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Abstract

I study the advantages of pollution permit markets over traditional standard regulations when the regulator has incomplete information on firms' emissions and costs of production and abatement (e.g., air pollution in large cities). Because the regulator only observes each firm's abatement technology but neither its emissions nor its output, there are cases in which standards can lead to lower emissions and, hence, welfare dominate permits. If permits are optimally combined with standards, in many cases this hybrid policy converges to the permits-alone policy but (almost) never to the standards-alone policy.

Keywords: asymmetric information, imperfect monitoring, pollution markets, permits

JEL classification: L51, Q28

1 Introduction

In recent years, environmental policy makers are paying more attention to pollution markets (i.e., tradable emission permits) as an alternative to the traditional command-and-control (CAC) approach of setting emission and technology standards. A notable example is the 1990 U.S. acid rain program that implemented a nationwide market for electric utilities' sulfur dioxide (SO_2) emissions (Schmalensee et al., 1998; Ellerman et al., 2000). In order to have a precise estimate of the SO_2 emissions that are coming out of the stack, the acid rain program requires each affected electric utility unit to install costly equipment that can continuously monitor emissions.[1]

This and other market experiences show that conventional permits markets are being implemented only in those cases where emissions can be closely monitored, which almost exclusively occurs in large stationary sources like electric power plants and refineries. It is not surprising then, that environmental authorities continue relying on CAC instruments to regulate emissions from smaller sources because compliance with such instruments only requires the authority to ensure that the regulated source has installed the required abatement technology or that its emissions per unit of output are equal or lower than a certain emissions rate standard. These observations raise a natural question: could the compliance flexibility of permit trading be extended to the regulation of sources whose emissions can only be imperfectly measured through the observation of their abatement technologies or emission rates (where the firm's output or utilization is not observable to infer emissions)?

This would be a second-best permits scheme in that sources are not trading emissions but

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[1]Another example with similar monitoring requirements is the Southern California RECLAIM program that implemented separated markets for nitrogen oxide (NO_x) and SO_2 emissions from power plants, refineries and other large stationary sources. This program did not include a market for volatile organic compounds (VOC) in large part because of the difficulties with monitoring actual emissions from smaller and heterogeneous sources (Harrison, 1999).

some proxy for emissions (e.g., maximum possible emissions), which raises the possibility that actual emissions under permits could eventually be higher (or lower) than under an alternative CAC regulation. In fact, emissions are likely to be higher if the trade pattern is such that lower-output firms sell permits to higher-output firms. The latter may explain, at least in part, why pollution markets are virtually absent in the control of air pollution in large cities where emissions come from many small (stationary and mobile) sources rather than a few large stationary sources.[2] Rather than disregard pollution markets as a policy tool, however, the challenge faced by policy makers is when and how to implement these markets using monitoring procedures that are similar to those under CAC regulation.

Most of the literature on environmental regulation under asymmetric information deals with the case in which firms' costs are privately known but emissions are publicly observed (see Lewis (1996) for a survey). While there is some literature considering imperfect observation of emissions (e.g., Segerson, 1988; Fullerton and West, 2002; Cremer and Gahvari, 2002),[3] there seems to be no work specifically studying the effect of imperfect information about emissions and costs on the design and performance of a permits market. Yet, it is interesting to observe that despite its limited information on each source's actual emissions (and costs), Santiago-Chile's environmental agency has already implemented a (small) market to control total suspended particulate (TSP) emissions from a group of about 600 stationary sources (Montero et al., 2002).[4] Based on estimates from annual inspection for technology parameters such as each

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[2] Note that there are some credit-based trading programs aimed at curbing air pollution in urban areas working in the US (Tietenberg, 1985). New sources (or expansion of existing ones) must acquire emission credits to cover their emissions through, for example, shutting down existing plants or scrapping old vehicles. Although these programs have the merit of involving small sources, they are very limited in scope in the sense that they are embedded within an existing CAC regulation and are particularly designed to prevent further deterioration of air quality from the entry of new sources.

[3] Segerson (1988) studies the control of emissions from (few) non-point sources using a "moral hazard in teams" approach. Fullerton and West (2002) consider the control of vehicle emissions using a combination of taxes on cars and on gasoline as an alternative to an (unavailable) tax on emissions. Cremer and Gahvari (2002) look at output and emission taxation under costly (rather than imperfect) monitoring.

[4] These 600 sources affected by the TSP program are responsible for only 5% of 2000 TSP emissions in Santiago. Remaining TSP sources are controlled through CAC regulation.

source's size and fuel type, Santiago's environmental regulator approximates each source's actual emissions by the maximum amount of emissions that the source could potentially emit in a given year. In particular, the observable firm's emission rate (i.e., emissions per unit of output) is multiplied by its maximum possible output to infer its maximum emissions for which the firm must buy permits.[5]

Motivated by Santiago's TSP permits program,[6] in this paper I study the instrument design and choice problem associated with the implementation of a pollution market in which the regulator faces several information constraints. In doing so, in the next section (Section 2) I develop a theoretical model for an industry of heterogenous firms that produce output and pollution as a by-product. Each firm has private information about its emissions, abatement costs, and production costs. I first show that the regulator cannot implement the first-best when emissions are imperfectly measured (even in the absence of budget constraints and perfect cost information). In Section 3, I derive the optimal design of the two policy instruments discussed above: standards and permits. I focus on these two (second-best) policies rather than on more optimal ones not only because the latter include the use of nonlinear instruments and transfers to firms that have not been used in practice (Stavins, 2003; Hahn et al., 2003), but more importantly, because I want specifically to explore whether permits can still provide an important welfare advantage over traditional CAC regulation when emissions are imperfectly monitored.

In Section 4, I discuss the conditions under which the permits policy is welfare superior to the standards policy. On the one hand, the permits policy retains the well known cost-effectiveness

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[5]As we shall see later, using the source's maximum emissions as a proxy does not prevent any adverse effects that the use of permits (instead of CAC regulation) could eventually have on aggregate emissions. The choice of proxy is an arbitrary matter because the number of permits being allocated can always be adjusted accordingly with no efficiency effects.

[6]Variations of the permit trading programs documented in USEPA (2001) face similar issues because these are programs that are not based on actual emissions (e.g., the averaging programs for mobile sources or the fireplace permit trading in Colorado).

property of conventional permits programs, namely that permit trading allows heterogenous firms to reduce their abatement and production costs. On the other hand, the permits policy can sometimes provide firms with incentives to choose combinations of output and abatement technology that may lead to higher aggregate emissions than under standards. Thus, when (abatement and production) cost heterogeneity across firms is large, the permits policy is likely to work better. In contrast, as heterogeneity disappears, the advantage of permits reduces, and standards might work better provided that they lead to lower emissions.[7]

Permits can result in higher emissions in two situations, and hence, can be potentially welfare dominated by standards. The first case is when firms with relatively large output ex-ante (i.e., before the regulation) are choosing low abatement (i.e., the case with a negative correlation between production and abatement costs). When this is so, permits induce primarily low-output firms to install abatement technologies, while standards force all firms to invest in abatement. The second case is when firms doing more abatement find it optimal to reduce output ex-post. In this case, the permits policy has the disadvantage of reducing the output of firms installing more abatement technologies relative to the output of those doing less abatement. This problem is less significant under standards because all firms are required to install similar technologies.

