the Kalman filter estimation follows Harvey (1989) and Hamilton (1994), generally using the notation of Schwartz and Smith (2000). To implement the Kalman filter we represent the model in a transition equation describing changes in the state variables through time and a measurement equation describing the relation between the state variables and observed futures prices.

We re-write the model in discrete-time to yield a transition equation:

$$\mathbf{x}_t = \mathbf{c} + \mathbf{G}\,\mathbf{x}_{t-1} + \boldsymbol{\omega}_t, \quad t = 1, \ldots n,$$

where $\mathbf{x}_t$ is a 2×1 vector of the two state variables, $\mathbf{x_t} = \begin{bmatrix} \chi_t \\ \xi_t \end{bmatrix}$, $\mathbf{c}$ is the 2×1 vector,

$\mathbf{c} = \begin{bmatrix} 0 \\ \mu_\xi \Delta t \end{bmatrix}$, $\mathbf{G}$ is the 2 × 2 matrix, $\mathbf{G} = \begin{bmatrix} e^{-\kappa\Delta t} & 0 \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{\omega}_t$, is the 2 × 1 vector of serially uncorrelated innovations, Δt is the time interval in years, i.e., for weekly observations Δt=1/52, and n is the number of periods in the data set. The innovations, $\boldsymbol{\omega}_t$, are normally distributed with zero mean and covariance matrix W,

$$\mathrm{Var}(\boldsymbol{\omega}_t) = W = \begin{bmatrix} \sigma_\chi^2 \Delta t & \rho_{\chi\xi}\sigma_\chi\sigma_\xi \Delta t \\ \rho_{\chi\xi}\sigma_\chi\sigma_\xi \Delta t & \sigma_\xi^2 \Delta t \end{bmatrix}.$$ [6]

We then write the measurement equation describing the relationship between the underlying state variables and observed futures prices:

$$\mathbf{y}_t = \mathbf{d} + \mathbf{F}_t'\mathbf{x}_t + \mathbf{v}_t, \quad t = 1, \dots n,$$

where $\mathbf{y}_t$ is a m×1 vector of log futures prices for m different maturities, $\mathbf{d}$ is a m×1 vector of functions of the model parameters, with $\mathbf{d} = \begin{bmatrix} A(T_1) \\ \vdots \\ A(T_m) \end{bmatrix}$,

$$A(T_i) = (\mu_\xi - \lambda_\xi)T_i - (1 - e^{-\kappa T_i})\frac{\lambda_\chi}{\kappa} + \tfrac{1}{2}\left[(1 - e^{-2\kappa T_i})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T_i + 2(1 - e^{-\kappa T_i})\frac{\rho_{\xi\chi}\sigma_\xi\sigma_\chi}{\kappa}\right],$$

T_i is the time to maturity of the i^{th} futures contract, $\mathbf{F}'$ is a m×2 matrix of functions of the model parameters, $\mathbf{F} = \begin{bmatrix} e^{-\kappa T_1} & 1 \\ \vdots & \vdots \\ e^{-\kappa T_m} & 1 \end{bmatrix}$, and $\mathbf{v}_t$ is a m×1 vector of uncorrelated, normally distributed residuals with mean zero and covariance matrix $\mathbf{V}$. We assume this matrix to

[6] Schwartz and Smith (2000 p. 901) write that W is given by their equation (3b) which is different from the matrix we show here. However, in the Appendix they construct the discrete time process which in the limit gives the continuous process. At the top of p. 911 they produce the matrix W which describes the variance of the serially uncorrelated innovations in the discrete time process, and this W matches the one we use here.

As their appendix shows, a sequence of innovations generated with this variance matrix W, translates into a future pair of state variables with the covariance matrix V_n also shown on p. 911. In continuous time the covariance matrix V_n becomes the covariance matrix for the state variables shown in equation (3b) on p. 901.

be diagonal. The coefficient vectors c and d_t, and matrices, G, W and F, are functions of the parameters of the continuous-time model. The matrix V does not involve the model parameters, only the variance of the noise in the measurement equation.

We follow the approach of Harvey (1989) to update the optimal estimator of the state variables at time t given information available at time t, $\mathbf{a}_{t|t}$, and the covariance matrix of the estimation error, $\mathbf{P}_{t|t}$. In the process, we derive the likelihood function to be maximized for the parameters of interest.[7] Given the optimal estimator of the state variables at time t-1, $\mathbf{a}_{t-1|t-1}$, and the covariance matrix of the estimation error, $\mathbf{P}_{t-1|t-1}$, the optimal forecasts $\mathbf{a}_{t|t}$ and $\mathbf{P}_{t|t}$ can be obtained using the prediction equations

$$\mathbf{a}_{t|t-1} = \mathbf{c} + \mathbf{G}\mathbf{a}_{t-1|t-1},$$

$$\mathbf{P}_{t|t-1} = \mathbf{G}\mathbf{P}_{t-1|t-1}\mathbf{G}' + \mathbf{W}$$

and the updating equations

$$\mathbf{a}_{t|t} = \mathbf{a}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{F}\mathbf{Q}_t^{-1}(\mathbf{y}_t - \mathbf{d}_t - \mathbf{F}'\mathbf{a}_{t|t-1}),$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{F}\mathbf{Q}_t^{-1}\mathbf{F}'\mathbf{P}_{t|t-1}$$

where

$$\mathbf{Q}_t = \mathbf{F}'\mathbf{P}_{t|t-1}\mathbf{F} + \mathbf{V}.$$

Note that the updating equations incorporate the information contained in $\mathbf{y}_t$, the variables observed at time t.

