

# Joint Allocation of Climate Control Mechanisms is the Cheapest Way to Reduce Global Climate Damage

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**Joint Allocation of Climate Control Mechanisms  
is the Cheapest Way to Reduce Global Climate Damage.**

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**Abstract.**

Coupling the modes of action of four different climate control mechanisms: emission reduction, CO<sub>2</sub> removal, adaptation, and geoengineering can greatly lower the damage of future global average temperature increase. This paper introduces a model for this joint action and solves it for the optimal allocation of available budget to the different control mechanisms over their entire range of action. The results provide intuition about the factors that influence efficient climate control deployment and points to research needed to improve understanding

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Both experts and the public understand that avoiding adverse climate impacts of global warming is a global imperative.<sup>1,2</sup> Communities such as New York, California, the European Community have adopted aspirational goals of becoming “carbon free” by mid-century dates.

Environmental experts warn that not enough is being done: emission reductions will not suffice,<sup>3</sup> concrete actions by government and industry are not being taken, and exploration of gigaton scale new technology approaches are discussed but not yet urgently pursued. Avoiding adverse climate change will require massive resources deployed efficiently among all available abatement measures to reverse global warming and its economic damage. Rational management of such a process requires the integration of technical and economic analysis to reveal the most efficient way to allocate available resources to the multiple climate control mechanisms. This paper provides a framework for such an analysis. It does not have

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quantitative fidelity but it identifies linkages that should be taken into account in any comprehensive climate control program.

The four climate control mechanisms are:

- *Measures* to reduce CO<sub>2</sub> emissions into the atmosphere;<sup>4</sup>
- *Measures to remove* CO<sub>2</sub> from the atmosphere;<sup>5</sup>
- *Planned Adaptation* measures intended to diminish adverse climate consequences from global temperature increase;<sup>6,7</sup> and
- *Geoengineering* measures to reduce radiation forcing, and hence lower temperature.<sup>8,9</sup>

Most studies have focused on emission reduction. Some studies have analyzed the combined action for several of the control mechanism: Tol<sup>10</sup> and Kane and Shorgren<sup>11</sup> have addressed the joint optimization of adaptation and emission reduction. Moreno-Cruz *et al.*<sup>12</sup> and Keller<sup>13</sup> have addressed the joint optimization of geoengineering and emission reduction. Recently, Mariia Balaia has considered the joint optimization of emission reduction, CO<sub>2</sub> removal, and geoengineering.<sup>14</sup> This is the first report to address all four control mechanisms.

### The conceptual model.

Global climate damage  $D[\delta T(t)]$  is a monotonically increasing function of global average temperature,  $\delta T = T - T_{pre}$  relative to an earlier, pre-industrial temperature,  $T_{pre}$ . All damage functions approach zero as  $D[\delta T(t)] \rightarrow \beta \delta T(t)^2$ , as  $\delta T(t) \rightarrow 0$ .<sup>15</sup> The conceptual model proposed here describes how the four climate control mechanisms work together to reduce the expected future damagedamage at some future time 't' according to the modified damage function:

$$D_{mod}[\delta T(t)] = (1 - \chi(t)) \langle D[\delta T_{\phi, \theta}(t)(1 - \lambda(t))] \rangle. \quad \text{Eq.(1)}$$

Each climate control mechanism imposes a different mode of action on damage: *Adaptation* reduces the damage by the factor  $(1 - \chi(t))$ , *geoengineering* reduces the temperature increase through diminishing radiation forcing by the factor  $(1 - \lambda(t))$ . Two mechanisms reduce the

temperature anomaly indirectly: the first reduces *emissions* into the atmosphere by the factor  $(1-\varphi(t))$ . The second *removes* CO<sub>2</sub> from the atmosphere, by the factor  $(1-\phi(t))$ .<sup>16</sup> The decrease in temperature from these two measures is denoted  $\delta T_{\varphi(t),\phi(t)}(t)$ . All four variables are in the range [0,1]. The angular bracket in Eq.(1), denotes an average of if any stochastic behavior is introduced in the model that influences the temperature increase  $\delta T(t)$ . If we assume an average annual global GDP of \$90 trillion and damages of 2% global GDP per 3 °C temperature then  $\beta = 2.22 \cdot 10^9 \text{ } \$/(\text{C}^0)^2$  and, from Eq.(1), in the low damage limit:

$$D_{\text{mod}}[\delta T(t)] = \beta(1-\chi(t))\langle \delta T_{\varphi,\phi}^2(t) \rangle (1-\lambda(t))^2. \quad \text{Eq.(2)}$$

