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Investment Model for Renewable Electricity Systems (IMRES): an Electricity Generation Capacity Expansion Formulation with Unit Commitment Constraints

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Investment Model for Renewable Electricity Systems (IMRES): an Electricity Generation Capacity Expansion Formulation with Unit Commitment Constraints

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Abstract

This paper describes the formulation of IMRES: a static electricity generation capacity expansion model with detailed unit commitment constraints in which decisions pertaining to investment, unit commitment and energy dispatch are taken jointly. The purpose of this model is to determine the minimum cost electricity generation capacity mix in systems with a high penetration of intermittent renewable energy resources, while accounting for the operational dynamics of thermal units and their impact on total system cost. The model is formulated as a 0-1 MILP, taking capacity decisions at the individual power plant level, operational decisions with an hourly time resolution, and accounting for techno-economic considerations such as ramp constraints, startup costs, and minimum stable outputs of thermal plants, among others. Additionally, the model offers the possibility of introducing in the system other dynamic elements such as storage or demand side management, that facilitate renewable integration and reduce the total system cost, as well as a limit on carbon emissions.

1 Introduction

The *Investment Model for Renewable Electricity Systems* (IMRES) aims at determining the minimum cost electricity generation capacity mix in systems with a high penetration of intermittent renewable resources[1]. The model uses a capacity expansion formulation with embedded unit commitment constraints, integrating the operational dynamics induced by a high net load variability characteristic of this type of power systems. The main advantage of this formulation is that its objective function accounts for capital costs, variable costs, as well as the costs associated with a more intense cycling regime. In addition, the model also includes detailed unit commitment constraints which relate the technical characteristics of thermal units to total system cost and capacity decisions [2].

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[1]The adjective intermittent refers to resources with high variability and low predictability, like wind power and solar photovoltaics.

From a centralized planning perspective, IMRES can help to determine the future investments needed to supply a future electricity demand at minimum cost. In the context of liberalized markets, IMRES can be used by regulators for *indicative energy planning* [3] in order to establish a long-term vision of where efficient markets should lead to.

Classic capacity expansion models such as screening curves models [1] only assess the economic trade-off between generating technologies with a high capital cost and a low variable costs, and technologies with lower capital cost but high variable cost. This approach does not account for other important factors such as start-up costs, the indivisibility of units, minimum stable output levels, ramp limits and reserve needs. The method presented with IMRES combines the economic assessment performed by classic approaches with the techno-economic analysis of unit commitment models, allowing a detailed study of the impact of technical constraints on cost.

IMRES can be viewed as a two component model (Table 1): the primary component decides which power plants to build; and the secondary component accounts for the operational decisions at the power plants. In its original form, renewables and storage capacity are taken as parameters, while capacity and operational decisions concerning the thermal capacity mix are treated as variables. However, as it will be shown later in this section, IMRES also allows to treat renewable capacities as decision variables, at the expense of a larger computational complexity and computing time.

Table 1: IMRES' General Structure and Reference to Equations

Minimize	Investment costs + Operational costs	(1)
s.t.:	operate-if-built coupling constraint	(2)
	demand balance equation	(3)
	unit commitment constraints	(4-11)
	renewable energy and emissions constraints	(12-15)
	storage constraints	(16-22)
	demand-side management constraints	(23-27)
	reserves constraints	(28-34)
	non-negativity/binary constraints	(35-47)

The time interval evaluated in IMRES is one year, divided into one-hour periods and representing a future year (e.g.: in 2050). In this sense, IMRES is a *static* model because its objective is not to determine when investments should take place over time, but rather to produce a snapshot of the minimum cost generation capacity mix under some pre-specified future conditions.

2 Notation

2.1 Indices and Sets

Table 2: Model Indices and Sets

$i \in \mathcal{I}$, where $\mathcal{I}$ is the set of generating units that can be potentially built
$j \in \mathcal{J}$, where $\mathcal{J}$ is the set of hours in the data series
$j' \in \mathcal{J}$, where $\mathcal{J}$ is the set of hours in the data series
$\mathcal{W} \subset \mathcal{I}$, where $\mathcal{W}$ is the subset for wind capacity
$\mathcal{S} \subset \mathcal{I}$, where $\mathcal{S}$ is the subset for solar photovoltaic capacity
$\mathcal{T} \subset \mathcal{I}$, where $\mathcal{T}$ is the subset of thermal power units (nuclear, coal, CCGTs and OCGTs)
$\mathcal{G} \subset \mathcal{I}$, where $\mathcal{G}$ is the subset of gas peaking units (OCGTs)
$\mathcal{N} \subset \mathcal{I}$, where $\mathcal{N}$ is the subset of nuclear units