Concerning the initial forecast, $\mathbf{a}_{1|0}$, required to start the Kalman recursion, we set the forecast of the initial value of the short-term component χ_0 equal to its unconditional mean of zero, and let the model estimate the mean of the initial value of the long-term

---

[7] See Harvey (1989) Section 3.2.

log-price, ξ_0. Since the long-term component has a unit root, $\mathbf{x}_0$ does not have a finite variance, and we use $\mathbf{W}/(1-\mathbf{G}(1,1)^2)$, where $\mathbf{G}(1,1) = \exp(-\kappa\Delta t)$ is the autoregressive coefficent of the stationary short-term component.[8]

This setup enables us to calculate a likelihood of observing the set of data examined given a particular set of model parameters. We vary the parameters to determine which set maximizes the likelihood. Estimation was done using the OPTIMUM module in GAUSS, and trying various combinations of initial values to check the robustness of the estimates. In all cases, essentially the same estimates were obtained, suggesting that we obtain a global maximum.

We estimated this short-term, long-term model using weekly oil futures price data from September 1989 to May 2006. We used prices on futures contracts with maturity out to 17-months. A number of considerations entered into this choice. First, as indicated in the introduction, most of the apparent short-run swings in price dissipate within this horizon, so 17-months may be far enough out to filter the short-run volatility. Second, data limitations during the window of estimation push in favor of this horizon. The NYMEX oil futures exchange has only gradually begun to offer a dense set of contracts for maturities of every month out longer than the 17-month horizon. The maturity of a contract shortens through time, so what is today the 5-month contract becomes the 4-month contract. And what becomes the 5-month contract was once the 6-month contract. A continuous series of prices for the constant maturity 5-month contract requires a regular set of contracts for each neighboring month's maturity so that as the maturity of one contract shortens to less than 5-months, it is replaced by a new contract that has

---

[8] Here we are following Hamilton's suggestion—Hamilton (1994, pp. 378-379).

shortened to 5-months. While there exist oil futures contracts out several years in maturity, there is not a series of contracts with a maturity for each month at that horizon. When the 2-year contract becomes a 23-month contract, there is no formerly 25-month contract becoming a 2-year contract. These interruptions make it difficult to construct a continuous constant maturity contract dataset at longer maturities. Through time as the oil futures market has evolved, the horizon where this is possible has gotten longer. But then the tradeoff is whether one can go back far enough to construct an adequately long time series. We chose the 17-month contract as a compromise that gives us a meaningfully long time series and an adequately long maturity.

Table 1 reports the model parameter estimates as well as the raw volatility on the spot price. Focusing on the estimation made using the 1, 5, 9, 13 and 17 month contracts, the estimated volatility for the long-term component, σ_ξ, is only 16.3%, or less than half the raw spot price volatility of 36.2%. The mean reversion parameter, κ, is estimated at 0.863, which translates to an estimated half-life for short-term swings in price of a little more than 9 months. This estimate for the half-life vitiates our earlier, casual observation on the life of short-term price swings and our choice of the 17 month contract horizon in the estimation.[9]

Implicit in the estimation of the model is the construction of a time series for the latent, long-run price of oil—i.e., the price that would obtain if the effects of the short-run component were removed. Figure 1 shows this series overlayed on the graph of the actual spot oil price. The effect of filtering out the short-term, transitory factor is evident. Many

---

[9] Although the model also produces estimates of the long-term drift parameter, μ_ξ, and of the price of risk for each factor, λ_ξ and λ_χ, as with forecasts for expected returns on other assets, these estimates are not very precise. We report them for completeness.

of the largest swings and reversals present in the spot oil price series disappear in this series. Of course the recent rise from around $25/bbl in 2003 to as high as $70/bbl is not represented as a short-run swing: the estimated long-run price rises as well. Indeed, at the end of this time in May 2006, the estimated long-run price is above the spot price. The volatility of this estimated long-run price series is the 16.3% estimated volatility of the long-term component.[10]

3. HAS SHORT-TERM VOLATILITY BEEN SUCCESSFULLY FILTERED?

How successful is the two-factor model in filtering out all of the short-run, transitory volatility? If it is successful, then the estimated latent, long-term price series should be a random walk, free of any swings and reversals. Is it?

Visual inspection of Figure 1 suggests that the estimated series may fail this test. Although the long-term price series doesn't exhibit price swings and reversals of the same magnitude as the spot price, tempered versions of many of these swings and reversals still seem evident. For example, the long-term series spikes during the Gulf War when the spot price spiked so sharply. The long-term spot price series shows a sharp spike and reversal in October 2000 when the spot price shows a similar movement. From September 2004 through the end of 2005 there are several short-run swings in the spot price of $10 barrel around the general upward trend. These apparent swings are mirrored in the estimated long-term price series. Are these casual observations truly signs of

[10] Volatility reported is the annualized weekly standard deviation of returns, where $R_t = \log(P_t/P_{t-1})$ and weekly volatilities are annualized as $\sigma_a=\sigma_w\sqrt{52}$.

remaining short-term volatility that was not successfully filtered out, or simply peculiar realizations in an essentially random walk pattern?

To address this question more rigorously, we applied a formal statistical test to the series. A random walk is a specific case of a unit process – one for which there is no serial correlation in the first differences. Accordingly, we applied a unit root test and then tested for serial correlation in the first differences of the process. The results are shown in Table 2. Consistent with the random walk specification, the unit root test fails to reject the null hypothesis of a unit root. However, the serial correlation test finds strong evidence of negative serial correlation in the first differences: the coefficients are negative and the p-values establish that these are significantly different from zero. This is inconsistent with the estimated long-term price series being a random walk, and seems to confirm that the two-factor model has not been completely successful in filtering out all of the short-term, transitory contributions to volatility.