Suppose the global climate is on a path over the time interval,  $t_0 < t < H$ , for the temperature anomaly to increase, from  $\delta T(t_0)$  to  $\delta T(H)$ . If no climate control mechanisms are deployed over this time interval the resulting temperature increase will be  $\delta T_0(H)$ . If the two mitigations control mechanism for emission reduction and removal are present, the atmospheric CO<sub>2</sub> concentration  $c_{\varphi,\phi}(t)$  is modified and evolves during this time interval as:

$$\frac{\partial c_{\varphi,\phi}(t)}{\partial t} = q(t)(1-\varphi(t)) - c_0(t_0)\phi(t), \quad \text{Eq.(3)}$$

based on the simple CO<sub>2</sub> removal strategy of removing a fraction,  $\phi(t)$ , of the concentration present at the initial time:  $c_0(t_0)$ . The resulting concentration change is

$$c_{\varphi,\phi}(t) = c_0(t_0) + \int_{t_0}^t d\tau [(1-\varphi(\tau))q(\tau) - \phi(\tau)c_0(t_0)], \quad \text{Eq. (4)}$$

$$c_{\varphi,\phi}(t) = c_0(t) - \int_{t_0}^t d\tau [\varphi(\tau)q(\tau) - \phi(\tau)c_0(t_0)], \quad \text{Eq.(5)}$$

where the quantity 'q(t)' denotes the emission rate of CO<sub>2</sub> into the atmosphere. (Economic and population growth are subsumed in the emission rate.)

Climate sensitivity,  $\delta T(t) - \delta T(t') = \epsilon \ln(c(t)/c(t'))$ , gives the temperature change resulting from a change in atmospheric concentration. In the presence the two emission control measures:

$$\delta T_{\varphi(t),\phi(t)}(H) - \delta T_0(H) = \varepsilon \ln \left( \frac{c_0(t_0) + \int_{t_0}^H d\tau q(\tau) - \int_{t_0}^H d\tau [\varphi(\tau)q(\tau) - c_0(t_0)\phi(\tau)]}{c_0(t_0) + \int_{t_0}^H d\tau q(\tau)} \right). \quad \text{Eq.(6)}$$

In low damage limit,  $\beta \delta T_{\varphi(t),\phi(t)}^2(t) \ll 1$ , the reduction in environmental damage from deployment of the four climate control mechanisms between  $t_0$  and the time horizon,  $H$ , is:

$$D_{\text{mod}}[\delta T(H)] \approx (1 - \chi(H)) \beta \left\langle \left( \delta T_0(H) + \varepsilon \ln \left( 1 - \frac{\left[ \int_{t_0}^H \varphi(\tau)q(\tau) + c_0(t_0)\phi(\tau) \right]}{c_0(t_0) + \int_{t_0}^H d\tau q(\tau)} \right) \right)^2 \right\rangle (1 - \lambda(H))^2. \quad \text{Eq.(7)}$$

### Optimization.

The objective is to determine the values of the climate control shares that minimizes the damage,  $\alpha(t) = \{\varphi(t), \phi(t), \lambda(t), \chi(t)\}$ , Eq.(7), subject to a budget constraint (or alternatively to determine the necessary budget to achieve a prescribed change in anticipated damage). [See SM—1]. The optimization process is different for a deterministic system and a system that exhibits stochastic behavior. This report considers only deterministic behavior; future work will report on application of dynamic programming to address the stochastic behavior and relaxing the assumption of perfect foresight.

The annual budget constraint,  $B(t)$ , is

$$B(t) = C_{\varphi}(\varphi(t)) + C_{\phi}(\phi(t)) + C_{\lambda}(\lambda(t)) + C_{\chi}(\chi(t)). \quad \text{Eq.(8)}$$

$C_{\alpha}(\alpha(t))$  denotes the annual cost for each abatement measure:  $\alpha = \{\varphi, \phi, \lambda, \chi\}$ .