2.2 Variables

Table 3: Model Variables

$y_{i \in \mathcal{T}} \in \{0, 1\}$	building decision for thermal power plant $i \in \mathcal{T}$
$y_{i \in \mathcal{W} \cup \mathcal{S}} \in \mathbb{R}_+$	building decision for renewable capacity $i \in \mathcal{W} \cup \mathcal{S}$
$x_{ij} \in \mathbb{R}_+$	output power of plant i during hour j
$u_{ij} \in \{0, 1\}$	commitment state of power plant i during hour j
$z_{ij} \in \{0, 1\}$	start-up decision of power plant i at hour j
$v_{ij} \in \mathbb{R}_+$	shut-down decision of power plant i at hour j
$w_{ij} \in \mathbb{R}_+$	output power over minimum output of plant i during hour j
$f_j \in \{0, 1\}$	charging/discharging state of the storage unit during hour j
$x_j^{STOR} \in \mathbb{R}_+$	output power of the storage unit during hour j
$l_j^{STOR} \in \mathbb{R}_+$	energy capacity of the storage unit during hour j
$p_j^{STOR} \in \mathbb{R}_+$	energy inflows to the storage unit (or hydro reservoir) during hour j
$x_j^{DSM} \in \mathbb{R}_+$	energy 'put back' from demand side management during hour j
$l_j^{DSM} \in \mathbb{R}_+$	total energy withheld in demand side management during hour j
$p_j^{DSM} \in \mathbb{R}_+$	energy withheld in demand side management during hour j
$r_j^{PRI\_U}$	upwards primary reserve requirement in hour j
$r_j^{PRI\_D}$	downwards primary reserve requirement in hour j
$r_j^{SEC\_U}$	upwards secondary reserve requirement in hour j
$r_j^{SEC\_D}$	downwards secondary reserve requirement in hour j
r_j^{TER}	tertiary reserve requirement in hour j
n_j	non-served energy in hour j

2.3 Parameters

Table 4: Model Parameters

D_j	electricity demand in hour j [GWh]
CF_j^{WIND}	capacity factor of wind power during hour j [%]
CF_j^{SOLAR}	capacity factor of solar power during hour j [%]
$C_{i\in\mathcal{T}}^{FOM}$	fixed cost for operations and maintenance for unit i [M$/year]
$C_{i\in\mathcal{W}\cup\mathcal{S}}^{FOM}$	fixed cost for operations and maintenance for renewable capacity i [M$/GW-year]
$C_{i\in\mathcal{T}}^{FCAP}$	annualized fixed cost for unit i [M$/year]
$C_{i\in\mathcal{W}\cup\mathcal{S}}^{FCAP}$	annualized fixed cost for renewable capacity i [M$/GW-year]
C_i^{VOM}	variable cost for operations and maintenance for unit i [M$/GWh]
C_i^{FUEL}	fuel cost per unit power output from unit i [M$/GWh]
C_i^{STUP}	start-up cost for unit i [M$/startup]
$VOLL$	value of lost load [M$/GWh]
$Y_{i\in\mathcal{W}\cup\mathcal{S}}$	externally fixed renewable capacity [GW], for runs with fixed renewable capacity
$\bar{E}$	limit on carbon emissions [tons]
E_i	carbon emissions per unit power output from power plant i [tons/GWh]
π^{CO_2}	price of carbon emissions [M$/ton]
$\bar{P}_i$	maximum power output for unit i [GW]
$\underline{P}_i$	minimum stable power output for unit i [GW]
$\bar{P}^{IN}$	maximum input power of storage unit (pumping capacity, in hydro) [GW]
$\bar{P}^{OUT}$	maximum output power of storage unit [GW]
S	self-discharge rate of the energy storage unit [p.u.]
$\bar{L}$	energy capacity of the aggregated storage unit [GWh]
$\underline{L}$	minimum energy level of the aggregated storage unit [GWh]
I_j	hydro inflows during hour j [GWh] (only if storage is pumped-hydro with inflows)
ϵ	efficiency of the storage pumping unit or charging efficiency [p.u.]
k	efficiency of the storage generating unit or discharging efficiency [p.u.]
H	maximum time to ‘put back’ energy withheld in DSM [hrs]
$\bar{P}^{DSM^+}$	maximum hourly rate at which demand can be ‘put back’ [GW]
$\bar{P}^{DSM^-}$	maximum hourly rate at which demand can be withheld [GW]
$\bar{R}_i^U$	maximum up-ramping capability for unit i [GW/hr]
$\bar{R}_i^D$	maximum down-ramping capability for unit i [GW/hr]
$\bar{M}_i^U$	minimum up time for unit i [hrs]
$\bar{M}_i^D$	minimum down time for unit i [hrs]
K^U	up reserves constant, when renewable capacity is fixed exogenously
K^D	down reserves constant, when renewable capacity is fixed exogenously
α	up primary reserves demand factor, when renewable capacity is fixed exogenously
β	down primary reserves demand factor, when renewable capacity is fixed exogenously
γ	up secondary reserves wind factor, when renewable capacity is fixed exogenously
δ	up primary reserves solar factor, when renewable capacity is fixed exogenously
ξ	down secondary reserves wind factor, when renewable capacity is fixed exogenously

θ	down secondary reserves solar factor, when renewable capacity is fixed exogenously
ζ	tertiary reserves wind factor, when renewable capacity is fixed exogenously
η	tertiary reserves solar factor, when renewable capacity is fixed exogenously
$K^{U'}$	up reserves constant, when renewable capacity is endogenous
$K^{D'}$	down reserves constant, when renewable capacity is endogenous
α'	up primary reserves demand factor, when renewable capacity is endogenous
β'	down primary reserves demand factor, when renewable capacity is endogenous
γ'	up secondary reserves wind factor, when renewable capacity is endogenous
δ'	up primary reserves solar factor, when renewable capacity is endogenous
ξ'	down secondary reserves wind factor, when renewable capacity is endogenous
θ'	down secondary reserves solar factor, when renewable capacity is endogenous
ζ'	tertiary reserves wind factor, when renewable capacity is endogenous
η'	tertiary reserves solar factor, when renewable capacity is endogenous