The instrument choice problem studied in this paper is similar in spirit and approach to the instrument choice dilemma considered by Weitzman (1974), but with important differences. Weitzman (1974) compares the relative advantage of a price instrument (taxes) over a quantity instrument when the regulator has imperfect information about the aggregate abatement cost curve (and possibly about the damage curve as well). Thus, cost heterogeneity across firms plays no role in Weitzman's analysis. Instead, I compare the performance of two different

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[7]Note that when emissions are accurately measured and the total number of permits is set at the efficient (i.e., first-best) level, permits are always welfare superior to standards even if the latter leads to lower aggregate emissions. When emissions are not accurately measured, permits are no longer a first-best policy.

quantity instruments and focus on the effect of cost heterogeneity on instrument performance.[8]

Since I ignore aggregate uncertainty, the permits policy here is no different than a tax set equal

to the permits price and applied over the same proxy emissions.[9]

In deciding whether to use permits or standards, the regulator is likely to face a trade-off

between cost-effectiveness and possible higher emissions, so in Section 5 I discuss the advantages

of implementing an optimal hybrid policy in which permits are combined with an optimally

chosen standard. While the introduction of the hybrid policy provides a net welfare increase

for several combinations of parameters, there are many cases in which the hybrid policy adds

nothing over the permits policy (i.e., the hybrid policy converges to the permits-alone policy).

Conversely, there are very few cases in which the hybrid policy converges to the standards-alone

policy. Final remarks are in Section 6.

2   The model

Consider a competitive market for an homogeneous good supplied by a continuum of firms of

mass 1. Each firm produces output q and emissions e of a uniform flow pollutant. To simplify

notation, I assume that when the firm does not utilize any pollution abatement device, then

$e = q$. Market inverse demand is given by $P = P(Q)$, where Q is total output and $P'(Q) \leq 0$.

Total damage from pollution is given by $D(E)$, where E are total emissions and $D'(E) > 0$.

Functions $P(Q)$ and $D(E)$ are known to the regulator.

A firm can abate pollution at a positive cost by installing technology x, which reduces

emissions from q to $e = (1 - x)q$. Hence, the firm's emission rate is $e/q = 1 - x$. Each firm

is represented by a pair of cost parameters (β, γ). A firm of type (β, γ) has a cost function

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[8]See also Roberton Williams (2002) for a comparison between quantity instruments in the presence of aggregate uncertainty.

[9]As I explain below, aggregate cost uncertainty can be easily added to the model with no changes in the comparison between permits and standards. Of course, permits and taxes will no longer be equivalent.

$C(q, x, \beta, \gamma)$, where β and γ are the firm's private information. To keep the model mathematically tractable, I assume that the cost function has the following quadratic form in the relevant output-abatement range

$$C(q, x, \beta, \gamma) = \frac{c}{2}q^2 + \beta q + \frac{k}{2}x^2 + \gamma x + vxq \qquad (1)$$

where c, k and v are known parameters common to all firms and $c > 0$, $k > 0$, $\Lambda \equiv ck - v^2 > 0$ and $v \gtreqless 0$. Note that the parameter v can be negative, for example, if switching to a cleaner fuel saves on fuel costs but involves such a large retrofitting cost (i.e., high k) that no firm switches to the cleaner and cheaper fuel unless regulated.

Function (1) incorporates two key cost parameters that are essential to understand firm behavior under permits and standards regulation. One of these cost parameters is the correlation between β and γ (that we shall denote by ρ), which captures whether firms with higher output ex-ante (i.e., before the regulation) are more or less likely to install more abatement x. The other cost parameter is v, which captures the effect of abatement on output ex-post (note that we have constrained v to be the same for all firms, so a negative value of v indicates that, on average, a larger x means a larger increase in q ex-post). As we shall see, the values of the cost parameters v and ρ play a fundamental role in the design and choice of policy instruments when emissions are not easily monitored.

Although the regulator does not observe firms' individual values for β and γ, we assume that she knows that they are distributed according to the cumulative joint distribution $F(\beta, \gamma)$ on $\beta \in [\underline{\beta}, \overline{\beta}]$ and $\gamma \in [\underline{\gamma}, \overline{\gamma}]$.[10] To simplify notation further and without any loss of generality I let $E[\beta] = E[\gamma] = 0$, where $E[\cdot]$ is the expected value operator.[11] I also use the following notation:

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[10] Note that we can easily add aggregate uncertainty to this formulation by simply letting $\beta^i = \beta^i + \theta$ and $\gamma^i = \gamma^i + \eta$, where θ and η are random variables common to all firms.

[11] Note that because β and γ are negative for some firms, one can argue that marginal costs can take negative

$\mathrm{Var}[\beta] \equiv \sigma_\beta^2$, $\mathrm{Var}[\gamma] \equiv \sigma_\gamma^2$, $\mathrm{Cov}[\beta, \gamma] \equiv \rho\sigma_\beta\sigma_\gamma$ and $F_{\beta\gamma} \equiv \partial^2 F(\beta, \gamma)/\partial\beta\partial\gamma$.

Firms behave competitively, taking the output clearing price P as given. Hence, in the absence of any environmental regulation, each firm will produce to the point where its marginal production cost equals the product price (i.e., $C_q(q, x, \beta, \gamma) = P$), and install no abatement technology (i.e., $x = 0$). Because production involves some pollution, this market equilibrium is not socially optimal. The regulator's problem is then to design a regulation that maximizes social welfare.

I let the regulator's social welfare function be

$$W = \int_0^Q P(z)dz - \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} C(q, x, \beta, \gamma)F_{\beta\gamma}d\gamma d\beta - D(E) \tag{2}$$

where $Q = \int_\beta \int_\gamma q(\beta, \gamma)F_{\beta\gamma}d\gamma d\beta$ is total output and $E = \int_\beta \int_\gamma (1 - x(\beta, \gamma))q(\beta, \gamma)F_{\beta\gamma}d\gamma d\beta$ is total emissions. In this welfare function, the regulator does not differentiate between consumer and producer surplus, and transfers from or to firms are lump-sum transfers between consumers and firms, with no welfare effects.[12]

The firms' outputs and abatement technologies that implement the social optimum or first-best outcome are given by the following two first-order conditions for each firm

$$q : P(Q) - C_q(q, x, \beta, \gamma) - D'(E) \cdot (1 - x) = 0 \tag{3}$$

$$x : -C_x(q, x, \beta, \gamma) - D'(E) \cdot (-q) = 0 \tag{4}$$

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values. This possibility is eliminated by assuming parameter values (including those in the demand and damage functions) that lead to interior solutions for q and x in which $\partial C/\partial q > 0$ and $\partial C/\partial x > 0$ for all β and γ. Furthermore, since these interior solution are assumed to fall within the range in which (1) is valid, what happens beyond this range is not relevant for the analysis of instrument design and choice that follows. Alternatively, one can let $\beta \in [\underline{\beta} > 0, \overline{\beta}]$ and $\gamma \in [\underline{\gamma} > 0, \overline{\gamma}]$ with additional constant terms in the optimal instrument designs but no change in the welfare comparisons.

[12] The model can be generalized by allowing the regulator to consider a weight $\mu \neq 1$ for firm profits and a shadow cost $\lambda > 0$ for public funds. However, this would not add much to our discussion unless permits are not grandfathered but auctioned off (see Goulder et al., 1997).

If rearranged, eq. (3) says that the benefits from consuming an additional (small) unit of output is equal to its cost of production and environmental effects. Eq (4), on the other hand, says that emissions should be reduced to the point where the marginal cost of emissions abatement (i.e., C_x/q) is equal to marginal damages (i.e., $D'(E)$).

