CEEPR Center for Energy and Environmental Policy Research

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04-005 WP February 2004

A Joint Center of the Department of Economics, Laboratory for Energy and the Environment, and Sloan School of Management

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Abstract

In this paper, we investigate the effect of market power on the equilibrium path of an emission permits market in which firms can bank current permits for use in later periods. In particular, we study the market equilibrium for a large (potentially dominant) firm and a competitive fringe with rational expectations. We characterize the equilibrium solution for different permits allocations. We find, for example, that if the large firm enjoys a dominant position in the after-banking market, this position gets extended to the market during the banking period regardless of the allocation of the stock (bank) of permits.

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1 Introduction

Emission permits trading usually refers to trades across space in the same period of time, but it can also refer to trades through time, typically by banking, i.e., the possibility of carrying over unused permits from one period for use in later periods.[1] Over the past decade, this latter dimension of emission permits trading has drawn increasing attention in the literature and proposals to decrease emission caps over time suggest a particular larger role for banking in the future.[2] A salient example is the US Acid Rain Program, where banking has been a major form of emissions trading (Ellerman et al, 2000; Ellerman and Montero, 2002). During the first five years of the program constituting Phase I, 1995-99, only 26.4 million of the 38.1 million permits (or allowances) distributed were used to cover emissions. The remaining 11.65 million allowances (30% of all the allowances distributed) were banked and has been gradually consumed during Phase II (2000 and beyond). As a result, the Phase II cap is expected to be reached sometimes between 2008 and 2010.

Several authors have studied the theoretical properties of intertemporal trading (Rubin, 1996; Cronshaw and Kruse, 1996; Schennach, 2000), but there is little work looking at the effect of market power on the equilibrium path. Because the evolution of a permits bank is closely related to the evolution of an exhaustible resource stock,[3] in this note we draw upon both the literature on permits markets and the literature on exhaustible resources to discuss whether and how a large (potentially dominant) firm can affect the market equilibrium path.[4]

We view each firm as a rational-expectation participant in a noncooperative dynamic

[1] Borrowing of permits from future vintages could also be included (and may be efficient to do so), but it has attracted much less attention than banking.

[2] An effective policy for reducing atmospheric greenhouse gas concentrations would likely include emission caps that would become more stringent over time.

[3] There are important differences though. First, the permits market still remains after the permits bank has been exhausted while the market for a typical exhaustible resource vanish after the total stock has been consumed. Second, extraction costs for permits are zero while they are generally positive for a typical exhautible resource. In addition, the demand for permits corresponds to a derived demand from the same firms that hold the permits while the demand for a typical exhaustible resource comes from a third party.

[4] In solving for the equilibrium we restrict attention to spot transactions in the permits market. The introduction of forward markets brings new elements to the model beyond the scope of this paper. See Liski and Montero (2004) for a discussion.

game. We characterize the market equilibrium path for different permits allocations, so we consider the possibility of market power both during and after the banking period. As in some exhaustible resource models, we identify time-consistency problems for some permits allocations in the sense that what is optimal for the large firm at time zero is no longer optimal at some future time. Without adopting a particular game structure, e.g., the Cournot structure of Salant (1976) or the Stackelberg structure of Newbery (1981), we provide a characterization of the (time-consistent) equilibrium path for allocations in which time inconsistency becomes an issue. In that respect, we find, for example, that if the large firm is able to exercise market power in the after-banking market, as described by Hahn (1985), this dominant position extends to the market during the banking period regardless of the permits allocation during the banking period.

The rest of the paper is organized as follows. In Section 2, we present the model and derive the equilibrium path (i.e., price and quantity paths) for a competitive market. In Section 3, we study the evolution of prices and quantities for a market composed of large firm and a competitive fringe. Final remarks are in Section 4.

2 The model

Consider an industry with a large number N of heterogenous plants whose emissions are regulated by a tradeable permits program with banking (a firm may own one or several plants). The regulator allocates a total of $A(t) = \sum_{i=1}^{N} a_i(t)$ allowances (or permits) in period t, where $a_i(t)$ is plant i's allocation at t (we will use capital letters for industry or group-level variables and small letters for plant-level variables). For (aggregate) banking to actually happen permits allocations must decrease over time (at least at a rate higher than the discount rate for some period of time). To follow the design of the US Acid Rain Program, we assume that during the first T_0 periods of the program the total number of permits allocated in each period is A_H and that thereafter is A_L, with $A_H \gg A_L$.[5]

Plant i's unrestricted or counterfactual emissions (i.e., emissions that would have been

[5]Alternatively one can let $(A_H - A_L)T_0$ be the initial "stock" allocation and A_L the per-period allocation.