3 Model Formulation

$$\min_{\substack{x,y,z\\u,n,f,p}} \sum_{i\in\mathcal{I}}(C_i^{FCAP}+C_i^{FOM})y_i + \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}((C_i^{VOM}+C_i^{FUEL}+\pi^{CO_2}E_i)x_{ij}+C_i^{STUP}z_{ij}) + \sum_{j\in\mathcal{J}} VOLL\ n_j \tag{1}$$

$$\text{s.t.}\quad x_{ij} \le \bar{P}_i\, y_i \qquad \forall i\in\mathcal{T},\ \forall j\in\mathcal{J} \tag{2}$$

$$\sum_{i\in\mathcal{I}} x_{ij} + x_j^{STOR} + p_j^{DSM} + n_j = D_j + p_j^{STOR} + x_j^{DSM} \qquad \forall j\in\mathcal{J} \tag{3}$$

$$u_{ij} - u_{ij-1} = z_{ij} - v_{ij} \qquad \forall i\in\mathcal{T},\ \forall j\in\mathcal{J}\setminus\{1\} \tag{4}$$

$$w_{ij} = x_{ij} - u_{ij}\, \underline{P}_i \qquad \forall i\in\mathcal{T},\ \forall j\in\mathcal{J} \tag{5}$$

$$w_{ij} - w_{ij-1} \le \bar{R}_i^U \qquad \forall i\in\mathcal{T},\ \forall j\in\mathcal{J}\setminus\{1\} \tag{6}$$

$$w_{ij-1} - w_{ij} \le \bar{R}_i^D \qquad \forall i\in\mathcal{T},\ \forall j\in\mathcal{J}\setminus\{1\} \tag{7}$$

$$w_{ij} \le u_{ij}(\bar{P}_i - \underline{P}_i) \qquad \forall i\in\mathcal{T},\ \forall j\in\mathcal{J} \tag{8}$$

$$u_{ij} \ge \sum_{j'>j-\bar{M}_i^U}^{j} z_{tj'} \qquad \forall i\in\mathcal{T},\ \forall j\in\mathcal{J} \tag{9}$$

$$1 - u_{ij} \ge \sum_{j'>j-\bar{M}_i^D}^{j} v_{tj'} \qquad \forall i\in\mathcal{T},\ \forall j\in\mathcal{J} \tag{10}$$

$$u_{ij} \ge y_i \qquad \forall i\in\mathcal{N}, \forall j\in\mathcal{J} \tag{11}$$

$$y_i = Y_i \qquad \forall i\in\mathcal{W}\cup\mathcal{S} \tag{12}$$

$$x_{ij} \le y_i\, CF_j^{WIND} \qquad \forall i\in\mathcal{W},\ \forall j\in\mathcal{J} \tag{13}$$

$$x_{ij} \le y_i\, CF_j^{SOLAR} \qquad \forall i\in\mathcal{S},\ \forall j\in\mathcal{J} \tag{14}$$

$$\sum_{i\in\mathcal{I}}\left(E_i \sum_{j\in\mathcal{J}} x_{ij}\right) \le \bar{E} \tag{15}$$

$$l_j^{STOR} = (1-S)\, l_{j-1}^{STOR} - x_j^{STOR}/k + I_j^{STOR} + \epsilon\, p_j^{STOR} \qquad \forall j \in \mathcal{J} \qquad (16)$$

$$x_j^{STOR} \leq f_j \bar{P}^{OUT} \qquad \forall j \in \mathcal{J} \qquad (17)$$

$$x_j^{STOR} \leq k\, l_j^{STOR} \qquad \forall j \in \mathcal{J} \qquad (18)$$

$$p_j^{STOR} \leq (1-f_j)\bar{P}^{IN} \qquad \forall j \in \mathcal{J} \qquad (19)$$

$$p_j^{STOR} \leq (\bar{L} - l_j^{STOR})/\epsilon \qquad \forall j \in \mathcal{J} \qquad (20)$$

$$l_j^{STOR} \leq \bar{L} \qquad \forall j \in \mathcal{J} \qquad (21)$$

$$l_j^{STOR} \geq \underline{L} \qquad \forall j \in \mathcal{J} \qquad (22)$$

$$\sum_{j'>j}^{j+H} x_{j'}^{DSM} \geq l_j^{DSM} \qquad \forall j \in \mathcal{J} \qquad (23)$$

$$l_j^{DSM} = l_{j-1}^{DSM} - x_j^{DSM} + p_j^{DSM} \qquad \forall j \in \mathcal{J} \qquad (24)$$

$$l_j^{DSM} \leq \bar{P}^{DSM^-} H \qquad \forall j \in \mathcal{J} \qquad (25)$$

$$x_j^{DSM} \leq \bar{P}^{DSM^+} \qquad \forall j \in \mathcal{J} \qquad (26)$$

$$p_j^{DSM} \leq \bar{P}^{DSM^-} \qquad \forall j \in \mathcal{J} \qquad (27)$$

$$r_j^{PRI\_U} \geq \max_{i \in \mathcal{T}} \bar{P}_i\, y_i \qquad \forall j \in \mathcal{J} \qquad (28)$$

$$r_j^{SEC\_U} \geq K^U[\alpha D_j^2 + \gamma(CF_j^{WIND}\, Y_{i\in\mathcal{W}})^2 + \delta(CF_j^{SOLAR}\, Y_{i\in\mathcal{S}})^2]^{1/2} \qquad \forall j \in \mathcal{J} \qquad (29)$$

$$r_j^{SEC\_D} \geq K^D[\beta D_j^2 + \xi(CF_j^{WIND}\, Y_{i\in\mathcal{W}})^2 + \theta(CF_j^{SOLAR}\, Y_{i\in\mathcal{S}})^2]^{1/2} \qquad \forall j \in \mathcal{J} \qquad (30)$$