Because of various information asymmetries between firms and the regulator, it is not clear that the latter can design an environmental policy that can attain the first-best resource alloca-tion. The regulator's problem then becomes to maximize (2) subject to information constraints (and sometimes administrative and political constraints as well). From the various possible types of information constraints, the case that has attracted most attention in the literature is that in which the regulator knows little or nothing about firms' costs (i.e., she may not know F) but can costlessly monitor each firm's actual emissions e (Kwerel, 1977; Dasgupta et al.,1980; Spulber, 1988; Lewis, 1996). These authors show that in many cases the first-best can be implemented, such as when the regulator announces a (non-linear) emissions tax schedule $\tau(E)$ equal to $D'(E)$.[13]

In this paper, however, I am interested in the problem where the regulator cannot directly observe firms' actual emissions $e = (1 - x)q$, although she can costlessly monitor firms' abate-ment technologies or emission rates x. As in Santiago's permits program, this information asymmetry will be present when both continuous monitoring equipment is prohibitively costly and individual output or utilization q is not observable.[14] Thus, if the regulator asks for an output report from the firm, we anticipate that the firm would misreport its output whenever

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[13]A competitive firm (β, γ) takes the price $P(Q)$ and tax rate $\tau(E) = D'(E)$ as given and maximize $\pi(q, x, \beta, \gamma) = P(Q)q - C(q, x, \beta, \gamma) - D'(E) \cdot (1 - x)q$ with respect to q and x. The firm's first order con-ditions for q and x are those given by (3) and (4).

[14]In Santiago's TSP program, the imperfect observability of output comes from the fact that the regulator does not observe (neither can estimate) the number of hours that boilers and ovens of industrial sources, and heaters of residential and commercial buildings are operated during the year. The regulation of mobile sources is another example where the regulator faces severe constraints to observe or estimate vehicle use (Fullerton and West, 2002).

this was to its advantage. In this case, the regulator cannot implement the social optimum regardless of the information she has about firm's costs.[15]

Even if the regulator has perfect knowledge of the firm's costs and, therefore, can ex-post deduce output based on this information and the observation of x, the fact that she cannot make the policy contingent on either emissions or output prevents her from implementing the first-best. In other words, the regulator cannot induce the optimal amounts of output and emissions with only one instrument (i.e., x).[16] Consequently, the regulator must necessarily content herself with less optimal policies.

3   Instrument design

Rather than considering a full range of policies, I focus here on those policies that are either currently implemented or have drawn some degree of attention from policy makers in the context of urban air pollution control (Stavins, 2003). I first study the optimal design of a traditional technology (or emission rate) standard and then the optimal design of a permits market. I leave the optimal design of the hybrid policy, which builds upon the individual designs, for Section 5. To keep the model tractable, I make two further simplifications regarding the demand curve and the damage function. I let $P(Q) = P$ (constant) and $D(E) = hE$, where $h > v$.[17]

To visualize how the two policy designs depart from the social optimum, it is useful to compute the first-best and keep it as a benchmark. Plugging the above assumptions into (3)

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[15] Consider the extreme situation in which regulator knows both β and γ. His optimal policy will be some function $T(x; \beta, \gamma)$ in the form of either a transfer from the firm or to the firm. Then, firm (β, γ) takes $P(Q)$ and $T(x; \beta, \gamma)$ as given and maximizes $\pi(q, x, \beta, \gamma) = P(Q)q - C(q, x, \beta, \gamma) - T(x; \beta, \gamma)$ with respect to q and x. It is not difficult to see that the firm's first order conditions for q and x will always differ from (3) and (4) for any function $T(x; \beta, \gamma)$.

[16] Since the regulator can have a good idea of total emissions E from air quality measures, one might argue that Holmström's (1982) approach to solving moral hazard problems in teams may apply here as well. However, in our context this approach is unfeasible because the large number of agents would require too much transfers; either from firms as penalties or to firms as subsidies.

[17] Because costs are always lower under the permits policy, a backward sloping demand would lead to higher output, and hence, to higher surplus under such policy. On the other hand, a convex damage function will make the policy leading to lower emissions look relatively more attractive.

and (4), the first-best outcome is given by

$$x^* = \frac{(P - h - \beta)(h - v) - \gamma c}{ck - (h - v)^2} \tag{5}$$

$$q^* = \frac{(P - h - \beta)k - (h - v)\gamma}{ck - (h - v)^2} \tag{6}$$

It is immediate that $\partial x^*/\partial\beta < 0$, $\partial x^*/\partial\gamma < 0$, $\partial q^*/\partial\beta < 0$ and $\partial q^*/\partial\gamma < 0$. As expected, higher production and abatement costs lead to lower abatement and output levels.

3.1   Standards

The regulator's problem here is to find the emission rate standard x_s to be required of all firms in order to maximize social welfare (subscript "s" denotes standards policy). The regulator knows that for any given x_s,[18] firm (β, γ) will maximize $\pi(q, x_s, \beta, \gamma) = Pq - C(q, x_s, \beta, \gamma)$. Hence, the firm's (β, γ) output decision will solve the first-order condition

$$P - cq - \beta - vx_s = 0$$

which provides the regulator with the firm's output q as a function of the standard x_s

$$q_s(x_s) \equiv q_s = \frac{P - \beta - vx_s}{c} \tag{7}$$

Since x_s is the same across firms, the firm's output clearly departs from its first-best level q^*, as q_s does not adapt to changes in γ.

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[18]It is implicitly assumed that k is large enough, so that the standard is binding for all firms even if $v < 0$. More precisely, it is assumed that $d\pi(x_s)/dx_s = -(kx_s + \gamma + vq) < 0$ for all γ.

Based on the welfare function (2), the regulator now solves

$$\max_{x_s} \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} [Pq_s - C(q_s, x_s) - h \cdot (1 - x_s)q_s] \, F_{\beta\gamma} d\gamma d\beta$$

By the envelope theorem, the first-order condition is

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} \left[-kx_s - \gamma - vq_s - h \cdot (1 - x_s)\frac{\partial q_s}{\partial x_s} + hq_s \right] F_{\beta\gamma} d\gamma d\beta = 0 \qquad (8)$$

By using (7) and $\partial q_s / \partial x_s = -v/c$ in (8), the first-order condition (8) reduces to

$$-ckx_s - v \cdot (P - vx_s) + h \cdot (1 - x_s)v + h \cdot (P - vx_s) = 0$$

which leads to the optimal standard

$$x_s^* = \frac{P \cdot (h - v) + hv}{\Lambda + 2vh} \qquad (9)$$

where $\Lambda \equiv ck - v^2 > 0$.

3.2   Permits

Since the regulator only observes x, the permits policy is not based on actual emissions e but on some proxy for emissions that we denote by $\widetilde{e}$. The regulator's problem is then to find the total number of permits $\widetilde{e}_0$ to be distributed among firms in order to maximize social welfare ($\widetilde{e}_0$ is also the numbers of permits received by each firm). Let R denote the equilibrium price of permits, which will be determined shortly.[19] The regulator knows that firm (β, γ) will take

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[19] Note that under a tax policy, the optimal price R will be the tax. If we add aggregate uncertainty to the model, the tax and permit policies will not be equivalent from an efficiency standpoint (Weitzman, 1974).

R as given and solve

$$\max_{q,x} \pi(q, x, \beta, \gamma) = Pq - C(q, x, \beta, \gamma) - R \cdot (\widetilde{e} - \widetilde{e}_0)$$

where $\widetilde{e} = (1 - x)\widetilde{q}$ are firm's proxy emissions and $\widetilde{q}$ is some arbitrarily output or utilization level that is common to all firms. For example, $\widetilde{q}$ could be set equal to the maximum possible output that could ever be observed, which would occur when $x = 0$ and $\beta = \underline{\beta}$. As we shall see shortly, the exact value of $\widetilde{q}$ turns out to be irrelevant because it simply works as a scaling factor. Note that if $\widetilde{e} < \widetilde{e}_0$, the firm will be a seller of permits.