There is no reliable operational data or engineering analysis to inform the choice of the functional form or magnitude of the cost function at global scale. However, arbitrary cost functions will reveal the linkage between the climate control mechanisms. For ease of comparison assume each of the cost functions has the same functional form, differing only in a scale factor,  $\tilde{C}_{\alpha}$ , thus  $C_{\alpha}(\alpha(t)) = \tilde{C}_{\alpha} f(\alpha(t))$ . Two different functional form are selected to

illustrate the range of response to unlike cost behaviors:

$$f_{\text{low}}(\alpha) = (\alpha/(1+\alpha))^2 \text{ and } f_{\text{high}}(\alpha) = (\alpha/(1-\alpha))^2 .$$

As an illustration, suppose the relative costs of the different control measures are: for *emission reduction*,  $\tilde{C}_\phi = 50 T$ ; for *CO<sub>2</sub> removal*,  $\tilde{C}_\phi = 100T$ ; for *geoengineering*,  $\tilde{C}_\lambda = 150T$ ; and for *adaptation*,  $\tilde{C}_\chi = 150T$ . The reader is free to select the dollar amount for the scale factor T; it is unknown but likely to be on the order of  $T \sim \$1$  trillion, and to choose ratios other than the illustrative values selected here.

### Cost of Meeting the Paris Agreement 2<sup>o</sup> C target.

The 2015 Paris 21<sup>st</sup> Conference of the Parties of the UN Framework Convention on Climate reduced the target for global warming at the end of the century to  $\delta T(2100) = 2^\circ\text{C}$ . In addition, the Agreement called on all countries to strengthen efforts to adapt to climate change.<sup>17</sup> The proposed model shows how to determine the optimal, i.e., lowest cost way, to achieve a desired lower global temperature increase by allocating available budget to the control mechanisms.

Consider the damage caused by a temperature increase realized at a future time, H, from an initial time  $t_0$  according to Eq.(7). Suppose the time period begins at  $t_0 = 2020$  and ends at  $H = 2100$ , and that the CO<sub>2</sub> emitted into the atmosphere occurs at a constant rate  $q = 2$  ppmv/yr during this eighty-year interval.<sup>18,19</sup> The resulting concentration increases from  $c(2020) = 400$  ppmv to  $c(2100) = 560$  ppmv. With climate sensitivity  $\varepsilon = 4.33$ , the temperature increase without climate controls will be  $\delta T_0(2100) = 3^\circ\text{C}$ . The assumption of constant emission means the control shares are constant throughout the period, determined by the key dimensionless parameter in Eq.(6),  $a \equiv q(H - t_0)/c_0(t_0) = 0.4$ , which is the ratio of the annual CO<sub>2</sub> emitted into the atmosphere over the period of interest to atmospheric concentration of CO<sub>2</sub> at the beginning of the period.

Table (1) gives the cost of reducing the temperature increase anticipated from  $\delta T(2100) = 3^\circ\text{C}$  to the Paris target  $\delta T(2100) = 2^\circ\text{C}$ . For comparison purposes the table includes the cost of meeting the 2<sup>o</sup> C target in 2020, if individual abatement measures are used separately.

Table (1) Budget and Allocation to Control Measures to reduce global temperature anomaly from $\delta T(2100) = 3^{\circ}\text{C}$ to $\delta T(2100) = 2^{\circ}\text{C}$ (reference cost allocation)					
Cost Functions	Budget Total	Emission Reduction $\varphi$	CO <sub>2</sub> Removal $\phi$	Geo-Engineering $\lambda$	Adaptation $\chi$
Low Cost	3.91 T	0.71T	2.73T	0.39T	0.08T
%	100%	18%	70%	10%	2%
Control Shares	--	0.135	0.198	0.054	0.024
Individual Control Measures Cost	8.79T	0.7218	--	--	--
	5.02T	--	0.2887	--	--
	9.38T	--	--	0.3333	--
	19.13T	--	--	--	0.5556
High Cost	7.13 T	1.33 T	3.74 T	1.61T	0.43T
%	100%	19%	52%	23%	6%
Control Shares	--	0.140	0.162	0.094	0.051
Individual Control Measures Cost	336.5 T	0.7218	--	--	--
	16.47T	--	0.2887	--	--
	37.49	--	--	0.3333	--
	234.5 T	--	--	--	0.5556