observed in the absence of the permits program) are denoted by u_i and its abatement costs by $c_i(q_i(t))$, where $q_i(t)$ are emissions reduced at period t (we assume that both u_i and $c_i(\cdot)$ remain unchanged over time). Thus, plant i's emissions at t are $e_i(t) = u_i - q_i(t)$. A fully competitive market solves the following infinite horizon intertemporal minimization problem (Rubin, 1996; Schennach, 2000)

$$\min \int_0^\infty \left(\sum_{i=1}^{N} c_i(q_i(t)) \right) \mathrm{e}^{-rt} dt \tag{1}$$

$$\text{s.t. } \dot{B}(t) = A(t) + Q(t) - U \tag{2}$$

$$B(0) = 0, -B(t) \leq 0 \tag{3}$$

where r is the risk-free discount rate, $B(t)$ is the stock (i.e., bank) of allowances at time t (dots denote time derivatives), $Q(t)$ is aggregate reduction and $U > A_L$ is aggregate counterfactual emissions.

The solution of (1)–(3) can be decomposed as follows. First, there is a static efficiency condition that must hold at all times (even after the bank of permits is exhausted): $c_i'(q_i(t)) = c_j'(q_j(t)) = P(t)$ for all $i \neq j$, where $P(t)$ is the equilibrium price of permits at t. To find the rest of the solution, we use the static efficiency condition to denote the industry least-cost curve by $C(Q(t))$. This implies that $C'(Q(t) = P(t)$. Hence, the (current-value) Lagrangian for the problem (1)–(3) can be written as

$$\mathcal{L} = C(Q(t)) + \lambda(A(t) + Q(t) - U) - \phi B(t)$$

where $\lambda(t)$ and $\phi(t)$ the multiplier functions.

Necessary conditions for optimality include satisfaction of (2), (3) and[6]

$$\frac{\partial \mathcal{L}}{\partial Q} = C'(Q(t)) + \lambda(t) = 0 \tag{4}$$

$$\dot{\lambda}(t) - r\lambda(t) = -\frac{\partial \mathcal{L}}{\partial B} = \phi(t) \tag{5}$$

$$\phi(t) \geq 0, \ \phi(t)B(t) = 0 \tag{6}$$

[6] See Kamien and Schwartz (1991).

In addition, taking the derivative of (4) with respect to time yields

$$\frac{d(C'(Q(t))}{dt} - rC(Q(t)) + \phi = 0 \tag{7}$$

When $B(t) > 0$, $\phi(t) = 0$ and marginal costs $C'(Q(t))$, and hence price $P(t)$, follow the Hotelling's rule and rise at the discount rate r (note that permits are "extracted" at zero cost). Expression (7) is commonly known as the (no) arbitrage condition.

Whether and when firms will bank permits depends upon the allocation of permits, the evolution of marginal cost functions and the discount rate. For example, a significant reduction of the permits allocation in the future, as in the SO_2 program, will result in a banking period of some length $T > T_0$ (to be determined shortly): firms bank permits during some period of time and gradually use them thereafter until the bank expires at T. After T, permits trading continues (to equate marginal costs across plants) but total emissions remain constant at A_L.

The full compliance condition establishes the total number of permits allocated during the banking period $[0, T]$ be equal to the accumulated emissions during such period, that is (this condition is equivalent to the exhaustion condition found in the depletable resources literature)

$$(A_H - A_L)T_0 + A_L T = \int_0^T E(t)dt \tag{8}$$

At T the terminal condition $E(T) = A(T)$ must also hold, which is

$$Q(T) = U - A_L \tag{9}$$

Combining (7), (8) and (9) we can solve for T^* (the superscript "*" indicates a perfectly competitive value), which in turn allows us to compute the competitive price and quantity paths; $P^*(t)$ and $Q^*(t)$, respectively.

A solution for T^* can be obtained if we assume some functional form for $C(Q)$. For example, if we assume that $C'(Q)$ is linear, as found by Ellerman and Montero (2002) for the SO_2 program, the abatement path during the banking period is

$$Q(t) = (U - A_L)\mathrm{e}^{-r(T^* - t)} \tag{10}$$

Replacing (10) into (8) and rearranging we have the following expression that solves for T^*

$$\frac{(A_H - A_L)T_0}{U - A_L} = T^* - \frac{1}{r}\left(1 - \mathrm{e}^{-rT^*}\right) \tag{11}$$

Let us next discuss the effect on the market equilibrium path when a large number of plants are owned by a single firm.