>From a firm's first-order conditions

$$x: \quad -kx - \gamma - vq + R\widetilde{q} = 0 \tag{10}$$

$$q: \quad P - cq - \beta - vx = 0 \tag{11}$$

we have that the optimal abatement and output responses to R and $\widetilde{q}$ (or, more precisely, to $R\widetilde{q}$) of firm (β, γ) are

$$x_p = \frac{R\widetilde{q}c - \gamma c - (P - \beta)v}{\Lambda} \tag{12}$$

$$q_p = \frac{P - \beta - vx_p}{c} \tag{13}$$

where the subscript "p" denotes permits policy. Comparing (12) and (13) with (5) and (6) illustrates the trade-off a regulator may face when implementing a permits program with imperfect observation of emissions. While $\partial q_p/\partial \beta$ and $\partial x_p/\partial \gamma$ are negative as in the first-best, regardless of the sign of v, $\partial q_p/\partial \gamma$ and $\partial x_p/\partial \beta$ are both positive when $v > 0$.[20]

Now we can solve the regulator's problem of finding the optimal $\widetilde{e}_0$. Since the market

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[20]Note that $\partial q_p/\partial \beta = -k/\Lambda$, $\partial x_p/\partial \beta = \partial q_p/\partial \gamma = v/\Lambda$ and $\partial x_p/\partial \gamma = -c/\Lambda$.

clearing condition is

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} \widetilde{e} F_{\beta\gamma} d\gamma d\beta = \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} (1 - x_p) \widetilde{q} F_{\beta\gamma} d\gamma d\beta = \widetilde{e}_0 \tag{14}$$

and x_p is a function of $R\widetilde{q}$ as indicated by (12), it is irrelevant whether we solve for $R\widetilde{q}$ or $\widetilde{e}_0/\widetilde{q}$. Permits purchases and sales are transfers with no net welfare effects, so we let the regulator solve

$$\max_{R\widetilde{q}} \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} \left[P q_p(x_p(R\widetilde{q})) - C(q_p(x_p(R\widetilde{q})), x_p(R\widetilde{q})) - h \cdot (1 - x_p(R\widetilde{q})) q_p(x_p(R\widetilde{q})) \right] F_{\beta\gamma} d\gamma d\beta$$

By the envelope theorem, the first-order condition is

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} \left[-(1 - x_p) h \frac{\partial q_p}{\partial (R\widetilde{q})} + h q_p \frac{\partial x_p}{\partial (R\widetilde{q})} - R\widetilde{q} \frac{\partial x_p}{\partial (R\widetilde{q})} \right] F_{\beta\gamma} d\gamma d\beta = 0 \tag{15}$$

By plugging (12), (13), $\partial q_p / \partial (R\widetilde{q}) = [\partial q_p / \partial x_p][\partial x_p / \partial (R\widetilde{q})]$, and $\partial q_p / \partial x_p = -v/c$ into (15), the first-order condition can be rearranged to obtain the optimal permits price

$$R^* \widetilde{q} = \frac{Ph(kc + v^2) + hv\Lambda}{(\Lambda + 2hv)c} \tag{16}$$

which, in turn, allows us to obtain the optimal permits allocation $\widetilde{e}_0^*/\widetilde{q}$ by simply substituting (16) into (12) and then into (14).

We can now replace the optimal $R^* \widetilde{q}$ in (12) and (13) to obtain expressions for q_p and x_p that are more readily comparable to q_s and x_s (see eqs. (7) and (9)). After some algebra, the following expressions are obtained

$$x_p = x_s^* + \frac{v\beta - c\gamma}{\Lambda} \tag{17}$$

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$$q_p = \frac{P - vx_s^*}{c} - \frac{k\beta - v\gamma}{\Lambda} = q_s - \frac{v^2\beta - cv\gamma}{c\Lambda} \qquad (18)$$

If firms are homogeneous (i.e., $\beta = \gamma = 0$ for all firms), then $x_p = x_s^*$ and $q_p = q_s$ (i.e., both policies provide the same welfare at a level that is always lower than the first-best level; see footnote 13). As firms become more heterogenous, x and q move in different magnitudes and sometimes directions depending on the policy choice, which will ultimately affect the welfare comparison between the two policies. Suppose, for example, that $v > 0$. As firms differ on their abatement costs γ, firms' abatement decisions x tend to remain closer to the social optimum x^* under the permits regulation than under the CAC regulation since $\partial x^*/\partial \gamma$ and $\partial x_p/\partial \gamma$ are both negative.[21] As firms differ on their production costs β, firms' abatement decisions remain closer to x^* under the CAC regulation, since $\partial x^*/\partial \beta$ and $\partial x_p/\partial \beta$ have opposite signs. A similar pattern can be found from analyzing firms' production decisions with regard to changes in β and γ. I study the implication on instrument choice of these and related issues more formally in the next section.

4   Instrument choice

The regulator must now decide wether to use permits or standards (or, eventually, a combination of both). Although by construction a hybrid policy cannot be welfare dominated by either single-instrument policy, it is useful first to compare single-instrument policies and then to explore the extra gains, if any, from implementing a hybrid policy.

For a regulator who is limited to use a single instrument, the difference in the social welfare

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[21] Note that $\partial x^*/\partial \gamma = \partial x_p/\partial \gamma$ for $v = 0$.

between the optimal permits policy and the optimal standards policy is

$$\Delta_{ps} = W_p(\widetilde{e}_0^*/\widetilde{q}) - W_s(x_s^*) \tag{19}$$

where $\widetilde{e}_0^*$ is the optimal number of permits normalized by some $\widetilde{q}$ and x_s^* is the optimal standard. The normative implication of (19) is that if $\Delta_{ps} > 0$, the regulator should implement the permits policy.

To explore under which conditions this is the case, we write (19) as

$$\Delta_{ps} = \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} [Pq_p - C(q_p, x_p) - (1-x_p)q_p h - Pq_s + C(q_s, x_s^*) + (1-x_s^*)q_s h] \, F_{\beta\gamma} d\gamma d\beta \tag{20}$$

where q_p, x_p, q_s and x_s^* can be expressed according to (17) and (18). Since $Q_p = Q_s = (P - vx_s^*)/c$, eq. (20) can be re-written as

$$\Delta_{ps} = \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\overline{\gamma}} [\{C(q_s, x_s^*) - C(q_p, x_p)\} + \{(1-x_s^*)q_s - (1-x_p)q_p\} \, h] \, F_{\beta\gamma} d\gamma d\beta \tag{21}$$

Recalling that $e = (1-x)q$, the first curly bracket of the right hand side of (21) is the difference in costs between the two policies, whereas the second curly bracket is the difference in emissions (which multiplied by h gives the difference in pollution damages).

If we plug (17) and (18) into (21), after some algebra (21) becomes

$$\Delta_{ps} = \frac{v^2\sigma_\beta^2 - 2cv\rho\sigma_\beta\sigma_\gamma + c^2\sigma_\gamma^2}{2c\Lambda} - \frac{h \cdot \left(kv\sigma_\beta^2 - (kc + v^2)\rho\sigma_\beta\sigma_\gamma + cv\sigma_\gamma^2\right)}{\Lambda^2} \tag{22}$$

and after collecting terms, it reduces to

$$\Delta_{ps} = A_1\sigma_\beta^2 + A_2\sigma_\gamma^2 + A_3\rho\sigma_\beta\sigma_\gamma \tag{23}$$

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where $A_1 \equiv (v^2\Lambda - 2ckhv)/2c\Lambda^2$, $A_2 \equiv (c\Lambda - 2chv)/2\Lambda^2$ and $A_3 \equiv (ckh + hv^2 - v\Lambda)/\Lambda^2 > 0$.