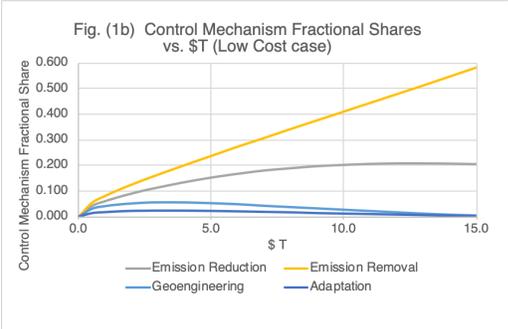
The budget required for low (high) costs 3.91T (7.13T) is expended over an eighty-year period. If T = \$trillions, these are small fractions of \$90 trillion annual global GDP; less than 0.14%. If the 2100 temperature target is lowered further to 1.5 °C in 2100, the budget required is larger. For the low-cost case, it increases to 7.3T; for the high cost case it increases to 17.1T. Table (1) illustrates the damage lowering achieved by optimization. Optimization among pairs or triplets of the four adaptation measures will close the gap between the result for individual and all measures. Allocating available budget optimally to accessible control measures saves considerable money.

These results are sensitive to the ratio of the control measure costs. Some will object to the high relative cost of geoengineering, which is believed to have quite low direct costs, although side effects are unknown. The following table shows the sensitivity of the results to different cost assumptions:

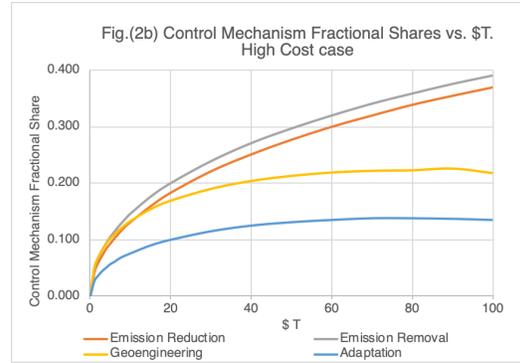
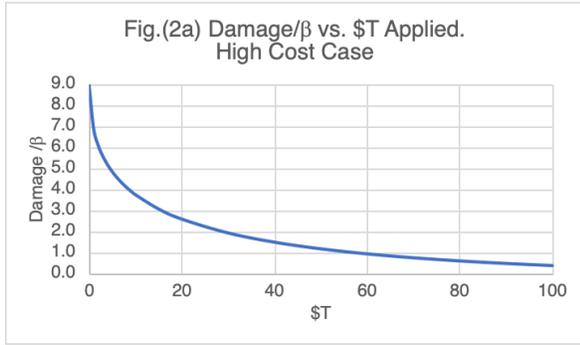
Table (2). Sensitivity of Reference of Budget Cost and Allocation to Change Relative Control Measure Cost. to reduce the temperature anomaly from $\delta T(2100) = 3^\circ\text{C}$ to $\delta T(2100) = 2^\circ\text{C}$						
Allocation	Reference		Low Geo. Eng.		All Low	
Budget	7.13T		6.12T		3.42T	
	Relative cost	Share	Relative cost	Share	Relative cost	Share
Emission Reduction $\varphi$	50	0.14	50	0.11	50	0.08
CO <sub>2</sub> Removal $\phi$	100	0.16	100	0.13	50	0.15
Geoengineering $\lambda$	150	0.09	50	0.19	50	0.13
Adaptation $\chi$	150	0.05	150	0.04	50	0.07

The Relationship between Budget Size and Damage Level.

For the Low-Cost functions each control mechanism approaches ¼ its scaling cost  $\tilde{C}_\alpha$  as  $\alpha \rightarrow 1$ . The reduction in damage as a function of the control budget is shown in Figure (1a). (The numerical values of the abatement share are reported in SM – 2.) Figure (1b) displays the damage reduction as a function of budget applied for all four control mechanisms.