Why might the estimated long-term price series still show signs of the short-term volatility that ought to have been filtered out? One obvious problem will arise if there is a specific form of measurement error in the observed prices on long maturity futures contracts that we have not been able to identify and control for properly. This might arise, for example, due to illiquidity in longer maturity contracts. Another possibility is that the short-term/long-term model does not describe the true dynamics and therefore does not predict the correct term structure, and this misspecification error is translated into a misestimated long-term price series. Alternative models for the term structure have been proposed by Routledge, Seppi, and Spatt (2000), by Casassus, Collin-Dufresne and

Routledge (2004) by Kogan, Livdan, and Yaron (2005) and by Tze, Foster, Ramaswamy and Stine (2005).

A particular case of the model misspecification arises if the curvature of the term structure changes with the maturity. A key assumption of the model is that all of the short-term, transient shocks dissipate at a constant rate, so that the curvature of the term structure is fixed throughout the term structure. If the curvature of the term structure changes with the maturity, then the model is mis-specified and the parameter estimates will be affected. This is especially relevant for the estimate of the long-term volatility parameter, since the model is essentially projecting the curvature of the term structure within the range of the maturities used for estimation, i.e., between 1- and 17-months, out to the infinite horizon. This projection of the curvature applies equally to the term structure of volatilities.

We re-estimated the model on two different subsets of contracts within the 17 month horizon: the shorter maturity contracts of 1, 3, 5, 7 and 9 months, and the longer maturity contracts of 9, 11, 13, 15 and 17 months. The parameter estimates are shown in Table 1. The rate of mean reversion is highly sensitive to the choice of contracts used to estimate the model: at the short end the estimated rate of mean reversion is 1.329 or a half-life of approximately 6 months, while at the long end the estimated rate of mean reversion is 0.757 or a half-life of 11 months. The estimated long-term volatility using the shorter maturity contracts is 17.5% and using the longer maturity contracts is 16.6%. This is not a large difference, although in both cases it is higher than the 16.3% arrived at using the full range of contracts.

We also ran the test for serial correlation on the long-term price series generated by each of the alternative estimations. The results are shown in Table 2. In all cases the long-term price series shows evidence of serial correlation.

4. A NAÏVE ESTIMATOR

By reducing to only two factors all of the dynamics in the term structure of futures prices, the short-run, long-run model is a significant simplification of the possible dynamics driving the oil price. Nevertheless, it is a mathematically demanding model for industry analysts to employ, and estimation of the parameters is correspondingly difficult. Few people understand the mechanics of the Kalman filter technique, and even fewer of them are capable of sharing that knowledge with less statistically inclined members of a corporate team involved in decision making. Moreover, although we have not touched on the complications here, there are certain subtle choices to be made and a number of issues and caveats that could be raised in favor of alternative approaches to the estimation. Quite often those with the mechanical knowledge of the Kalman filter technique and those who have the familiarity with key market issues relevant to choosing among these approaches are different groups of people and it may not be practical to assume they can be brought together.

So the question arises, how much extra information does one get as a result of deploying this complicated machine? Or put another way, is there a simpler foundation for deriving the same results that practicing analysts would find more appealing? How large of a compromise is made if a simpler estimator is used?

The premise of the short-run, long-run model is that the effect of the variations in the short-run factor is largely felt in the price of short maturity futures contracts. The effect of these variations in the short-run factor die out gradually as one looks to contracts with greater and greater maturity. Therefore, variation in the price of contracts with a relatively long maturity reflects primarily the effect of volatility in the latent, long-term factor. The solid curved line in Figure 2, Panel A shows the estimated model volatilities for futures prices as a function of the maturity of the contract using the formula $\sigma_T^2 = e^{-2\kappa T}\sigma_\chi^2 + \sigma_\xi^2 + 2e^{-\kappa T}\rho_{\chi\xi}\sigma_\chi\sigma_\xi$. The solid horizontal line is the long-term volatility to which the solid curved line asymptotes. Also shown in the figure are the actual volatilities for futures contracts of various maturities. The solid diamonds show the maturities used in the estimation. The '×' marks show other maturities.

A naïve estimator, then, could be to simply view the volatility in the futures price on a long maturity contract as a direct observation of the volatility of the latent, long-run price. This naïve inference would contain some bias, because even the volatility of a long-maturity futures price will reflect some residual amount of the short-run volatility that has not yet dissipated. But so long as this residual amount is small, the bias is small. Theoretically, the longer the maturity of the contract used, the less the bias. In actual practice there may be measured sources of volatility due to institutional issues in the futures price that may not dissipate with maturity, or even that may increase if one gets to long maturities with insufficient liquidity.

Since the longest maturity contract used in our estimation is the 17 month contract, it seems appropriate to use the volatility on the raw 17-month futures price as the naïve estimator for the long-term volatility. How much bias is left in this naïve model? How does it compare against the estimated latent, long-run price from the full blown 2-factor model? How much extra filtration of short-run volatility is provided by supplementing this raw 17-month volatility with a model of the term structure of volatilities and data on shorter maturity contracts?