Since A_1, A_2 and ρ can be either positive, negative, or zero,[22] the sign of Δ_{ps} will depend upon the value of the different parameters. As the heterogeneity across firms decreases (i.e., σ_β^2 and σ_γ^2 approach zero), however, the welfare difference between the two policies tends to disappear.[23]

The ambiguous sign of Δ_{ps} should not be surprising given the trade-off we identified in the previous section. Expression (22) illustrates the ambiguous sign of Δ_{ps} more clearly. The first term is the difference in costs between the two policies. Since $-1 \leq \rho \leq 1$, this term is always positive, which indicates that the optimal permits policy is never more costly (in terms of abatement and production) than the optimal standards policy. This is simply because permits provide firms with such flexibility that if the standards outcome were the cheapest compliance solution, firms will have (cost-savings) incentives to engage in permit trading until reaching such outcome. The second term is the difference in damages, which can either be positive, negative, or zero, depending on the value of the different parameters of the cost function. Hence, a permits policy will always lead to cost savings, while it can lead to either higher or lower emissions.[24]

The magnitude of Δ_{ps} depends on the value of the different parameters of the model, but its sign is governed by the key cost parameters v and ρ. If $v = \rho = 0$, for example, the second term of (22) is zero, so $\Delta_{ps} > 0$ and permits do better than standards. But if $v > 0$ and $\rho < 0$, this second term becomes negative, and depending on the value of h relative to other parameters, Δ_{ps} may become negative as well (it would be so, for example, for $\sigma_\beta = \sigma_\gamma$, $v = 0$ and $\rho < -c/h$). More generally, one can draw a line $\ell_{\Delta=0}$ in the (v, ρ)–space that solves

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[22] Recall that for interior solutions in all cases we must have $ck > (h-v)^2$, $ck > v^2$, and $h > v$.

[23] Note that the welfare difference remains if we were to consider homogeneous sources subject to common cost shocks (denoted by β and γ), which are observed by firmst but not by the regulator.

[24] Recall that in the perfect monitoring case (where permits implement the first-best solution), an "optimal" standards policy would lead to more emissions than the first-best level.

$\Delta_{ps}(v, \rho) = 0$. These results can be summarized in the following proposition

Proposition 1 *The permits policy can lead to either higher or lower welfare than the standards policy. A necessary condition for permits to be welfare dominated by standards is that either $\rho < 0$ or $v > 0$.*

To understand this proposition, one must first recognize that a permits program can create incentives for shifting output from cleaner to dirtier firms, resulting in higher total emissions. The latter could happen under two circumstances. The first case is when firms with relatively large output ex-ante (i.e., before the regulation) are choosing low abatement, i.e., when there is a negative correlation between production and abatement costs ($\rho < 0$). When this is so, permits induce mostly low-output firms to install abatement technologies, while standards force all firms to invest in abatement. The second case is when firms doing more abatement find it optimal to reduce output ex-post, i.e., when there is a positive interaction between emissions and output ($v > 0$). In this case, the permits policy has the disadvantage of decreasing the output of firms installing more abatement technologies relative to the output of those doing less abatement. This problem is minor under standards because all firms are required to install similar technologies.

Contrary to what occurs when emissions are perfectly monitored, Proposition 1 indicates that neither permits nor standards is the appropriate policy choice in all cases. Because of this ambiguity, there seems to be room for a hybrid policy to improve upon either single-instrument policy. In fact, in the classic problem of prices versus quantities under aggregate uncertainty pioneered by Weitzman (1974), Roberts and Spence (1980) shows that a hybrid policy that combines permits with taxes/subsidies is always superior to either permits or taxes/subsidies alone. The reason for this result is that in Weitzman (1974) taxes are always superior in terms of costs (lower expected compliance costs) while permits are always superior in terms of emissions

(higher expected environmental benefits). In our problem, however, permits are always superior in terms of costs but standards are not always superior in terms of emissions, so it remains to be seen whether and when a hybrid policy would provide a net welfare gain.

5 The hybrid policy

The regulator's design problem here is to find the allocation $\widetilde{e}_0^h$ of permits (together with an utilization factor $\widetilde{q}^h$) and an emission standard x_s^h that maximize social welfare. The superscript "h" stands for hybrid policy. Under this hybrid policy, firms are free to trade permits, but each of them cannot abate less than x_s^h. As in the permits-alone policy, these permits will have an equilibrium price $R^h \widetilde{q}^h$ (the scaling factor $\widetilde{q}^h$ can be the same as in the permits-alone policy; its actual value is not relevant).

Depending on the values of β and γ, the standard x_s^h will be binding for some firms; the rest will abate beyond the standard and sell additional permits. Thus, we can divide the universe of firms in two groups: those that comply with the permits policy ($x^h \equiv x_p^h$ and $q^h \equiv q_p^h$) and those that comply with the standards policy ($x^h \equiv x_s^h$ and $q^h \equiv q_s^h$). Note that each firm in the latter group must still cover its (proxy) emissions with permits, which will ultimately affect the equilibrium price of permits. The values of β and γ of those firms that are on the border that divides the two groups of firms satisfy $x_p^h = x_s^h$. From the first order condition (12), we know that

$$x_p^h = \frac{R^h \widetilde{q}^h c - \gamma c - (P - \beta)v}{\Lambda}$$

which combined with $x_p^h = x_s^h$ gives us the boundary

$$\gamma = \frac{v}{c}\beta - \frac{\Lambda}{c}x_s^h + R^h \widetilde{q}^h - \frac{Pv}{c} \equiv \widehat{\gamma}(\beta) \tag{24}$$

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Because the initial allocation of permits does not affect behavior, firms with $\gamma \geq \widehat{\gamma}(\beta)$ will behave as in the standard-alone policy, and firms with $\gamma < \widehat{\gamma}(\beta)$ will behave as in the permits-alone policy. Then, the regulator's problem can be written as

$$\max_{x_s^h, \widetilde{e}_0^h / \widetilde{q}^h} \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\widehat{\gamma}(\beta)} \left[P q_p^h - C(q_p^h, x_p^h, \beta, \gamma) - h(1 - x_p^h) q_p^h \right] F_{\beta\gamma} d\gamma d\beta +$$

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\widehat{\gamma}(\beta)}^{\overline{\gamma}} \left[P q_s^h - C(q_s^h, x_s^h, \beta, \gamma) - h(1 - x_s^h) q_s^h \right] F_{\beta\gamma} d\gamma d\beta \quad (25)$$

where q_s^h, x_p^h and q_p^h can be obtained, respectively, from (7), (12) and (13) (simply add the superscript "h"). The regulator ensures full compliance, so the market clearing condition where all firms must cover their emissions with permits is

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\widehat{\gamma}(\beta)} (1 - x_p^h) \widetilde{q}^h F_{\beta\gamma} d\gamma d\beta + \int_{\underline{\beta}}^{\overline{\beta}} \int_{\widehat{\gamma}(\beta)}^{\overline{\gamma}} (1 - x_s^h) \widetilde{q}^h F_{\beta\gamma} d\gamma d\beta = \widetilde{e}_0^h$$