$$r_j^{TER} \geq \zeta\, Y_{i\in\mathcal{W}} + \eta\, Y_{i\in\mathcal{S}} \qquad \forall j \in J \qquad (31)$$

$$\sum_{i\in\mathcal{I}}(u_{ij}\ \bar{P}_i - x_{ij}) + f_j \bar{P}^{OUT} - x_j^{STOR} + p_j^{STOR} + \\ + \bar{P}^{DSM^-} + x_j^{DSM} - p_j^{DSM} \geq r_j^{PRI\_U} + r_j^{SEC\_U} \qquad \forall j \in \mathcal{J} \qquad (32)$$

$$\sum_{i\in\mathcal{I}}(x_{ij} - u_{i,j}\ \underline{P}_i) + (1-f_j)\bar{P}^{IN} + x_j^{STOR} - p_j^{STOR} + \\ + \bar{P}^{DSM^+} - x_j^{DSM} + p_j^{DSM} \geq r_j^{SEC\_D} \qquad \forall j \in \mathcal{J} \qquad (33)$$

$$\sum_{i\in\mathcal{I}}(y_i - u_{ij})\bar{P}_i) \geq r_j^{TER} \qquad \forall j \in \mathcal{J} \qquad (34)$$

$$x_{ij} \geq 0 \qquad \forall i \in \mathcal{I},\ \forall j \in \mathcal{J} \qquad (35)$$

$$v_{ij} \geq 0 \qquad \forall i \in \mathcal{I},\ \forall j \in \mathcal{J} \qquad (36)$$

$$w_{i,j} \geq 0 \qquad \forall i \in \mathcal{I},\ \forall j \in \mathcal{J} \qquad (37)$$

$$l_j^{STOR} \geq 0 \qquad \forall j \in \mathcal{J} \qquad (38)$$

$$l_j^{DSM} \geq 0 \qquad \forall j \in \mathcal{J} \qquad (39)$$

$$p_j^{STOR} \geq 0 \qquad \forall j \in \mathcal{J} \qquad (40)$$

$$p_j^{DSM} \geq 0 \qquad \forall j \in \mathcal{J} \qquad (41)$$

$$n_j \geq 0 \qquad \forall j \in \mathcal{J} \qquad (42)$$

$$y_i \in \{0,1\} \qquad \forall i \in \mathcal{T} \qquad (43)$$

$$y_i \in \mathbb{R}_+ \qquad \forall i \in \mathcal{W} \cup \mathcal{S} \qquad (44)$$

$$f_i \in \{0,1\} \qquad \forall i \in \mathcal{I} \quad (45)$$

$$u_{ij} \in \{0,1\} \qquad \forall i \in \mathcal{T},\ \forall j \in \mathcal{J} \quad (46)$$

$$z_{ij} \in \{0,1\} \qquad \forall i \in \mathcal{T},\ \forall j \in \mathcal{J} \quad (47)$$

4 Description of the Model

4.1 Indices and Sets

Two indices are used in this model: $i \in \mathcal{I}$ and $j \in \mathcal{J}$. $\mathcal{I}$ denotes the set of individual generating units that can be built, containing all the units from the different thermal technologies considered –nuclear, coal, combined cycle gas turbines (CCGTs), open cycle gas turbines (OCGTs)–, as well as wind and solar photovoltaic capacity. In addition, $\mathcal{W} \subset \mathcal{I}$ denotes the subset of wind capacity; $\mathcal{S} \subset \mathcal{I}$, denotes the subset of solar photovoltaic capacity; $\mathcal{T} \subset \mathcal{I}$, denotes the subset of thermal power units –nuclear, coal, CCGTs and OCGTs–; and $\mathcal{G} \subset \mathcal{I}$, denotes the subset of gas-fired power plants –CCGTs and OCGTs–. $\mathcal{J}$ denotes the set of hours in a year or, alternatively, the total number of hours contained in the weeks sampled, used in the unit commitment component of the model (see section 4.5 below).

Building decisions are modeled with binary variables $y_{i\in\mathcal{T}} \in \{0,1\}$ for thermal generators, and real variables $y_{i\in\mathcal{W}\cup\mathcal{S}} \in \mathbb{R}_+$ for renewable resources; unit commitment decisions are denoted by $u_{i,j} \in \{0,1\}$; start-up decisions are denoted by $z_{ij} \in \{0,1\}$; shut-down decisions are denoted by $v_{ij} \in \mathbb{R}_+$; power output decisions are denoted by $x_{ij} \in \mathbb{R}_+$; energy inflows to the storage unit are denoted by p_j^{STOR}; energy withheld through DSM is denoted by p_j^{DSM}; and non-served energy is denoted by $n_j \in \mathbb{R}_+$. An extra variable $w_{ij} \in \mathbb{R}_+$ (where $w_{ij} = x_{ij} - \underline{P}_i u_{ij}$), is introduced for all $i \in \mathcal{T}$ to separate the total output of each power plant between its minimum stable level and the remainder output level to facilitate the formulation of ramping rate constraints.