Assuming that the model estimate of 16.3% for the long-term volatility is correct, short-term factors contribute approximately 20 percentage points to bring the observed spot price volatility up to 36.2%. The naïve estimator of the long-term volatility is the 17.4% volatility on the 17-month contract. This is only 1.1 percentage points greater than the 16.3% model estimate for the long-run volatility. In this case the naïve estimator would have identified 94% of the short-term contribution to the spot volatility identified by the more complicated model.[11]

Panels B and C of Figure 2 show the same comparison of model and observed volatilities when the model is estimated either using the short maturities of 1, 3, 5, 7 and 9 months or using the long maturities of 9, 11, 13, 15 and 17 months. Both of these estimations produce a higher long-term volatility—17.5% and 16.6%, respectively—and therefore attribute less of the raw spot volatility to the short-term factor. If we use either of these as a benchmark, then the naïve estimator of the raw 17-month contract volatility—17.4%--appears to filter more of the short-term, transient volatility. Indeed, in the case of the short maturities, the naïve estimator is one-tenth of a percentage point lower than the estimate from the full short-term, long-term model.

---

[11] We chose the 17-month contract price as the naïve estimator because it was the longest maturity used in our estimation. Others may prefer a contract maturity that is more of a focal point such as the 1-year futures price. This, too, does reasonably well. The 1-year contract has a volatility of 19.3%. This is only 3 percentage points greater than the 16.3% model estimate for the long-term volatility. In this case the naïve estimator would have identified 85% of the short-term contribution to the spot volatility identified by the more complicated model.

Another way to evaluate the naïve estimator is to visually compare the 17-month future price series against the estimated long-term price series as is done in Figure 3. The dynamics of the two series are remarkably similar. Many of the transitory price spikes that seem apparent in the spot price series, and that appear to have some residual influence in the 17-month futures price series, show themselves also in the long-term price series. In a simple visual inspection it is difficult to discern the superiority of the estimated long-term price series over the 17-month futures price series with respect to having filtered out short-run, transient factors.

As we noted earlier, valuation equations or models based on the more widely understood single-factor, geometric Brownian motion model can be reliably used to value options in long-term oil related investments, so long as the right inputs are used. The right inputs are not those taken from the simple dynamics of the spot price. The effects of short-term, transitory fluctuations in the spot price must be first filtered out. The right volatility estimate to use in such a model would be the estimate of the long-run volatility like that obtained from the short-run, long-run model. The results of this paper suggest that a volatility estimate based on the raw volatility from a long-maturity futures contract would be a reasonable substitute.[12]

[12] The valuation of the long-term assets using a single factor model also requires (i) an estimate of the current level of the long-term price, and (ii) an estimate of the risk-neutral drift. Both of these can be derived from the two-factor model, although there may be reasons for looking elsewhere for estimates of these two parameters. We don't address alternative estimators for these two elements of the valuation. We focus exclusively on the volatility parameter.

5. CONCLUSIONS

The short-term, long-term model of oil prices clearly helps us understand the distinction between the daily volatility observed in the spot price and the long-run volatility in forecasted prices. Estimation of the model provides a markedly improved measure of the correct long-term volatility that is useful for most major investment decisions linked to the oil price. We have shown that although this improvement is significant, there remains some amount of the transient volatility still contained in the model's estimate of the long-term volatility. We have also shown that a naïve estimator using the raw volatility on the longest maturity futures contract succeeds in filtering out most of the transient volatility caught by the full blown model. This naïve estimator is significantly easier to work with. It requires no complicated statistical knowledge and no implementation of difficult estimation procedures. The formula for estimation can be simply written in a single cell of an Excel spreadsheet.

The success of the naïve estimator also provides an opening for the resolution of the conflict over the right price to use in calculating proven reserves for financial reporting purposes. While many of the oil companies have properly complained about the volatility in the spot price and the curious results that follow from that, an alternative that is sometimes proposed is to leave them discretion to utilize whatever price forecast they deem best. Regulators are obviously cautious about granting that kind of discretion. This paper shows, however, that longer maturity futures contracts contain very little of the short-term transitory volatility that is at the heart of the objection to end-of-year spot pricing. They provide a viable alternative that is both simple and effective.

References

Baker, Malcolm P., E. Scott Mayfield and John E. Parsons, 1998, Alternative models of uncertain commodity prices for use with modern asset pricing methods, *Energy Journal* 19, 115-148.

Casassus, J., P. Collin-Dufresne and B. Routledge, 2004, Equilibrium commodity prices with irreversible investment and non-linear technologies, CMU working paper.

Dow, J. C. R., 1941, The inaccuracy of expectations: a statistical study of the Liverpool cotton futures market 1921/2-1937/8, *Economica* 8, 162-75.

Gibson, Rajna and Eduardo S. Schwartz, 1990, Stochastic convenience yield and the pricing of oil contingent claims, *Journal of Finance* 45, 959–976.

Gibson, Rajna and Eduardo S. Schwartz, 1991, Valuation of long term oil-linked assets, *Stochastic Models and Option Values*, D. Lund and B. Øksendal, eds., Elsevier Science (North Holland), 73-101.

Hamilton, James D., 1994, *Time Series Analysis*, Princeton: Princeton University Press.

Harvey, Andrew C., 1989, *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge, UK: Cambridge University Press.

Kogan, Leonid, Dmitry Livdan, and Amir Yaron, 2005, Futures prices in a production economy with investment constraints, MIT working paper.

McDonald, Robert L., 2003, *Derivatives Markets*, Boston: Addison-Wesley.

Routledge, B., D. Seppi and C. Spatt, 2000, Equilibrium forward curves for commodities, *Journal of Finance* 55, 1297-1338.

Schwartz, Eduardo S., 1997, The stochastic behavior of commodity prices: implications for valuation and hedging, *Journal of Finance* 52, 923-973.