Since $\widetilde{q}^h$ is a constant and x_p^h depends on $R^h \widetilde{q}^h$, it is clear again that it is irrelevant whether we solve for $R^h \widetilde{q}^h$ or $\widetilde{e}_0^h / \widetilde{q}^h$. As before, we solve for $R^h \widetilde{q}^h$. Following the derivations for the single-instrument policies and allowing for corner solutions, the first order conditions for the optimal hybrid policy are

$$R^h \widetilde{q}^h : \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\widehat{\gamma}(\beta)} \left[-(1 - x_p^h) h \frac{\partial q_p^h}{\partial (R^h \widetilde{q}^h)} + h q_p^h \frac{\partial x_p^h}{\partial (R^h \widetilde{q}^h)} - R^h \widetilde{q}^h \frac{\partial x_p^h}{\partial (R^h \widetilde{q}^h)} \right] F_{\beta\gamma} d\gamma d\beta$$

$$+ \int_{\underline{\beta}}^{\overline{\beta}} \left[P q_p^h - C(q_p^h, x_p^h, \beta, \widehat{\gamma}(\beta)) - h(1 - x_p^h) q_p^h \right] F_\beta(\beta, \widehat{\gamma}(\beta)) d\beta$$

$$- \int_{\underline{\beta}}^{\overline{\beta}} \left[P q_s^h - C(q_s^h, x_s^h, \beta, \widehat{\gamma}(\beta)) - h(1 - x_s^h) q_s^h \right] F_\beta(\beta, \widehat{\gamma}(\beta)) d\beta \leq 0$$

$$[= 0 \text{ if } R^h \widetilde{q}^h > 0; < 0 \text{ if } R^h \widetilde{q}^h = 0] \quad (26)$$

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$$x_s^h : \int_{\underline{\beta}}^{\overline{\beta}} \int_{\widehat{\gamma}(\beta)}^{\overline{\gamma}} \left[-kx_s^h - \gamma - vq_s^h - (1 - x_s^h)h\frac{\partial q_s^h}{\partial x_s^h} + hq_s^h \right] F_{\beta\gamma} d\gamma d\beta$$

$$- \frac{\Lambda}{c} \int_{\underline{\beta}}^{\overline{\beta}} \left[Pq_p^h - C(q_p^h, x_p^h, \beta, \widehat{\gamma}(\beta)) - h(1 - x_p^h)q_p^h \right] F_\beta(\beta, \widehat{\gamma}(\beta)) d\beta$$

$$+ \frac{\Lambda}{c} \int_{\underline{\beta}}^{\overline{\beta}} \left[Pq_s^h - C(q_s^h, x_s^h, \beta, \widehat{\gamma}(\beta)) - h(1 - x_s^h)q_s^h \right] F_\beta(\beta, \widehat{\gamma}(\beta)) d\beta \leq 0$$

$$[= 0 \text{ if } x_s^h > 0; \ < 0 \text{ if } x_s^h = 0] \quad (27)$$

Since at $\widehat{\gamma}(\beta)$, we have that $x_s^h = x_p^h$ and, hence, $q_s^h = q_p^h$, so that (26) and (27) reduce, respectively, to

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\widehat{\gamma}(\beta)} \left[-(1 - x_p^h)h\frac{\partial q_p^h}{\partial x_p^h} + hq_p^h - R^h\widetilde{q}^h \right] F_{\beta\gamma} d\gamma d\beta \leq 0 \quad (28)$$

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\widehat{\gamma}(\beta)}^{\overline{\gamma}} \left[-kx_s^h - \gamma - vq_s^h - (1 - x_s^h)h\frac{\partial q_s^h}{\partial x_s^h} + hq_s^h \right] F_{\beta\gamma} d\gamma d\beta \leq 0 \quad (29)$$

where $\partial q_p^h / \partial x_p^h = \partial q_s^h / \partial x_s^h = -v/c$.

The solution of the optimal hybrid policy can result in a corner solution if either (28) or (29) is negative at the optimum. When (28) is negative, the hybrid policy converges to the standards-alone policy (i.e., $R^h\widetilde{q}^h = 0$ and $x_s^h = x_s^*$), and when (29) is negative, the hybrid policy converges to the permits-alone policy (i.e., $R^h\widetilde{q}^h = R^*\widetilde{q}$ and $x_s^h = 0$). Formal exploration of these possibilities lead to the following proposition

Proposition 2 *The optimal hybrid policy converges to the optimal permits-alone policy for many combinations of v and ρ (e.g., $v = 0$ and $\rho = 0$; $v < 0$ and $\rho = 1$). On the other hand, the optimal hybrid policy may converge to the optimal standards-alone policy when abatement cost heterogeneity is significantly smaller than production cost heterogeneity and there is a negative correlation between abatement costs and production costs (e.g., $-\underline{\gamma}/\overline{\beta} < (h - v)/c$ and $\rho = -1$).*

Proof. See the Appendix. ∎

Contrary to other models (e.g., Roberts and Spence, 1980; Kwerel, 1980), in this model the combination of instruments does not necessarily lead to higher welfare. Since permits are not only less costly than standards but also can lead to lower emissions, it is reasonable to think that in such cases standards may add nothing to the policy design. Take, for example, the simple case in which $v < 0$ and $\rho = 1$. To see why here the introduction of a standard in addition to permits is not welfare increasing, consider the effect of a very mild standard (i.e., x_s^h close to cero) that is just binding for one firm (all the other firms are reducing beyond the standard). When $\rho = 1$, the binding firm is exactly the one with the highest abatement and production costs (i.e., $\beta = \overline{\beta}$ and $\gamma = \overline{\gamma}$). The introduction of the standard is clearly inefficient because the extra reduction is coming from the most expensive firm.

It is less obvious that in some cases, although very few, permits may also add nothing to the policy design.[25] Now take the case in which $v > 0$ and $\rho = -1$. To see why the introduction of some permits in addition to the standard is not welfare increasing, consider the effect of issuing an arbitrarily small amount of permits (i.e., $\widetilde{e}_0^h$ close to cero) that are just binding for one firm in the sense that this is the only firm reducing beyond the standard. When $\rho = -1$, the binding firm is exactly the one with the lowest abatement cost (i.e., $\gamma = \underline{\gamma}$) but with the highest production cost (i.e., $\beta = \overline{\beta}$). The introduction of permits will be inefficient without sufficient abatement cost heterogeneity (i.e., $-\underline{\gamma}/\overline{\beta}$ sufficiently high) such that the savings in abatement costs more than offset the increase in production costs.

The exact shape of the region in which the hybrid policy dominates either single-instrument policy depends on the parameter values. A simple numerical exercise may be useful. In Figure 1, line $\ell_{h=p}$ indicates the combinations of v and ρ for which the hybrid policy just converges to

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[25] If the damage function is convex in the level of pollution, we should expect to have a few more cases in which the hybrid policy converges to the standards-alone policy.

the permits policy for the following parameters values: $P = k = c = 4$, $h = 2$, $\overline{\beta} = 2$, $\underline{\beta} = -2$, $\overline{\gamma} = 1$, $\underline{\gamma} = -1$.[26] The figure also includes the line $\ell_{\Delta=0}$ (i.e., combinations of v and ρ that yield $\Delta_{ps} = 0$) and the line $\ell_{\Delta E=0}$ (i.e., combinations of v and ρ for which the permits policy and the standards policy yield the exact same level of emissions). One can distinguish three regions in the figure. The first region (H=P>S)—to the left of $\ell_{h=p}$—includes all those combinations of v and ρ for which the hybrid policy coincides with the permits policy, which in turn, dominates the standards policy. For example, if $v = -0.5$ and $\rho = 0.6$, the first row of Table 1 shows that social net benefits (W) are 33% higher under the permits policy than under the standards policy. Note also that in some places of this region, the hybrid policy does not improve upon the permits-alone policy despite the fact that emissions are higher than under a standards-alone policy. The logic behind this result is that the introduction of some binding standard (in combination with permits) would not only reduce emissions but also increase production and abatement costs. And in this particular region, the latter effect dominates.