Since each of the High Cost functions are singular completion of any of the four control options ( $\alpha = 1$ ) is not possible at any finite budget. The reduction in damage as a function of the control budget is shown in Figure(2a). (The numerical values of the abatement share are reported in SM – 2.) Figure (2b) displays the damage reduction as a function of budget applied for all four control mechanisms.



### Time Dependent Emissions.

The preceding calculation assumes a constant emission rate of  $q = 2$  ppmv throughout the eighty-year period 2020 to 2100 that assures a single, time independent, participation share for each of the four control variables. Comparing different time dependent emission trajectories as shown in Eq.(9) identifies lower cost emission control strategies; however, the optimization involves time dependent participation rates.

Over longer time intervals comparing different time dependent emission trajectories as shown in Eq.(7) will identify lower cost emission control strategies. This requires generalization of the dynamical analysis to time dependent emissions with an optimal control solution for the control shares  $\alpha(t) = \{\varphi(t), \phi(t), \lambda(t), \chi(t)\}$ .<sup>20,21</sup> Such generalization would permit analysis of emission trajectories resulting from alternative policies and the stochastic nature of climate damage, the possibility of ‘tipping points,’ ‘catastrophic’ environmental events, and ‘fat tail’ outcomes, which, quite properly have received significant attention.<sup>22</sup>

As an example, assume that the time interval  $[t_0, H]$  is divided into two equal periods that have different emission constant rates,  $q_1$  and  $q_2$ . Eq.(9) shows the resulting temperature increase at the time horizon,  $H$ , with the two participation shares:  $\varphi_1$  and  $\varphi_2$  :

$$\delta T_{\varphi, \phi}(H) = \delta T_0(H) + \varepsilon \ln \left( \frac{1 - ((H - t_0)/2c_0(t_0)) [q_1(1 - \varphi_1) + q_2(1 - \varphi_2)] - \phi(H)}{1 + (q_1(H - t_0)/2c_0(t_0))(q_1 + q_2)} \right). \quad (9)$$

Table (2) gives the sensitivity of  $\delta T(H=2100)$  to combinations of the emission rates  $q_1$  and  $q_2$ . For the choice of parameter, the effects are small but point out the following trend: greater total

emission increases the temperature anomaly,  $(0.2, 0.3) > (0.2, 0.2)$ , and concentrating the emission reduces the temperature anomaly  $(0.3, 0.1) < (0.2, 0.2)$ , (the order does not matter because there is no discounting between periods).

Table (3) Temperature Increase from $\delta T(2020) = 3^{\circ}\text{C}$ to $\delta T(2100) = 2^{\circ}\text{C}$ at Different Emission Rates (ppmv) in Equally Divided period with Associated Control Shares and Budgets							
Low Cost B 4.245 T		$\delta T(H) \text{ }^{\circ}\text{C}$	Emission Reduction $\varphi_1$	Emission Reduction $\varphi_2$	CO <sub>2</sub> Removal $\phi$	Geo- Engineering $\lambda$	Adaptation $\chi$
$q_1$	$q_2$						
0.2	0.2	4.00	0.0580	0.0580	0.2254	0.0590	0.0259
0.3	0.1	3.95	0.0960	0.0263	0.2217	0.0576	0.0254
0.2	0.3	4.13	0.0561	0.0934	0.2121	0.0653	0.0283
0.2	0.1	3.85	0.0589	0.0268	0.2315	0.5160	0.0231
High Cost Budget 7.94 T							
$q_1$	$q_2$	$\delta T(H) \text{ }^{\circ}\text{C}$	Emission Reduction $\varphi_1$	Emission Reduction $\varphi_2$	CO <sub>2</sub> Removal $\phi$	Geo- Engineering $\lambda$	Adaptation $\chi$
0.2	0.2						
0.3	0.1	3.96	0.1230	0.0518	0.1723	0.1020	0.0563
0.2	0.3	4.05	0.0871	0.1179	0.1660	0.1048	0.0580
0.2	0.1	3.90	0.0963	0.0551	0.1799	0.1005	0.0554