3 Banking with market power

Consider now a permits market with banking in which there is a large (potentially dominant) firm and a competitive fringe.[7] Firms play a noncooperative dynamic game. For the purpose of this note we do not need be specific about the structure of the game. One could, for example, adopt a Cournot structure in which at each period each firm decides its quantities (permits purchases/sales and abatement level) so as to maximize its own profits while taking as given the actions of the remaining firms. Regardless of the structure adopted, the action of any small firm does not affect the price and, consequently, they are price takers.

Although similar versions of this problem has been already solved for a typical exhaustible resource under different set of assumptions (Salant, 1976; Gilbert 1978; Newbery, 1982), the proposed solutions do not immediately apply to a permits bank for several reasons. Extraction costs for permits are zero. In addition, costs of storage for permits are zero so speculators (and firms in the fringe) will make sure that prices neither jump nor grow at rate higher than r. This also gives the large firm the possibility to buy (or sell) a stock of permits from the fringe and store them for future use at no cost other than the opportunity cost of selling them earlier.

While several exhaustible resource models are already equiped with these zero-cost assumptions, there are more fundamental differences between a stock of permits and a stock of an exhaustible resource. In a permits market the large producer can still exercise market power after its stock (i.e., bank) and that of the fringe have been exhausted. So,

[7] Based on the analysis of Lewis and Schmalensee (1980) for an oligopolistic market, considering two or more large firms and a competitive fringe should not qualitatively alter the main result of this section.

contrary to what would occur in a typical exhaustible resource market, it may be possible that the large firm can still use its strategic position of the end of the banking period to exercise some market power during the banking period even if it does not receive any permits from the stock $(A_H - A_L)T_0$ but only an allocation flow throughout. Furthermore, because the demand for permits does not come from a third party (e.g., consumers) but internally from the fringe and the large producer, the large firm's decision problem is not only the choice of a permits sale/purchase path (or a price path supported by a sales path) but also of an abatement (or demand) path.

To study the large firm's problem, let f index the competitive fringe and m the large producer that attempts to manipulate the market. Abatement costs are denoted by $C_f(Q_f(t))$ and $C_m(Q_m(t))$, respectively. Total permits allocations are as before. However, it is useful to make an artificial distinction here between stock and flow allocations: the depletable stock is the cumulative number of permits allocated above the long-term goal of A_L. The total flow (or per period) allocation is A_L beginning in $t = 0$ and the total stock allocation is $(A_H - A_L)T_0$. The fringe receives a fraction θA_L of the flow allocation and $\mu(A_H - A_L)T_0$ of the stock allocation, so the large firm receives $(1 - \theta)A_L$ and $(1 - \mu)(A_H - A_L)T_0$, respectively.

Depending on the allocations (and cost structures), the large firm can, in principle, manipulate the market during and after the banking period. As explained by Hahn (1985), the after-banking manipulation is only profitable if the allocation θ and costs are such that the large producer is either a net seller or buyer of permits after the bank has exhausted. In other words, the large firm does not find it profitable to manipulate the after-banking market if it receives a flow allocation exactly equal to the number of permits that it would have demanded in a competitive after-banking market. To facilitate the exposition as to how the equilibrium solution changes with the permits allocation, we will consider two benchmark cases.

3.1 Large firm with all the stock

Let us first consider the case in which $\mu = 0$ and θ is such that there is no after-banking manipulation. The latter implies that the after-banking equilibrium price will be as in the competitive solution, i.e., $P^*(T^*)$. When $\mu = 0$, the fringe does not build a bank on its own but buys permits from the large producer from the very the first period. The large firm, on the other hand, finds it profitable to build and manage a permits bank. For this particular allocation in which small firms have no stocks, the equilibrium of the game can be found applying conventional dynamic programming principles. This is because the binding contract solution is time-consistent (and hence, there is no difference between Cournot and Stackelberg structures that one could eventually adopt). Then, the equilibrium is found by solving the large firm's following optimization problem[8]

$$\max \quad \int_0^\infty [P(t)X(t) - C_m(Q_m(t))]\mathrm{e}^{-rt}dt \tag{12}$$

$$\text{s.t.} \quad P(t) = C_f'(Q_f(t)) \tag{13}$$

$$X(t) = U_f(t) - Q_f(t) - A_f(t) \tag{14}$$

$$\dot{B}_m(t) = A_m(t) - U_m(t) + Q_m(t) - X(t) \qquad [\lambda_m(t)] \tag{15}$$

$$B_m(t) \geq 0 \qquad [\phi_m(t)] \tag{16}$$

$$B_m(0) = 0 \tag{17}$$

where $X(t)$ is the number of permits sold by the large firm in period t,[9] $B_m(t)$ is the large firm's bank and λ_m and ϕ_m are the multiplier functions associated to the different constraints.