4.2 Objective Function

The objective function in this model (1) minimizes the total generation cost of the system, which is the sum of fixed costs (annualized capital cost plus fixed O&M: $C_i^{FCAP} + C_i^{FOM}$), variable costs[2] (fuel cost plus variable O&M: $C_i^{FUEL} + C_i^{VOM}$), start-up costs (C_i^{STUP}) and the value of lost load ($VOLL$). IMRES can be thought of as a two-component model: 1) a component deciding on individual plant and renewable capacity building decisions; and 2) a component deciding on the operation of the generating capacity, including startup, commitment and energy dispatch decisions. Note however that these components are considered jointly and the optimization is performed over the whole problem at once.

$$\min_{\substack{x,y,z\\u,n,f,p}} \quad \sum_{i\in\mathcal{I}} \left(C_i^{FCAP} + C_i^{FOM}\right) y_i + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \left(\left(C_i^{VOM} + C_i^{FUEL} + \pi^{CO_2} E_i\right) x_{ij} + C_i^{STUP} z_{ij}\right) + \sum_{j\in\mathcal{J}} VOLL\, n_j \tag{1}$$

[2]The present formulation uses an affine variable cost function that could be replaced in future implementations by a piecewise linear cost function to increase the accuracy of the representation of the plants' cost structure.

4.3 Net Load Approximation

IMRES is formulated as a high-dimension 0-1 MILP. Therefore, for a real large-sized power system with a peak load in the order of tens of gigawatts, a large number of candidate plants would be necessary to determine the optimal expansion solution such that it is not constrained by the number of candidate plants available for each generating technology. Additionally, solving the unit commitment for the full year with each feasible combination of candidate plants presents a huge computational challenge, as each candidate plant in the initial set would generate multiples of 8,760 operational binary variables. This can produce instances with millions of binary variables. Problems like this one are very hard to solve using commercial solvers such as CPLEX given their high dimensionality. In addition, traditional decomposition techniques such as Benders' cannot be used with this formulation as binary commitment variables in the second component (the subproblem in a decomposed formulation) interfere with closing the duality gap [8].

Nevertheless, the deterministic nature of IMRES and its dimensionality limitations do not preclude it from including the characteristic variability of power systems with high shares of renewables. This hurdle is overcome by introducing a four-week approximation of a one-year net load series to reflect the joint variability of demand and renewable energy output (note that longer time series could also be used in this process to account for a larger variability spectrum). For cases when IMRES is run with an exogenously-determined renewable capacity, the week selection method is based on choosing the four weeks that approximate the net load duration curve the best, using least-squares. A full description of this heuristic method, its performance and its comparative advantage against other selection methods is presented in [6].

Alternatively, when the optimal renewable capacity is determined endogenously by IMRES, other improved dimensionality reduction techniques that break the circularity problem in selecting weeks based on knowing ex-ante de $NLDC$ and the renewable capacity installed should be used with the proposed formulation. One possible technique is to select a set of weeks that produce a *robust* energy error across a number of different renewable energy capacity scenarios.

4.4 Value of Lost Load

In the same way as in other capacity expansion models, the value of lost load ($VOLL$) used in IMRES affects critically the total non-served energy in the system and building decisions of peaking units. The choice of $VOLL$ can attend to different criteria and there are several methods to determine its value.

To give an example, the rule of thumb for establishing reliability criteria in power systems is that there can only be at most one day with non-served energy over a time span of ten years. If we calculate the per year ratio of this limitation, we have a maximum average of 2.4 hours per year with non-served energy. In a classic capacity expansion model with screening curves [1], non-served energy can be introduced as an additional technology with fixed cost equal to zero and variable cost equal to the $VOLL$. The intersection between the cost function of non-served energy and the cost function of the peaking technology determines the number of hours in a year for which it is cheaper to curtail demand rather than supply the full peak. If the maximum number of hours with non-served energy is fixed by the reliability criteria of choice (for the criteria just discussed, 2.4 hours), we can use this number to derive an analytical expression for the $VOLL$[3]:

$$VOLL = \frac{C_i^{FCAP} + C_i^{FOM}}{2.4} + C_i^{VOM} + C_i^{FUEL}, \quad i \in \mathcal{G} \qquad [M\$/GWh] \tag{48}$$

Conventional values of lost load range between 1,000 and 10,000 $/MWh. However, the method just described typically produces values much higher than 10,000 $/MWh. For instance, for a peaking technology with a fixed annual cost of 100,000 $/MW, the resulting value of lost load calculated with screening curves is above 40,000 $/MWh.

Alternatively, we can assume that some demand is sensitive to price, avoiding the price going above some certain threshold below the values presented above. In reality, this is achieved through contracts with special customer groups that are willing to reduce their demand during peak hours, or with grid elements that can supply electricity on an ad-hoc basis. Elements within this category are emergency generators located in critical infrastructures and public facilities such as hospitals, government offices, etc, used for back-up power in case of blackouts, staying idle when the system is operating normally. These generators could potentially be used to deliver electricity when prices are high, without jeopardizing their back-up generator functionality. Typically, back-up generators are fueled with expensive diesel, and if they are used in the mode just described, the system $VOLL$ would take the value of the variable cost of these generators ($\sim$ 500 $/MWh).

4.5 Accounting for Unit Commitment

The main two decision components in IMRES (building and operating) are linked with a coupling constraint imposing the condition that only units that have been built can generate (2).

$$x_{ij} \leq \bar{P}_i \, y_i \qquad \forall i \in \mathcal{T}, \ \forall j \in \mathcal{J} \tag{2}$$

The model is subject to the classic constraints included in a unit commitment model: demand balance constraints; constraints on the commitment state; constraints on the minimum and maximum output of the plants; constraints on the ramping rates; and constraints on the minimum up and down times. The demand balance constraint (3) establishes the equilibrium between the load in the system and the total power generated for all the time periods considered, including the option of having non-served energy. This equation includes the possibility of having energy storage and demand side management (DSM) in the system (see sections 4.8 and 4.9).