Schwartz, Eduardo S., 1998, Valuing long-term commodity assets, *Journal of Energy Finance and Development* 3(2), 85-99.

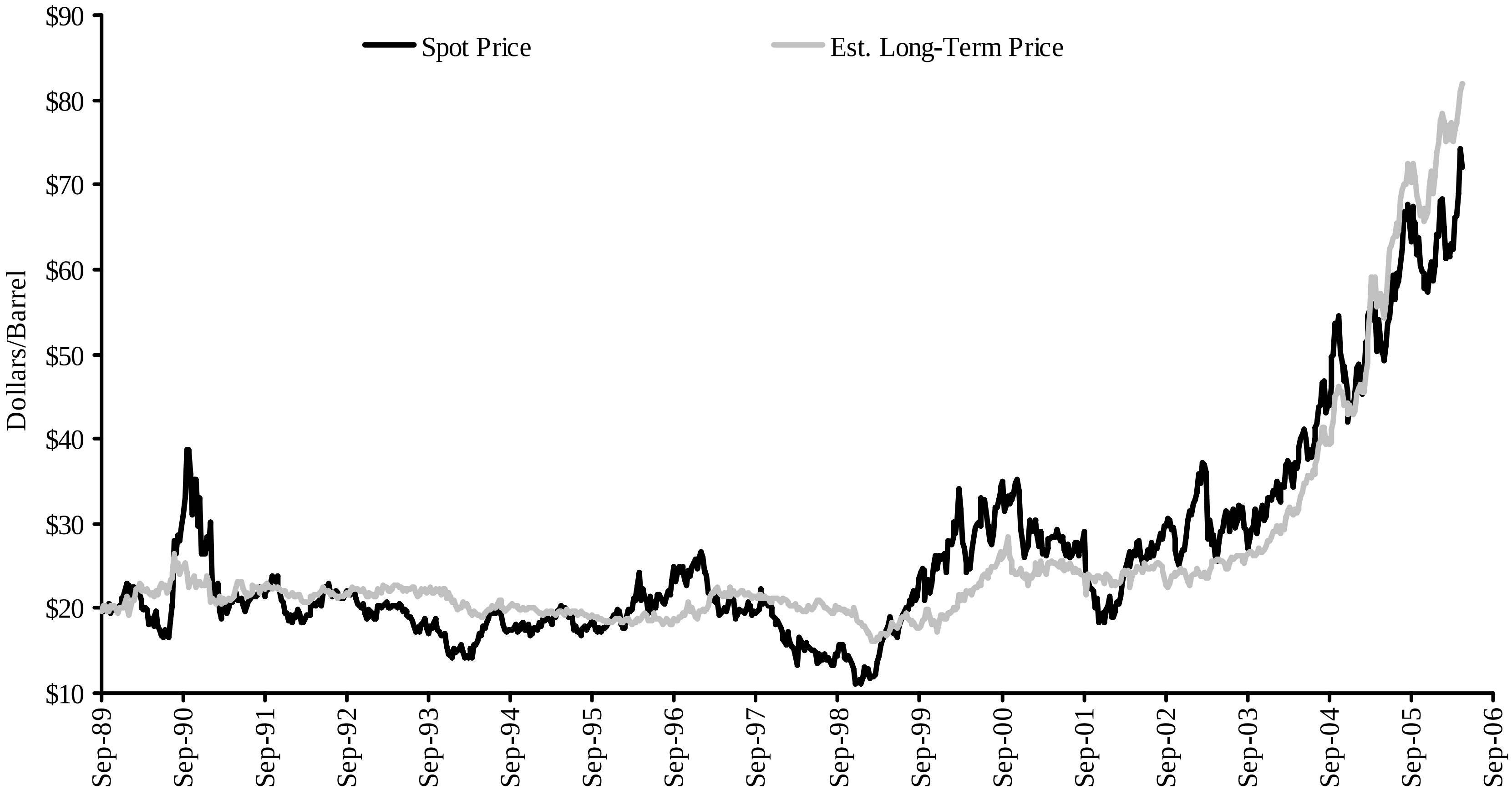
Schwartz, Eduardo S. and James E. Smith, 2000, Short-term variations and long-term dynamics in commodity prices, *Management Science* 46, 893-911.

Tze, Choong, Dean Foster, Krishna Ramaswamy and Robert Stine, 2005, A dynamic model for the forward curve, Wharton School, Rodney White Center for Financial Research Working Paper 34-05.

Williams, Jeffrey C. and Brian D. Wright, 1991, *Storage and Commodity Markets*, Cambridge UK: Cambridge University Press.

Working, Holbrook, 1942, Quotations on commodity futures as price forecasts, *Econometrica* 10, 39-52.