The second region (H>P>S)—between the lines $\ell_{h=p}$ and $\ell_{\Delta=0}$—includes all those combinations for which the hybrid policy is superior to the permits-alone policy, which in turn, is superior to the standards-alone policy. For example, if $v = 0.6$ and $\rho = 0.6$, the welfare gain from implementing the hybrid policy (Δ_h) is 12.6% of Δ_{ps}, as shown in the second row of Table 1. It is interesting to observe that despite the fact that welfare may not increase by much, policy designs are quite different (the hybrid policy includes a standard that is almost half the one in the standards-alone policy; though the equilibrium permit price does not vary much).

Finally, the third region (H>S>P) —to the right and below of $\ell_{\Delta=0}$— includes those combinations of v and ρ for which the hybrid policy dominates the standards-alone policy, which in turn, dominates the permits-alone policy. The third row of the table indicates that for $v = 0.7$

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[26] The simulation is carried out with only four type of firms: $(\overline{\beta},\overline{\gamma}),(\overline{\beta},\underline{\gamma})$, $(\underline{\beta},\overline{\gamma})$ and $(\underline{\beta},\underline{\gamma})$. Also, the value of the different parameters limit the range of v to $[-0.5, 0.7]$.
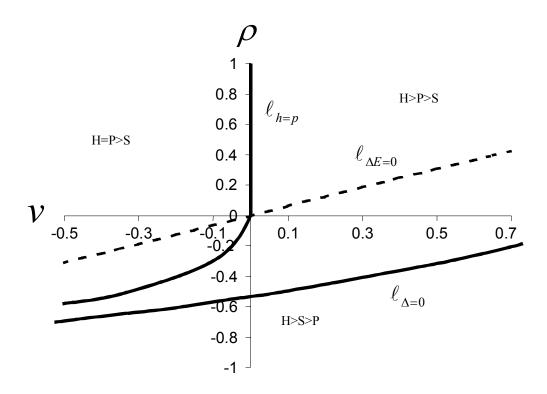
Figure 1: The optimality of hybrid and single-instrument policies to changes in the correlation (ρ) and interaction (v) parameters.

and $\rho = -0.5$, the gain from implementing the hybrid policy, as opposed to the standards-alone policy, is substantial, 32.5% of $|\Delta_{ps}|$.[27]

Table 1. Hybrid and single-instrument policies: design and welfare

v	ρ	x_s	$R\widetilde{q}$	x_s^h	$R^h\widetilde{q}^h$	W_s	Δ_{ps}	Δ_h
-0.5	0.6	0.65	2.08	0	2.08	123.64	41.08	0
0.6	0.5	0.38	2.07	0.18	1.99	82.04	13.66	1.72
0.7	-0.5	0.36	2.10	0.21	1.49	79.74	-6.37	2.07

6   Conclusions

I have developed a model to study the design and performance of pollution markets (i.e., tradable permits) when the regulator has imperfect information on firms' costs and emissions. A salient example is the control of air pollution in large cities where emissions come from many small (stationary and mobile) sources for which continuous monitoring is prohibitively costly and, therefore, the regulator must rely on monitoring procedures similar to those used in the traditional command-and-control (CAC) approach of setting technology and emission standards.

In such cases the superiority of permits over CAC regulation is no longer evident. Permits do retain the well known cost-effectiveness property of conventional permits programs but sometimes can provide firms with incentives to choose combinations of output and abatement technologies that may lead to higher aggregate emissions than under standards, something that would not occur if emissions were accurately measured. Thus, when (abatement and production) cost heterogeneity across firms is large, the permits policy is likely to work better. In contrast, as heterogeneity disappears, the advantage of permits reduces, and standards might

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[27]Note that despite the fact that $\sigma_\gamma = 0.5\sigma_\beta$, there is no region in Figure 1 where the hybrid policy converges to the standards-alone policy.

work better provided that they lead to lower emissions. Because of this trade-off between cost savings and possible higher emissions, I also examine the advantages of a hybrid policy that optimally combines permits and standards. I find that in many cases the hybrid policy converges to the permits-alone policy but it almost never converges to the standards-alone policy.

The results of the paper should also shed light into the design of permits market that face other types of monitoring constraints. Consider, for example, a permits market (even with observable output and emissions) in which the damages caused by the pollution from different sources are heterogenous and not easily observed.[28] If permits are traded from the less damaging to the more damaging sources, overall damages could increase. If the cost structure of affected sources is likely to cause such a trading pattern, the results of this paper would suggest the introduction a hybrid policy in which sources can freely trade permits subject to the restriction that no firm can emit more than a certain standard.

In addition, the results of the paper are valuable in thinking about the expansion of existing permits programs. Since sources under the TSP permits program of Santiago are currently responsible for less than 5% of total TSP emissions, the model developed here can be used to study the efficient incorporation of other TSP sources that today are subject to command and control regulation. A good candidate is powered-diesel buses, which are responsible for almost 40% of today's total TSP emissions. According to Cifuentes (1999), buses that abate emissions by switching to natural gas are likely to reduce utilization relative to buses that stay on diesel and that older, less-utilized buses are more likely to switch to natural gas. Since switching to natural gas is a major abatement alternative, both of these observations would suggest that the optimal way to integrate buses into the TSP program is by imposing, in addition to the allocation of permits, an emission standard specific to buses. It may also be optimal to use

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[28]I thank one of the referees for pointing out this case and its relevance for the Los Angeles' RECLAIM market.

different utilization factors ($\widetilde{q}$) for each group of sources. These and related design issues deserve further research.

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Appendix: Proof of Proposition 2

The proof is divided in four parts. The first three parts demonstrate that there exist several combinations of v and ρ for which the hybrid policy converges to the permits-alone policy (note that a more general proof would require specifying $F(\beta, \gamma)$, but this, in turn, would make the proof particular to the specified distribution function). Part 1 demonstrates so for $v = \rho = 0$. Part 2 demonstrates that for $v = 0$ and $\rho \neq 0$, the hybrid policy converges to the permits-alone policy only if $\rho > 0$. Part 3 demonstrates that for $v \neq 0$ and $\rho = 1$, the hybrid policy always converges to the permits-alone policy if $v < 0$ but not necessarily so if $v > 0$. Part 4, which finishes the proof, demonstrates that for $\rho = -1$ and some values of v the hybrid policy converges to the standards-alone policy if $-\underline{\gamma}/\overline{\beta} < (h-v)/c$.

Part 1. For $\rho = 0$, $F(\beta, \gamma)$ can be expressed as $F^{\beta}(\beta)F^{\gamma}(\gamma)$ where F^{β} and F^{γ} are independent cumulative distribution functions for β and γ respectively (f^{β} and f^{γ} are their corresponding probability density functions). Thus, when $v = \rho = 0$, $\widehat{\gamma}$ is independent of β and the first order conditions (28) and (29) become, respectively

$$\int_{\underline{\beta}}^{\overline{\beta}} \left[hq_p^h - R^h \widetilde{q}^h \right] F^{\gamma}(\widehat{\gamma}) f^{\beta} d\beta \leq 0 \tag{30}$$

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\widehat{\gamma}}^{\overline{\gamma}} \left[-kx_s^h - \gamma + hq_s^h \right] f^{\beta}(\beta) f^{\gamma}(\gamma) d\gamma d\beta \leq 0 \tag{31}$$

where $q_p^h = q_s^h = (P - \beta)/c$. Before solving for $R^h \widetilde{q}^h$ and x_s^h, note that from first order condition (11) we have that $-kx_p^h - \gamma + R^h \widetilde{q}^h = 0$ for $\gamma \leq \widehat{\gamma}$ and that $-kx_s^h - \gamma + R^h \widetilde{q}^h < 0$ for $\gamma > \widehat{\gamma}$

(otherwise the standard would not be binding). Since (30) cannot be positive, its solution yields $R^h \tilde{q}^h = Ph/c = R^* \tilde{q}$ (see (16)). On the other hand, developing (31) we obtain

$$\int_{\hat{\gamma}}^{\overline{\gamma}} \left[-kx_s^h - \gamma + \frac{Ph}{c} \right] f^\gamma(\gamma) d\gamma \leq 0$$

But since $Ph/c = R^h \tilde{q}^h$ and $-kx_s^h - \gamma + R^h \tilde{q}^h < 0$, the solution of (31) is a corner solution with $x_s^h = 0$.