$$\sum_{i \in \mathcal{I}} x_{ij} + x_j^{STOR} + p_j^{DSM} + n_j = D_j + p_j^{STOR} + x_j^{DSM} \qquad \forall j \in \mathcal{J} \tag{3}$$

State constraints (4) link commitment states with start-up and shut-down decisions. Note that even if $v_{i,j}$ has been defined in the positive real domain, it will only adopt binary values as the commitment states and start-up decisions are all binary.

$$u_{ij} - u_{ij-1} = z_{ij} - v_{ij} \qquad \forall i \in \mathcal{T}, \ \forall j \in \mathcal{J} \setminus \{1\} \tag{4}$$

[3] Note, however, that this calculation is only valid for the static case or for a case with monotonic demand growth. Under demand uncertainty, the $VOLL$ would need to be adjusted such that the expected operating profits of the units that are necessary to meet the required reliability criteria are equal to their investment cost.

Constraints on ramping rates (6-7) account for the physical limitations imposed by power plants' thermal and mechanical inertias. These equations are constructed using a set of auxiliary variables ($w_{i,j}, i \in \mathcal{T}, j \in \mathcal{J}$) (5) to prevent the constraints from becoming active when off-line power plants start-up and jump from zero output ($x_{ij} = 0$) to the minimum output ($x_{ij} = \underline{P}_i$), or when power plants shut down.

$$w_{ij} = x_{ij} - u_{ij}\ \underline{P}_i \qquad \forall i \in \mathcal{T},\ \forall j \in \mathcal{J} \tag{5}$$

$$w_{ij} - w_{ij-1} \leq \bar{R}_i^U \qquad \forall i \in \mathcal{T},\ \forall j \in \mathcal{J} \setminus \{1\} \tag{6}$$

$$w_{ij-1} - w_{ij} \leq \bar{R}_i^D \qquad \forall i \in \mathcal{T},\ \forall j \in \mathcal{J} \setminus \{1\} \tag{7}$$

Constraints accounting for the minimum and maximum output limits of the units (8) are thus defined in terms of the interval between each unit's minimum and maximum output levels:

$$w_{ij} \leq u_{ij} \left(\bar{P}_i - \underline{P}_i\right) \qquad \forall i \in \mathcal{T},\ \forall j \in \mathcal{J} \tag{8}$$

Constraints on the minimum up and down times are implemented following the formulation in [4]. In this formulation $\bar{M}_i^U$ and $\bar{M}_i^D$ represent the minimum time that a power plant has to remain on or off after a start-up or shut-down respectively, and $j' \in \mathcal{J}$ is an auxiliary index for the hours in the time series:

$$u_{ij} \geq \sum_{j' > j - \bar{M}_i^U}^{j} z_{tj'} \qquad \forall i \in \mathcal{T},\ \forall j \in \mathcal{J} \tag{9}$$

$$1 - u_{ij} \geq \sum_{j' > j - \bar{M}_i^D}^{j} v_{tj'} \qquad \forall i \in \mathcal{T},\ \forall j \in \mathcal{J} \tag{10}$$

Lastly, IMRES allows to selectively include a restriction on nuclear plants cycling, as it is done in some power systems for safety reasons. This constraint imposes the condition that all nuclear power plants built have to be permanently on-line (11).

$$u_{ij} \geq y_i \qquad \forall i \in \mathcal{N},\ \forall j \in \mathcal{J} \tag{11}$$

4.6 Accounting for Renewables

Wind and solar PV capacity can be treated in the model either as variables ($y_i,\ i \in \mathcal{W} \cup \mathcal{S}$) or as parameters, introducing the extra constraint ($y_i = Y_i,\ i \in \mathcal{W} \cup \mathcal{S}$) depending on the analysis. Wind and solar energy output is treated as a function of each technology's capacity factor and the total wind and solar capacity deployed. Capacity factors (CF) reflect the availability of wind or solar resources for a specific hour at a certain location. Hence, for a particular hour of the year, the output of the total wind or solar power in our system will be determined by the product of each technology's total capacity installed and its respective CF (49-50).

$$x_{ij} = y_i\ CF_j^{WIND} \qquad \forall i \in \mathcal{W},\ \forall j \in \mathcal{J} \tag{49}$$

$$x_{ij} = y_i\ CF_j^{SOLAR} \qquad \forall i \in \mathcal{S},\ \forall j \in \mathcal{J} \tag{50}$$

The model also offers the possibility of introducing curtailment as an extra degree of freedom to guarantee that generation exactly meets hourly demand. For both wind and solar power, this feature is implemented through substituting the equality constraints (49-50) by inequality constraints (13-14):

$$x_{ij} \leq y_i \, CF_j^{WIND} \qquad \forall i \in \mathcal{W}, \ \forall j \in \mathcal{J} \tag{13}$$

$$x_{ij} \leq y_i \, CF_j^{SOLAR} \qquad \forall i \in \mathcal{S}, \ \forall j \in \mathcal{J} \tag{14}$$

4.7 Accounting for Carbon Emissions

Each unit has a parameter E_i reflecting an estimation of its specific carbon emissions in tCO_2/GWh associated with producing electricity. The effect of these carbon emissions is modeled by either introducing a carbon price π^{CO_2} [M\$/tnCO_2] in the objective function that affects the total variable cost of each technology, or by introducing a cap on emissions $\bar{E}$, implemented as an extra constraint (15).

$$\sum_{i \in \mathcal{I}} \left(E_i \sum_{j \in \mathcal{J}} x_{ij} \right) \leq \bar{E} \tag{15}$$

Note that if the latter option is used, the dual variable of the emissions constraints can be interpreted as the carbon price that units should be charged in a carbon price scenario with an emissions target of $\bar{E}$ equivalent tons of CO_2 .