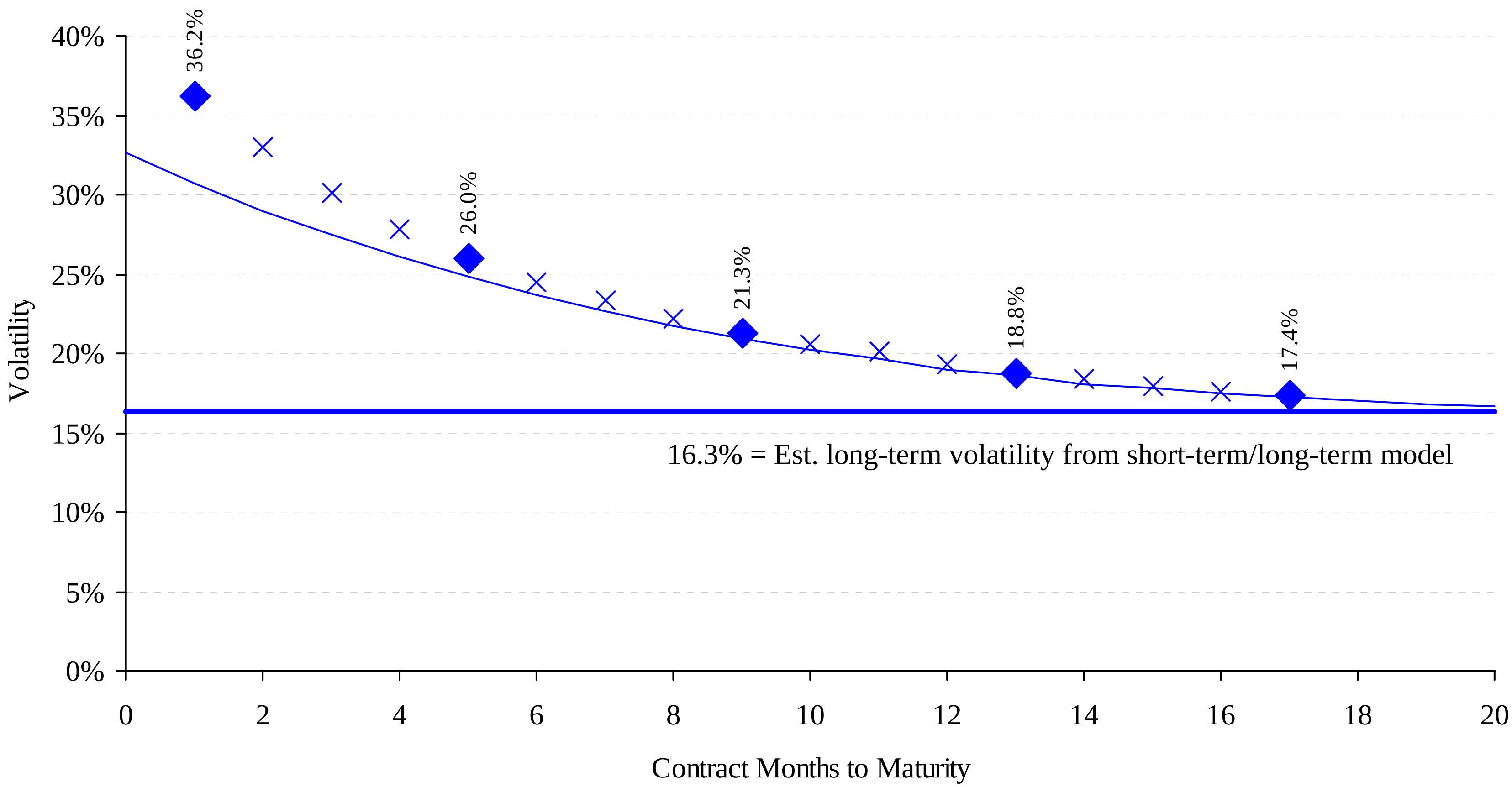
Figure 1. Spot Oil Price Overlayed with an Estimated Long-Term Price Series



The spot oil price weekly from September 1989 to May 2006 where the 1-month NYMES futures price is used as the spot price. The estimated long-term price series is calculated using the short-term,.long-term model in Schwartz and Smith (2000). Five futures contracts are used—the 1-month, 5-month, 9-month, 13-month and 17-mpnth contracts. Model parameter estimates are shown in Table 1.

Figure 2. Volatility of Oil Futures Prices by Contract Maturity

Panel A: Estimation Using the 1, 5, 9, 13, and 17-Month Contracts



The horizontal line is the estimated long-term volatility as reported in Table 1. The curved line shows model volatilities for futures by maturity per the formula in Schwartz and Smith (2000): $\sigma_T = e^{-2\kappa T}\sigma_\chi^2 + \sigma_\xi^2 + 2e^{-\kappa T}\rho_{\chi\xi}\sigma_\chi\sigma_\xi$ Points marked individually are the observed volatilities for futures prices of the corresponding maturities; diamonds mark the maturities used in the estimation, and crosses mark other maturities. Reported values are annualized weekly standard deviation of returns, where $R_t = \log(P_t/P_{t-1})$ and weekly volatilities are annualized as $\sigma_a = \sigma_w\sqrt{52}$.

Figure 2. Observed and Modeled Volatility of Oil Futures Prices by Contract Maturity