Since firms in the fringe are price takers, it is irrelevant whether the leader solves for $P(t)$ or $Q_f(t)$. Replacing (13) and (14) in the objective function to form the corresponding (current-value) Lagrangian $\mathcal{L}$, the necessary conditions for optimality include satisfaction

[8] Note that the solution implicitly takes into account agents' rational expectations. For example, we rule out a solution in which there is large price jump; rational firms would buy more today in anticipation to the price jump.

[9] If the dominant firm acts as a monopsonist then $X(t) < 0$.

of (13)–(17) and

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial Q_f} &= [C_f''(Q_f(t))X(t) - C_f'(Q_f(t))] + \lambda_m(t) = 0 && (18)\\
\frac{\partial \mathcal{L}}{\partial Q_m} &= -C_m'(Q_m(t)) + \lambda_m(t) = 0 && (19)\\
\dot{\lambda}_m(t) - r\lambda_m(t) &= -\frac{\partial \mathcal{L}}{\partial B_m} = -\ \phi_m(t),\ \phi_m \geq 0,\ \ \phi_m B_m = 0 && (20)
\end{aligned}$$

From (18) and (19) we obtain

$$[C_f'(Q_f(t)) - C_f''(Q_f(t))X(t) - C_m'(Q_m(t))] = 0 \tag{21}$$

Eq. (21) shows that marginal revenues ($MR = C_f' - C_f''X$)[10] and marginal costs ($MC = C_m'$) are equal in all periods. Combining (19) and (20) shows that the marginal cost must rise at the rate of interest unless the bank is zero.

The characterization of the price path $P(t)$ during the banking period can be obtained from (18). Taking the derivative with respect to time, letting $\dot{\lambda}_m = r\lambda_m$, and rearranging yields

$$\dot{P}(t) = rP(t) + \dot{C}_f''(Q_f(t))X(t) - rC_f''(Q_f(t))X(t) - C_f''\dot{Q}_f(t) \tag{22}$$

Although it is not possible to provide a precise characterization of $P(t)$ without a functional form for $C_f(\cdot)$, we can provide a general characterization about how it evolves over time. Because there are no storage costs, we know that arbitrage prevents prices from increasing at anything higher than the discount rate r, i.e., $\dot{P}(t)/P(t) \leq r$. We also know that since marginal cost $C_m'(Q_m(t))$ must increase at the rate of interest (otherwise the large firm could rearrange its reduction pattern and save on compliance costs), we must also have that marginal revenues $C_f'(Q_f(t)) - C_f''(Q_f(t))X(t)$ rise at the rate of interest. Provided that $C_f'(Q_f(t) = P(t)$ and $X(t) = U_f(t) - Q_f(t) - A_f(t)$, marginal revenues R_m' can be re-written as

$$R_m'(t) = P(t)\left(1 - \frac{1}{\epsilon(t)}\right) \tag{23}$$

[10]Note that since $C_f''(Q_f(t)) = \partial P(Q_f(t))/\partial Q_f(t)$ and large firm's revenues are $P(X(t))X(t)$, its marginal revenues can be expressed as $P(t) - P'(X(t))X(t)$.

where

$$\epsilon(t) = \left(\frac{dC'_f(Q_f(t))}{dQ_f(t)} \frac{X(t)}{P(t)} \right)^{-1} = -\frac{dX(t)}{dP(t)} \frac{P(t)}{X(t)} \tag{24}$$

is the fringe demand elasticity (defined positive). Since ϵ increases with price because $C'_f(0) = 0$ and $C'_f(U_f) = \bar{P} < \infty$, (23) indicates that in equilibrium prices $P(t)$ increase at lower rate than $R'_m(t)$.[11] Consistent with Salant (1976) and Newbery (1981), when the fringe has no stock, it is optimal for the large firm to let prices rise at rate strictly lower than the discount rate, i.e., $\dot{P}(t)/P(t) < r$.[12]

Both the competitive and monopoly price paths are depicted in Figure 1. The time at which the large firm's bank exhausts is denoted by T^m. Because of the exhaustion condition (8) and $\dot{P}(t)/P(t) < r$, the monopoly path must start above the competitive price and must cross it from above before exhaustion. Figure 1 also shows, as in the exhaustible resource literature, that the large firm extends the banking period compared to what would have been observed under perfect competition. The shape of the quantity path $Q(t) = Q_f(t) + Q_m(t)$, which can be derived from the price path, eq. (21) and the exhaustion condition, is similar to that of the price path.