4.8 Accounting for Hydro and Storage Capacity

IMRES aggregates pumped-hydro storage and other forms of energy storage capacity (mostly electro-chemical and chemical) in a single energy storage unit with the ability to increase and release the energy stored. The formulation used in this model is similar to that used in Cerisola et al. [7], where a constant energy-flow ratio for each hydro unit is considered and the storage level is expressed in terms of stored energy in GWh. The equation representing the storage level of the reservoir is as follows:

$$l_j^{STOR} = (1 - S) \, l_{j-1}^{STOR} - x_j^{STOR}/k + I_j^{STOR} + \epsilon \, p_j^{STOR} \qquad \forall j \in \mathcal{J} \tag{16}$$

–where $l_j^{STOR} \in \mathbb{R}_+$ represents the energy level of the storage unit at hour j; $p_j^{STOR} \in \mathbb{R}_+$ represents the pumped energy during hour j; ϵ is the efficiency of the pumping unit, or the charging efficiency; k is the efficiency of the generating unit, or the discharging efficiency; S is the self-discharge rate of the storage unit; and I_j are the energy inflows (water inflows from precipitation, in the case of hydro storage) during hour j.

Additionally, the energy storage unit is subject to constraints on the maximum and minimum level that the energy storage unit can reach, and the maximum speed of charging and discharging the unit. These constraints are respectively:

$$x_j^{STOR} \leq k \, l_j^{STOR} \qquad \forall j \in \mathcal{J} \tag{17}$$

$$x_j^{STOR} \leq f_j \bar{P}^{OUT} \qquad \forall j \in \mathcal{J} \tag{18}$$

$$p_j^{STOR} \leq (1 - f_j) \, \bar{P}^{IN} \qquad \forall j \in \mathcal{J} \tag{19}$$

$$p_j^{STOR} \leq (\bar{L} - l_j^{STOR})/\epsilon \qquad \forall j \in \mathcal{J} \qquad (20)$$
$$l_j^{STOR} \leq \bar{L} \qquad \forall j \in \mathcal{J} \qquad (21)$$
$$l_j^{STOR} \geq \underline{L} \qquad \forall j \in \mathcal{J} \qquad (22)$$

–where $\bar{L}$ and $\underline{L}$ denote the maximum and minimum storage level of the aggregated storage unit; $\bar{P}^{IN}$ and $\bar{P}^{OUT}$ are the maximum speed to discharge and charge the unit; and the variable $f_j \in \{0,1\}$ denotes the charging or discharging state of the storage unit ($f_j = 0$ indicates that the storage unit is charging, and $f_j = 1$ indicates that the storage unit is discharging).

An alternative to aggregating all storage units into a single unit is to treat them separately, splitting them into different units. Each unit can be distinguished from the other through different values of storage capacity, maximum power delivery or roundtrip efficiency. However, introducing many storage units in the model will have a direct impact on the model's computational time, as the solver would need to decide at every hour between using one storage unit or the other.

4.9 Accounting for Demand Side Management

Demand side management (DSM) is a strategy designed to shift demand by withholding a fraction of the demand and serving it a later time, when it is cheaper for the system to supply that energy. DSM programs try to achieve net load peak shaving and provide reserves, which could reduce fuel costs and defer investments.

In IMRES, DSM has been implemented as a capability to shift part of the demand at a given hour throughout the following H hours. The implementation of DSM is somewhat similar to the implementation of energy storage as the amount of demand that remains to be supplied is recorded at a 'virtual storage' unit. This virtual storage can only store a maximum of $\bar{P}^{DSM^-}H$ gigawatt-hours of energy, where $\bar{P}^{DSM^-}$ is the maximum energy that can be withheld in one hour. Accordingly, the equation modeling the behavior of DSM is:

$$\sum_{j'>j}^{j+H} x_{j'}^{DSM} \geq l_j^{DSM} \qquad \forall j \in \mathcal{J} \qquad (23)$$
$$l_j^{DSM} = l_{j-1}^{DSM} - x_j^{DSM} + p_j^{DSM} \qquad \forall j \in \mathcal{J} \qquad (24)$$
$$l_j^{DSM} \leq \bar{P}^{DSM^-}H \qquad \forall j \in \mathcal{J} \qquad (25)$$
$$x_j^{DSM} \leq \bar{P}^{DSM^+} \qquad \forall j \in \mathcal{J} \qquad (26)$$
$$p_j^{DSM} \leq \bar{P}^{DSM^-} \qquad \forall j \in \mathcal{J} \qquad (27)$$

–where $p_j^{DSM} \in \mathbb{R}_+$ denotes the energy withheld during hour j; $l_j^{DSM} \in \mathbb{R}_+$ denotes the total energy withheld or, alternatively, the energy level of the virtual storage at hour j; $\bar{P}^{DSM^-}$ denotes the hourly capacity to withhold demand; $\bar{P}^{DSM^+}$ denotes the hourly capacity to 'put back' demand; and x_j^{DSM} denotes the demand being 'put back' at hour j.

4.10 Reserves Constraints

IMRES uses the general reserves classification proposed in Milligan et al. [5], but with the European naming convention (primary, secondary and tertiary reserves). According to this report, *primary reserves* are those that are activated immediately after the system frequency deviates from the reference value following a system event, and must be able to be fully operational for 30 seconds; *secondary reserves* are fast-responding units controlled by Automatic Generation Control (AGC), responding 30 seconds after a system event occurs, and must be able to be fully operational for 15 minutes; and *tertiary reserves* have a slower response and are used to restore primary and secondary reserves back to their initial state.