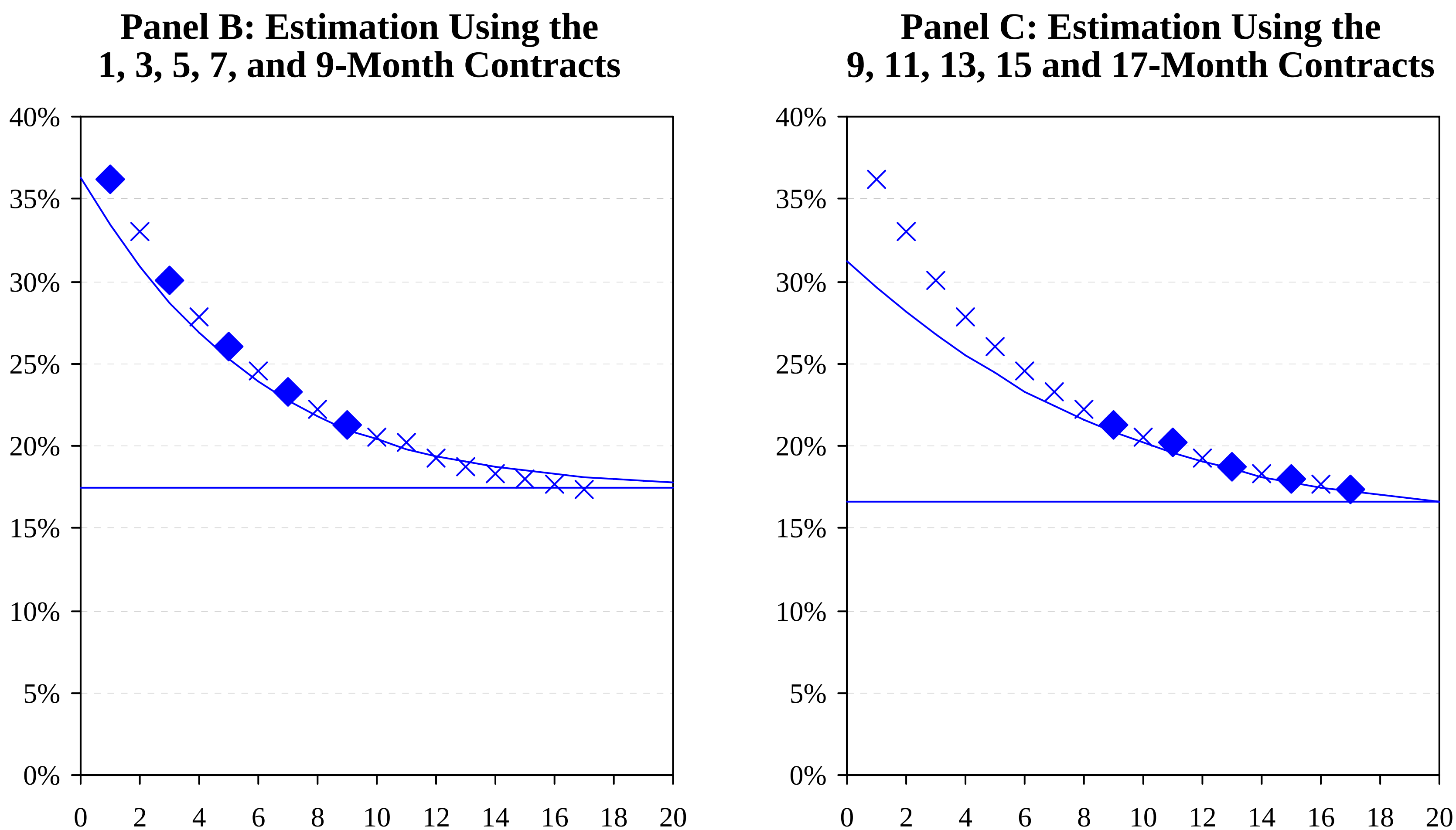
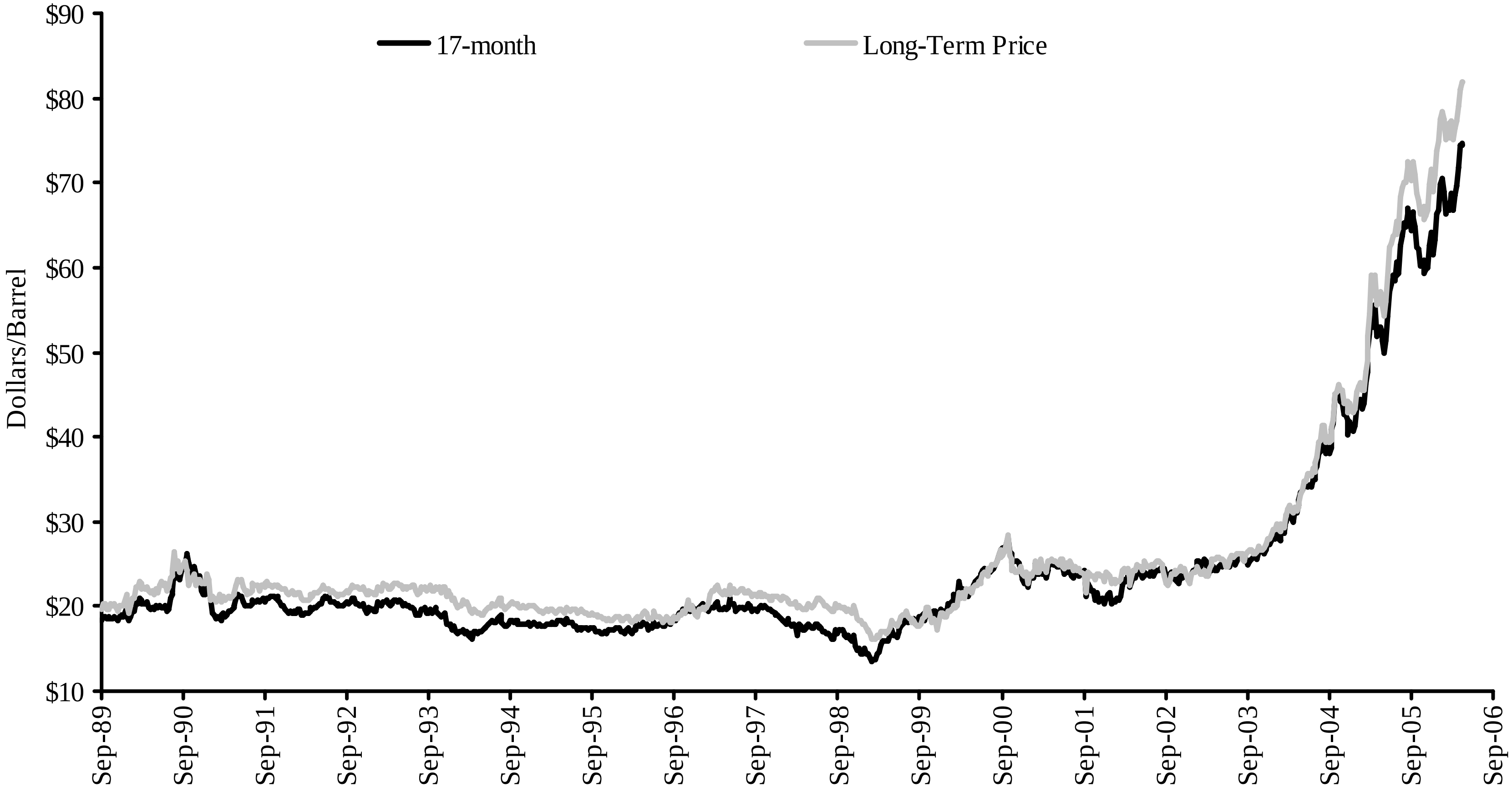