It is straightforward to extend the above analysis to the case in which θ is such that the large firm is also able to exercise market power after the bank has been exhausted (i.e., after T). If the large firm is a net seller in the after-banking market, the choke price $P^m \equiv P(T^m)$ will be higher than $P^*(T^*)$ but the rate of price increase will be still lower than the rate of interest. The "choke" price can be readily estimated by solving (21) subject to (14) and $Q_m(T^m) = U(T^m) - A_L - Q_f(T^m)$. If, on the other hand, the firm is a net buyer in the after-banking market, $P^m < P^*(T^*)$ and, again, $\dot{P}(t)/P(t) < r$. In summ, when the entire stock is allocated to the large firm, the problem of time inconsistency does not arise and, hence, the equilibrium solution can be found with conventional dynamic programming techniques.

[11] Note that the monopoly and competitive solution would coincide if the fringe's demand for permits (which derives directly from the marginal cost curve) were isoelastic (see Stiglitz, 1976). Such demand structure, however, is not possible here because both the number of permits demanded at $P = 0$ by any fringe member is finite (equal to its unrestricted or counterfactual emissions) and the demand for permits falls to zero above some $\bar{P}$.

[12] Note that if marginal cost curves are linear, $\dot{Q}_f(t)/Q_f(t) = \dot{P}(t)/P(t)$. Replacing this into (22) leads to $\dot{P}(t)/P(t) = r/2$.

3.2 Large firm with no stock

Let us now consider the opposite case in which the fringe holds all the stock, i.e., $\mu = 1$, and θ is such that the large producer is a seller of permits at the end of the banking period.[13] Since the large firm receives no stock, one might argue that in equilibrium the large firm builds no bank and the fringe's bank expires at the choke price P^m. This equilibrium candidate would exhibit competitive pricing during the banking period in the sense that prices would grow at the rate of interest all the way up to the choke price P^m.

If the large firm could sign binding contracts (or alternatively, actions were restricted to a one-time move at the beginning of the game), we show in the Appendix that the above candidate is indeed the equilibrium of the game. Part of the explanation of why competitive pricing during the banking period is atractive for the large firm is because it speeds up the exhaustion of the stock of permits moving forward its after-banking profits (i.e., profits after T^m). It is useful for later discussion to notice that this equilibrium has the large firm as a net buyer during the first part of the banking period and as a net seller during the latter part. Then, there is a time t' before the end of the banking period where the large firm does not trade permits but just covers its emissions with its own allocation.[14]

In the absence of binding contracts (i.e., where players make their decisions period after period), however, competitive pricing during the banking period is not time consistent. Hence, it cannot be part of the equilibrium of a game in which small firms have rational expectations, and therefore, correctly anticipate the deviation incentives that the large firm will face in the future. This time consistency problem arises because the large firm becomes a net permits seller before the end of the banking period.

More precisely, if the players were to follow competitive pricing during the banking period there will be a time $t'' > t'$ at which the fringe stock is so small that the fringe

[13] Same qualitative results apply if the dominant firm is a monopsonist at the end of the banking period (the end or "choke" price will be lower than the competitive price).

[14] The latter does not need be the case if we allow for a one-time transaction at the beginning of the game. But this would only exacerbate the time inconsistency problems that we identify below because the large agent will find itself with a large stock of permits right after the one-time transaction.

demand would absorb this stock in a period if the large agent unexpectedly revised its sales path and refused to supply.[15] The large agent could then bank the supply it was supposed to bring to the market that period and reoptimize its sales path so as to follow a monopoly path such as the one described in the previous section (i.e., there will be a price jump followed by prices growing at rate lower than r). Given that at t'' (and thereafter) the large agent is a net seller irrespective of whether it follows competitive pricing or monopoly pricing, it should be clear that it does not have incentives to stick to competitive pricing all the way to the end of the banking period. Consequently, the (time-consistent) equilibrium path must necessarily have the large firm conserving enough permits to keep a stock that it will consume and sell after all firms in the fringe have exhausted theirs, regardless whether it received some of the stock $(A_H - A_L)T_0$ or not.

Without introducing more structure to the game, this characterization of the equilibrium solution gives us enough information to qualitatively depict equilibrium price and quantity paths, $P(t)$ and $Q(t)$, respectively. As in Salant (1976) and Newbery (1981), there will be three distinctive phases. As shown in Figures 2 and 3, during phase A, price $P(t)$ rises at the interest rate r and quantities $Q_f(t)$ and $Q_m(t)$ rise accordingly. During this phase, which is commonly denoted as the competitive phase, the fringe is a net seller and, hence, the large firm is a net buyer of permits.[16] During the competitive phase the fringe consumes its stock and the large firms builds its own, so at T^f the fringe's bank is exhausted but the large firm's bank is positive.