Primary up reserve requirements, are denoted $r_j^{PRI\_U}$, and are proportional to the capacity of the largest generating unit. Secondary up reserve requirements, denoted $r_j^{SEC\_U}$, and secondary down reserve requirements, denoted $r_j^{SEC\_D}$, are proportional to the demand level in hour j, and to the wind and solar PV output. Finally, tertiary reserve requirements, are denoted r_j^{TER}, are proportional to the wind and solar PV capacity installed.

$$r_j^{PRI\_U} \geq \max_{i \in \mathcal{T}} \bar{P}_i \, y_i \qquad \forall j \in \mathcal{J} \qquad (28)$$

$$r_j^{SEC\_U} \geq K^U \left[\alpha D_j^2 + \gamma \left(CF_j^{WIND} \, Y_{i \in \mathcal{W}}\right)^2 + \delta \left(CF_j^{SOLAR} \, Y_{i \in \mathcal{S}}\right)^2\right]^{1/2} \qquad \forall j \in \mathcal{J} \qquad (29)$$

$$r_j^{SEC\_D} \geq K^D \left[\beta D_j^2 + \xi \left(CF_j^{WIND} \, Y_{i \in \mathcal{W}}\right)^2 + \theta \left(CF_j^{SOLAR} \, Y_{i \in \mathcal{S}}\right)^2\right]^{1/2} \qquad \forall j \in \mathcal{J} \qquad (30)$$

$$r_j^{TER} \geq \zeta \, Y_{i \in \mathcal{W}} + \eta \, Y_{i \in \mathcal{S}} \qquad \forall j \in \mathcal{J} \qquad (31)$$

When renewable capacity is endogenously determined by IMRES, the expressions for secondary reserve requirements need to be simplified in order to avoid nonlinearities. One straightforward solution to this problem is to substitute the geometric sum of the different factors (i.e., demand, wind output and solar output) in equations (29) and (29) by their arithmetic sum[4], adjusting the different parameters to match the original secondary reserve requirement function as closely as possible:

$$r_j^{SEC\_U} \geq K^{U'} \left[\alpha' D_j + \gamma' \left(CF_j^{WIND} \, y_{i \in \mathcal{W}}\right) + \delta' \left(CF_j^{SOLAR} \, y_{i \in \mathcal{S}}\right)\right] \qquad \forall j \in \mathcal{J} \qquad (51)$$

$$r_j^{SEC\_D} \geq K^{D'} \left[\beta' D_j + \xi \left(CF_j^{WIND} \, y_{i \in \mathcal{W}}\right) + \theta' \left(CF_j^{SOLAR} \, y_{i \in \mathcal{S}}\right)\right] \qquad \forall j \in \mathcal{J} \qquad (52)$$

The provision of operating reserves is modeled in IMRES through requiring the system to have a certain amount of spinning and non-spinning reserves ready to be deployed at every hour. Spinning reserves can be provided by committed thermal power plants, storage units and DSM. Accordingly, spinning reserves are formulated accounting for the capacity margin offered by these on-line elements (32-33).

$$\sum_{i \in \mathcal{I}} \left(u_{ij} \, \bar{P}_i - x_{ij}\right) + f_j \bar{P}^{OUT} - x_j^{STOR} + p_j^{STOR} + \bar{P}^{DSM^-} + x_j^{DSM} - p_j^{DSM} \geq r_j^{PRI\_U} + r_j^{SEC\_U} \qquad \forall j \in \mathcal{J} \qquad (32)$$

[4]Note that other more complicated piecewise quadratic approximations would match more closely the original function. Here a linear approximation is used for simplicity.

$$\sum_{i\in\mathcal{I}} (x_{ij} - u_{ij}\ \underline{P}_i) + (1-f_j)\,\bar{P}^{IN} + x_j^{STOR} - p_j^{STOR} + \bar{P}^{DSM^+} - x_j^{DSM} + p_j^{DSM} \quad \geq$$
$$r_j^{SEC\_D} \qquad \forall j \in \mathcal{J} \qquad (33)$$

Finally, non-spinning reserves are modeled accounting for the off-line capacity in the system that can be turned on to replace secondary reserves (34).

$$\sum_{i\in\mathcal{I}} (y_i - u_{i,j})\,\bar{P}_i \geq r_j^{TER} \qquad \forall j \in \mathcal{J} \qquad (34)$$

5 Applications

The formulation of IMRES offers a great versatility in terms of which constraints are included in the model, which are left out, and which decisions variables are free and which ones are fixed and parameterized. Therefore, it could serve as modeling framework for a large set of experiments. A list is presented below with a sample of possible tasks that could be performed with IMRES:

- Greenfield capacity expansion: determining the optimal electricity generation capacity mix at a given future time.
- Brownfield capacity expansion: determining the future investment needs to supply a growing electricity demand at minimum cost, fixing the variables representing the plants that are already built in the system and leaving free those variables associated with potential new investments.
- Analyzing the effect of constraints on the total system cost through selectively applying and removing these constraints.
- Calculating the value of *flexibility* options (storage, DSM, flexible power plants, etc) by comparing the total system cost with and without each option, to establish R&D priorities.
- Analyzing the effect of more stringent CO_2 emissions constraints on the optimal generation mix.
- Calculating the profitability of individual units through obtaining the prices and the quantities produced for the different services provided.
- Analyzing the effect of different renewable deployment levels for a fixed generation capacity mix.

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