### The Social Cost of Carbon, (SCC).

Integrated assessment models (IAM) simulate the environmental and economic effects of CO<sub>2</sub> trajectories emitted into the atmosphere that raise global mean temperature and result in damage to economic production.<sup>23</sup> IAMs focus on emission reduction and thus estimate the social cost of carbon based on emissions. IAMs do not directly address other control mechanisms, although there has been some effort to include adaptation directly in the analysis.<sup>14,24</sup> Some IAM studies introduce “backstop technologies” into the emission reduction analysis, so as to include new technical possibilities such as carbon air capture, but the coupling of the mode of action of different control mechanisms is hidden.<sup>25,26</sup>

The SCC investigates the incremental damage created by the addition of a marginal quantity of CO<sub>2</sub> into the atmosphere.<sup>27,28,29</sup> In terms of the model examined here, the SCC at time ‘t’ is the discounted present value, at rate  $\gamma$ , of the partial derivative of the damage with respect to

emissions over the time interval:  $[t_0, t]$ . According to Eq.(6),

$$SCC_B(t) = \int_{t_0}^{t-t_0} dt e^{-\gamma(t-t_0)} \left[ \partial D[\delta T(\tau)] / \partial q(\tau) \right] = \int_{t_0}^t dt e^{-(\tau-t_0)} (1-\chi(B)) 2\beta [\delta T(t_0) + (\epsilon \ln(1+(\tau-t_0)q/c_0(t_0)(1-\phi(B)) - \phi(B))) \frac{\epsilon [(\tau-t_0)/c_0(t_0)](1-\phi(B))}{(1+(\tau-t_0)q/c_0(t_0)(1-\phi(B)) - \phi(B))} (1-\lambda(B))^2] \quad \text{Eq.(10)}$$

Following the previous development the SCC is evaluated for the interval end time  $H = 2100$  relative to initial time  $t_0 = 2020$  with constant emission rate throughout the interval,  $q = 2$  ppmv and initial atmospheric concentration  $c_0(t_0) = 400$  ppmv. The annual budget applied to the four climate control mechanisms,  $B$ , is assumed constant over the 80 year interval

$$SCC_B(H) = \int_0^{H-t_0} dt e^{-\gamma t} (1-\chi(B)) 2\beta [\delta T(t_0) + (\epsilon \ln(1+tq/c_0(t_0)(1-\phi(B)) - \phi(B))) \frac{\epsilon tq/c_0(t_0)(1-\phi(B))}{(1+tq/c_0(t_0)(1-\phi(B)) - \phi(B))q} (1-\lambda(B))^2] \quad \text{Eq.(11)}$$

For simplicity the control variables which are dependent on time and budget,  $\alpha(t, B)$  are denoted  $\alpha(B)$  for simplicity. Emission of one  $q$ (ppmv) is equal to  $q_m \sim 1.25 \cdot 10^{10}$  metric tonnes of  $CO_2$ .

The discounted Damage is:

$$W_B = \int_{t_0}^H dt e^{-(t-t_0)} D_{mod}[\delta T(t)] = \int_{t_0}^H dt e^{-(t-t_0)} (1-\chi(B)) \beta [\delta T(t_0) + \epsilon \ln(1+(t-t_0)q/c_0(t_0)(1-\phi(B)) - \phi(B)) (1-\lambda(B))^2] \quad \text{Eq.(12)}$$

The positive values of SSC reflects that as. "q" increases  $\delta T(H)$  increases and hence the damage,  $D_{mod}[\delta T(H)]$  grows. The results for  $SCC_B(2100)$  are  $SCC_0(2100) = 0.02625\beta$  if  $B = 0$ , and  $SCC_7(2100) = 0.2323$  if  $B = 7T$ , which is close to the value required to lower anticipated  $\delta T(2100) = 2^\circ C$  from an initial temperature  $\delta T(2020) = 2^\circ C$  as reported above. The SCC is not sensitive to the control budget applied

The discounted damage values are  $W_0(2100) = 20.75 \beta$  with  $B = 0$ , and  $W_7(2100) = 6.214 \beta$  with control budget  $B = 7.0 T$  applied. The result shows again the significant damage reduction that results for optimal application of budget applied to joint control.