In phase B, which is commonly denoted as the monopoly phase, $P(t)$ rises at a rate strictly lower than r and $Q(t) = Q_f(t) + Q_m(t)$ also grows at a rate strictly lower than under the competitive case since $Q_f(t)$ follows the price path. Furthermore, from the full compliance (or exhaustion) condition, the observed path $Q(t)$ crosses the competitive path $Q^*(t)$ sometime during this phase. At T^m, the leader's bank is exhausted; after which prices remain constant at $P^m > P^*$. Finally, at T^m players reach phase C, i.e., the long-run equilibrium phase.

[15] Note that this cannot happen right at t' because the price jump that will follow the supply shortage will lead the fringe to abate more and, hence, consume fewer permits.

[16] Note that for the fringe to be indifferent between selling permits today and tommorrow prices must grow at the rate of interest; the same does not apply when the fringe is buying permits.

Understanding that finding the exact equilibrium values of T^f and T^m requires more structure to game (e.g., adopt a Cournot approach as in Salant (1976)), we have demonstrated that the exercise of market power in the after-banking market "allows" the large firm to extend its dominant position to the banking period even if it receives none of the permits stock $(A_H - A_L)T_0$ (By extension of the dominant position we mean that there is a monopoly phase during the banking period). The extension of this dominant position is not something that the large firm likes in the first place, but it is a consequence of the time inconsistency associated to competitive pricing during the banking period (followed by a dominant seller). As the stock allocation received by the large firm increases, the time-consistency problem diminishes and with that the large firm's losses from following an equilibrium path different from the binding contract path. Furthermore, the problem may dissapear completely even before the entire stock is allocated to the large firm.

4 Final Remarks

We have investigated the effect of market power on the equilibrium path of an emission permits market in which firms bank current permits for gradual use in later periods. In particular, we have studied the market equilibrium for a large (potentially dominant) firm and a competitive fringe with rational expectations. Each firm receives a stock (i.e., bank) and flow allocation of permits. We identified permits allocations for which the binding contract solution (i.e., full-commitment solution) is time inconsistent in the sense that what is optimal for the large firm today is no longer optimal tomorrow. Without imposing any particular game structure, we provided a characterization of the time-consistent equilibrium solution and explain how it differs from the binding contract solution. We showed, for example, that if the large firm does not receive any stock allocation but receives a flow allocation that allows it to enjoy a dominant position in the after-banking market, this dominant position gets extended to the banking period despite the large firm is worse off; otherwise the equilibrium path would be time inconsistent.

5 Appendix: the commitment solution in case $\mu = 0$

The solution to the following problem characterizes the behavior of the fringe (dependence on time is suppressed):

$$\max_{Q_f(t), X_f} \int_0^\infty \left(PX_f - C_f(Q_f)\right) e^{-rt} dt \tag{25}$$

$$\text{s.t. } \dot{B}_f = A_f + Q_f + X_f - U_f \qquad [\lambda_f] \tag{26}$$

$$B_f \geq 0 \qquad [\phi_f] \tag{27}$$

$$B_f(0) > 0, \{P(t) : t \in [0, \infty)\} \text{ given.} \tag{28}$$

Variable X_f is the sales by the fringe and, in equilibrium, satisfies $X_f = -X_m$. The current-value Lagrangian is $L = \mathcal{H} + \phi_f B_f$ where the current-value Hamiltonian is $\mathcal{H} = PX_f - C_f(Q_f) + \lambda_f(A_f + Q_f + X_f - U_f)$. The necessary conditions are summarized by the system

$$\dot{B}_f = A_f + Q_f(P) + X_f - U_f \tag{29}$$

$$\dot{P} = rP - \phi_f \tag{30}$$

where $\phi_f = 0$ if $B_f(t) > 0$ and $\phi_f B_f(t) = 0$.

Since the large agent is explicitly Stackelberg leader, he can take the system (29)-(30) and $X_f = -X_m$ as constraints and solve

$$\max_{Q_m, X_m} \int_0^\infty [PX_m - C_m(Q_m)] \mathrm{e}^{-rt} dt \tag{31}$$

$$\text{s.t.} \quad \dot{B}_m = A_m + Q_m - X_m - U_m \qquad [\lambda_m] \tag{32}$$

$$\dot{B}_f = A_f + Q_f(P) + X_m - U_f \qquad [\beta] \tag{33}$$

$$\dot{P} = rP - \phi_f \qquad [\gamma] \tag{34}$$

$$B_m \geq 0 \qquad [\phi_m] \tag{35}$$

$$B_m(0) = 0, B_f(0) > 0, P(0) \text{ free} \tag{36}$$

The current-value Lagrangian for this problem is $L = \mathcal{H} + \phi_m B_m$, where $\mathcal{H} = PX_m - C_m(Q_m) + \lambda_m[A_m + Q_m - X - U_m] + \beta[A_f + Q_f + X - U_f] + \gamma[rP - \phi_f\ ]$ is the current-value Hamiltonian. Necessary conditions include:

$$P - \lambda_m + \beta = 0 \tag{37}$$

$$-C_m'(Q_m) + \lambda_m = 0 \tag{38}$$

$$\dot{\lambda}_m = r\lambda_m - \phi_m, \phi_m \geq 0, \phi_m B_m = 0 \tag{39}$$

$$\dot{\beta} = r\beta \tag{40}$$

$$\dot{\gamma} = r\gamma - X_m - \beta Q_f'(P\ ) - r\gamma \tag{41}$$

$$\gamma(0) = 0. \tag{42}$$

The initial condition for (34) is lacking which necessitates the condition (42) (see Simaan and Cruz, 1973). The commitment solution along which prices grow at the rate of interest and the large firm does not accumulate a bank can be characterized as follows: for all $t \in [0, T^m]$,

$$C_m'(Q_m) = C_m'(Q_m(0))e^{rt} < P(0)e^{rt}$$
$$C_f'(Q_f) = C_f'(Q_f(0))e^{rt} = P(0)e^{rt}$$
$$X_m = A_m + Q_m - U_m$$
$$B_m = 0$$
$$B_f(T^m) = 0$$

The initial values $\{Q_m(0),Q_f(0)\}$and time T^m can be found such that the stock $B_f(0)$ is exhausted exactly at $t = T^m$ and $P(0)e^{rT^m} = P^m$. The solution satisfies (37)-(42) for $t < T^m$ if we set $\phi_m = 0$. For $t \geq T^m$, we should set $\phi_m = r\lambda_m > 0$ and $\phi_f = rP^m > 0$.

Time-consistency would require that $\gamma(t) = 0$ for all t (see Simaan and Cruz, 1973), which is clearly not the case: since $\beta = C_m'(Q_m) - P < 0, X_m \neq 0$, and $Q_f'(P) > 0$, the shadow value of the constraint (34) is changing over time, $\dot{\gamma} = -X_m - \beta Q_f'(P) = -A_m - Q_m + U_m - [C_m'(Q_m) - P]Q_f'(P\ ) \neq 0$. If the leader had a chance to reoptimize at some $t' > 0$, he would like to set $\gamma(t') = 0$ and possibly choose $P(t') \neq P(0)e^{rt'}$. This revision for the price must occur when B_f is small enough, as we argue in the text.

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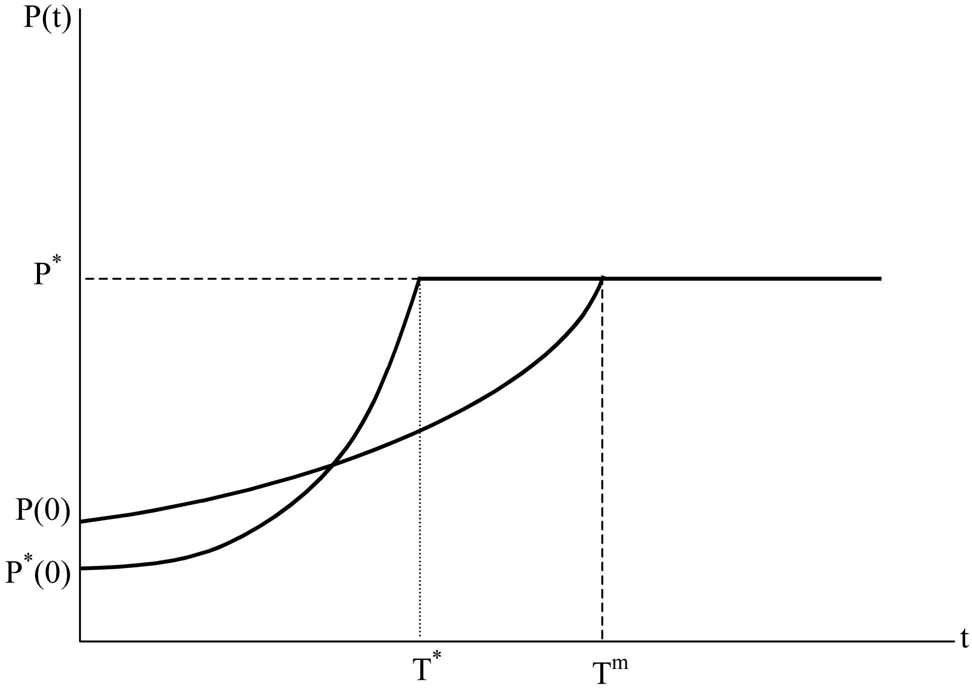