Figure 3. The 17-Month Futures Price Versus the Estimated Long-Term Price



The 17-month oil futures price weekly from September 1989 to May 2006. The estimated long-term price series is calculated using the short-term, long-term model in Schwartz and Smith (2000). Five futures contracts are used—the 1-month, 5-month, 9-month, 13-month and 17-mpnth contracts. Model parameter estimates are shown in Table 1.

Table 1. Parameter Estimates for the Short-Term, Long-Term Model

Parameter		Contract Months Used in Estimation: 1, 5, 9, 13, 17	1, 3, 5, 7, 9	9, 11, 13, 15, 17
Volatility of Long-Term Price (estimated)	σ_ξ	16.3%	17.5%	16.6%
		(38.567)	(40.640)	(40.076)
Volatility of Short-Term Factor (estimated)	σ_χ	31.4%	31.9%	31.1%
		(37.612)	(38.530)	(40.396)
Correlation of Volatilities (estimated)	$\rho_{\xi\chi}$	-18.6%	-0.3%	-26.1%
		(-2.038)	(-0.143)	(-7.212)
Volatility of Spot Price, Model (calculated)		34.1%	36.3%	33.3%
Volatility of Spot Price (raw data)		36.2%	36.2%	36.2%
Speed of Mean Reversion (estimated)	κ	0.863	1.329	0.757
		(35.158)	(49.243)	(42.854)
Half-Life of Short-Term Factor (calculated)		0.80	0.52	0.92
Risk-Neutral Instantaneous Drift of the Long-Term Price (estimated)	μ_ξ-λ_ξ	0.6%	-3.7%	1.4%
		(2.333)	(-11.772)	(6.175)
Instantaneous Drift of the Long-Term Price (estimated)	μ_ξ	9.0%	9.2%	8.9%
		(2.462)	(1.971)	(2.179)
Growth Rate of the Long-Term Price (calculated)		10.3%	10.7%	10.2%
Long-Term Risk Premium (calculated)		8.4%	12.8%	7.5%
Short-Term Risk Premium (estimated)	λ_χ	14.1%	9.9%	14.3%
		(1.829)	(1.713)	(1.536)

Parameter estimates are for the short-term, long-term model in Schwartz and Smith (2000) using weekly futures price data from September 1989 to May 2006. In each estimation five futures contracts are used with the months of the contracts shown at the top of the column. T-statistics are shown in parentheses. The model has seven parameters to be estimated. Other parameters shown in the table are functions of these seven, and so are simply calculated and displayed for convenience and with no t-stat shown.The model spot price volatility is a function of the volatility of the long- and short-term factors and the correlation coefficient. The half-life of short-term price movements is a restatement of the speed of mean reversion. The growth rate of the long-term price is a function of the estimated instantaneous drift and the estimated long-term volatility. The long-term risk premium is a function of the risk-neutral instantaneous drift and the instantaneous drift parameters. Not shown are our estimates for the residuals in the measurement equations, i.e., for each of the 5 futures contracts, and our estimate for the initial long-run component.

Table 2. Test Results for Unit Root and Serial Correlation in First Differences

	Model Estimated Long-Term Price Series			Raw Futures Price Series			
	Contract Months Used in Estimation						
	1, 5, 9, 13, 17	1, 3, 5, 7, 9	9, 11, 13, 15, 17	Spot, 1-Mos	9-Mos	1-Year	17-Mos
Unit Root Test							
Test Statistic (6 lags)	1.38	0.73	1.32	-1.91	-0.30	0.13	0.64
Test Statistic (20 lags)	1.57	0.73	1.59	-1.83	-0.25	0.15	0.65
Serial Correlation Test (1st Difference, 1 lag)							
Coefficient	-0.118	-0.085	-0.130	-0.153	-0.113	-0.123	-0.142
Q-Stat	11.540	6.043	14.152	19.581	10.610	12.698	16.900
P-Value	0.0007	0.0140	0.0002	0.0000	0.0011	0.0004	0.0000

The critical values for the unit root test are -3.4 at 5% significance and -3.1 at 10% significance. Although none of the time series can reject the unit root, whether for the estimated long-term price or for the actual futures contracts. However, the test statistic is closest to rejection at the shortest maturity contract. As the maturity of the contract increases, the test statistic moves further from rejection, with the estimated long-term price series being furthest from rejection. If a time series is a random walk, then in addition to having a unit root, there should be no serial correlation in the first differences. All of the time series fail this test: in all cases there is strong evidence of serial correlation.