Part 2. When $v = 0$ and $\rho \neq 0$, $\hat{\gamma}$ is independent of β and since $R^h \tilde{q}^h$ must be necessarily positive (otherwise (28) would be positive), the first order condition (28) reduces to

$$R^h \tilde{q}^h - \frac{Ph}{c} + \frac{h \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\hat{\gamma}} \beta F_{\gamma\beta} d\gamma d\beta}{c \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\hat{\gamma}} F_{\gamma\beta} d\gamma d\beta} = 0 \tag{32}$$

where its last term, which we will denote by ξ to save on notation, is positive (negative) for $\rho < 0$ ($\rho > 0$) and zero otherwise.

On the other hand, the first order condition (29) reduces to

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\hat{\gamma}}^{\overline{\gamma}} \left[-kx_s^h - \gamma + \frac{Ph}{c} - \frac{h\beta}{c} \right] F_{\gamma\beta} d\gamma d\beta \leq 0 \tag{33}$$

After replacing Ph/c according to (32), (33) can be rewritten as

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\hat{\gamma}}^{\overline{\gamma}} \left[-kx_s^h - \gamma + R^h \tilde{q}^h + \xi \right] F_{\gamma\beta} d\gamma d\beta - \frac{h}{c} \int_{\underline{\beta}}^{\overline{\beta}} \int_{\hat{\gamma}}^{\overline{\gamma}} \beta F_{\gamma\beta} d\gamma d\beta \leq 0 \tag{34}$$

where its last term (without the negative sign in front) has the exact opposite sign of ξ. Now, from condition (11) we know that $-kx_s^h - \gamma + R^h \tilde{q}^h < 0$ for $\gamma > \hat{\gamma}$, so if $\rho < 0$ there must be an interior solution (i.e., $x_s^h > 0$) because ξ is positive. To see this, let us imagine we increase the standard x_s^h from cero to the point where we reach the very first firm for which

there is no difference between complying with the standard and buying permits. At this point $-kx_s^h - \gamma + R^h\widetilde{q}^h = 0$ but $\xi > 0$, so condition (34) would be positive. It would be beneficial then to increase x_s^h a bit further until (34) reaches cero. Conversely, if $\rho > 0$, ξ is negative, so the solution of (34) is indeed a corner solution with $x_s^h = 0$.

Part 3. When $v \neq 0$ and $\rho \neq 0$, it is useful to first to express $q_p^h(R^h\widetilde{q}^h, \beta, \gamma)$, $x_p^h(R^h\widetilde{q}^h, \beta, \gamma)$ and $q_s^h(x_s^h, \beta)$ according to (7), (12) and (13) (simply add the superscript "h") and then substitute them into (28) and (29) to obtain, respectively, the first order conditions

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\gamma}}^{\widehat{\gamma}(\beta)} \left[(R^*\widetilde{q} - R^h\widetilde{q}^h)\left(\frac{\Lambda + 2hv}{\Lambda}\right) + \frac{2hv}{\Lambda}\gamma - \frac{h(ck + v^2)}{c\Lambda}\beta \right] F_{\gamma\beta}d\gamma d\beta \leq 0 \qquad (35)$$

$$\int_{\underline{\beta}}^{\overline{\beta}} \int_{\widehat{\gamma}(\beta)}^{\overline{\gamma}} \left[(x_s^* - x_s^h)\left(\frac{\Lambda + 2hv}{c}\right) - \gamma - \frac{(h - v)}{c}\beta \right] F_{\gamma\beta}d\gamma d\beta \leq 0 \qquad (36)$$

To demonstrate now that the hybrid policy converges to the permits-alone policy if $\rho = 1$ and $v < 0$, it is sufficient to show that the extra benefits from introducing $x_s^h > 0$ that is just binding for a marginal firm are negative when $R^h\widetilde{q}^h = R^*\widetilde{q}$. For $\rho = 1$ and $v < 0$, the cost parameters of the marginal firm are $\overline{\beta} > 0$ and $\overline{\gamma} > 0$ and the extra benefits are (from (36))

$$(x_s^* - x_s^h)\left(\frac{\Lambda + 2hv}{c}\right) - \overline{\gamma} - \frac{(h - v)}{c}\overline{\beta} \qquad (37)$$

Since this marginal firm is indifferent between permits and standards, i.e., $\widehat{\gamma}(\overline{\beta}) = \overline{\gamma}$, we have from (24) that

$$\overline{\gamma} = \frac{v}{c}\overline{\beta} - \frac{\Lambda}{c}x_s^h + R^*\widetilde{q} - \frac{Pv}{c} \qquad (38)$$

We also know from (9) and (16) that $R^*\widetilde{q}c = x_s^*\Lambda + Pv$. Using this to substitute $R^*\widetilde{q}$ into (38),

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we obtain an expression for $x_s^* - x_s^h$ that replaced into (37) leads to

$$\frac{2hv}{\Lambda}\overline{\gamma} - \frac{h(ck+v^2)}{c\Lambda}\overline{\beta} < 0 \tag{39}$$

which demonstrates that the extra benefits are negative.

For $\rho = 1$ and $v > 0$, on the other hand, the marginal firm is still $(\overline{\beta}, \overline{\gamma})$ and the extra benefits are also given by (39). But because $v > 0$, these extra benefits can now be either negative or positive, in which case it is optimal to implement a hybrid policy with $x_s^h > 0$ (using a similar procedure , it can be demonstrated that when $\rho = 1$, the authority always wants to use some permits).

Part 4. Let us now demonstrate that the hybrid policy may converge to the standards-alone policy for $\rho = -1$ and some values of v.[29] As analogous to Part 3, this will be the case if the extra benefits from introducing $R^h\widetilde{q}^h > 0$ that is just binding for a marginal firm are negative when $x_s^h = x_s^*$. When $\rho = -1$, the cost parameters of the marginal firm are $\overline{\beta} > 0$ and $\underline{\gamma} < 0$ and the extra benefits are (from (35))

$$(R^*\widetilde{q} - R^h\widetilde{q}^h)\left(\frac{\Lambda + 2hv}{\Lambda}\right) + \frac{2hv}{\Lambda}\underline{\gamma} - \frac{h(ck+v^2)}{c\Lambda}\overline{\beta} \tag{40}$$

Following the same procedure as in Part 3 to obtain an expression for $R^*\widetilde{q} - R^h\widetilde{q}^h$, (40) reduces to $-\underline{\gamma} - (h-v)\overline{\beta}/c$, which is negative (and hybrid policy converges to standards-alone policy) if $-\underline{\gamma}/\overline{\beta} < (h-v)/c$.■

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[29] Note that to work with interior solutions v cannot be any arbitrary value.