Figure 1: Price path for a dominant firm with all the permits stock

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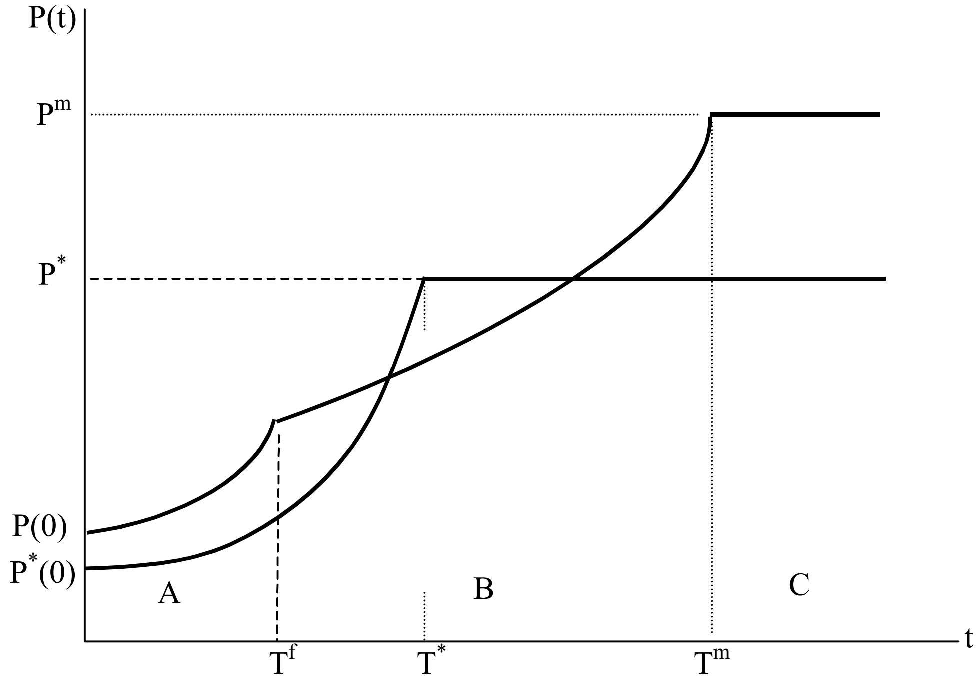

Figure 2: Effect of market power on the price path

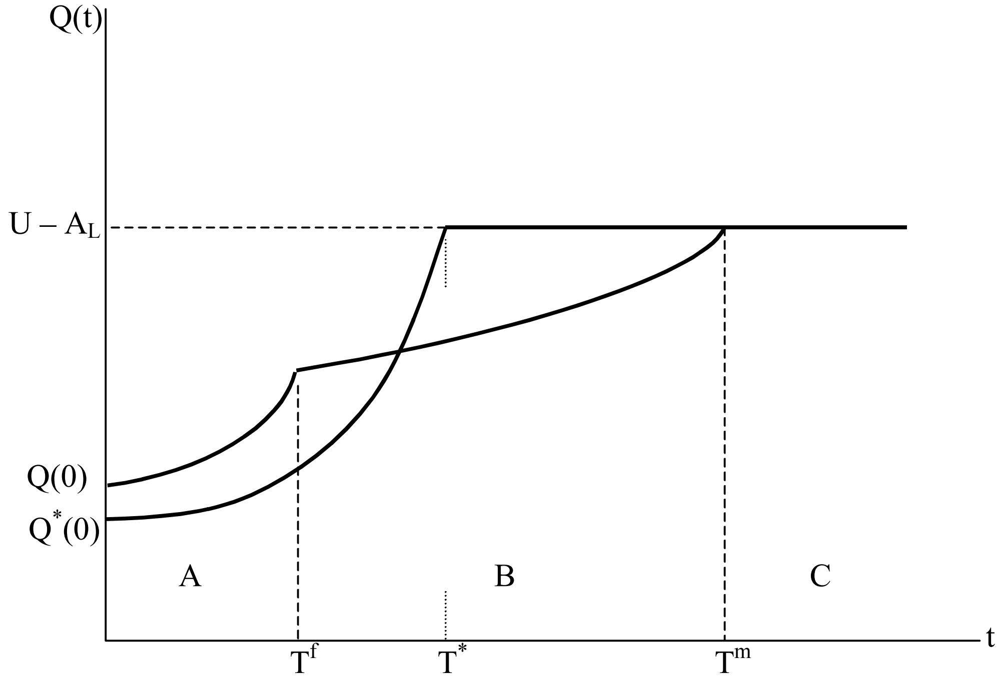

Figure 3: Effect of market power on the abatement (quantity) path