## Conclusion.

The model demonstrates the interplay between different climate control measures that act to reverse global warming. The main point is climate control is a vector with four components, and efficient control requires that the components be considered jointly.

The numerical results reported here depend entirely on the arbitrary choice of the form of the adopted cost functions and scaling parameters. Thus, the model calculation adds urgency to determining these control costs, especially for geoengineering, which many believe to be low.<sup>30</sup> Legitimate question may be raised about the value for the scaling cost selected, and some readers will disregard the entire framework because of the reliance on fictitious numbers in the illustrative applications. However, the intent is for this presentation to encourage work to construct empirically based cost function and to remind readers the absence of knowledge about cost function does not mean that the qualitative features of the climate mechanism coupling presented here do not exist and may be ignored in considering new comprehensive climate policies or ambitious new R&D programs. Currently work is underway to develop an optimization method for time dependent emission rates allowing comparison of different emission policies and a continuous time programming method to remove the artificial perfect forward knowledge implied by the optimization used here.<sup>1</sup>

Stopping the increase in global warming below 2 °C by the end of the century requires more than reducing emissions – it will require significant, e.g., CO<sub>2</sub> reduction from the atmosphere as well as adaptation programs, and possibly geoengineering. These efforts must be managed jointly if resources are not to be wasted.

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<sup>1</sup> This work is being undertaken with my MIT colleagues Henri Drake, Alan Edelman and Ronald Rivest.

Supplementary Material

**SM – 1. The optimization problem is**

$$\text{Minimize: } W\{\varphi(t), \phi(t), \lambda(t), \chi(t)\} = \int_{t_0}^H dt e^{-r(t-\tau)} (1-\chi(t)) \left\langle D\left[\delta T_{\varphi(t), \phi(t)}(t)(1-\lambda(t))\right] \right\rangle,$$

$$D_{\text{mod}}[\delta T(t)] \approx (1-\chi(t)) \beta \left( \delta T_0(t) + \varepsilon \ln \left( 1 - \frac{\int_{t_0}^t \varphi(\tau) q(\tau) + c_0(t_0) \phi(t)}{c_0(t_0) + \int_{t_0}^t d\tau q(\tau)} \right) \right) (1-\lambda(t))^2$$

subject to  $\{B = C_\chi(\varphi(t)) + C_\phi(\phi(t)) + C_\lambda(\lambda(t)) + C_\chi(\chi(t)),$

$0 < \varphi(t) < 1, 0 < \phi(t) < 1, 0 < \lambda(t) < 1, 0 < \chi(t) < 1\}$

In the interval we consider  $[t_0, H] = [2020, 2100]$  falls in the low damage limit

$D[\cdot] = \beta \delta T_{\varphi(t), \phi(t)}(\cdot)^2$ . Thus, the more general damage function  $D[\cdot] = 1 - \text{Exp}[-\beta \delta T[\cdot]]$  gives the same results as the low damage limit results quoted in the paper.

**SM – 2. Numerical results for the optimization shares presented in Figures (1a), (1b), (2a) and (2b)**

SM-2 Budget \$ Trillions vs. Damage/ $\beta = \delta T^2$ and Four Abatement Shares (High Cost Case)					
Annual Budget \$ Trillions	D(B)/ $\beta = \delta T(2100)$	$\varphi$ Emission Reduction	$\phi$ Emission Removal	$\lambda$ Geo engineering	$\chi$ Adaptation
0	9.000	0.000	0.000	0.000	0.000
1	6.892	0.042	0.049	0.051	0.027
2	6.096	0.060	0.068	0.070	0.037
3	5.588	0.073	0.083	0.083	0.045
4	5.192	0.085	0.095	0.094	0.051
5	4.867	0.094	0.106	0.103	0.057
6	4.591	0.103	0.115	0.110	0.061
7	4.350	0.111	0.124	0.117	0.066
8	4.137	0.119	0.132	0.123	0.070
9	3.946	0.126	0.140	0.129	0.073
10	3.773	0.132	0.147	0.134	0.076
15	3.098	0.160	0.177	0.155	0.090
20	2.617	0.184	0.201	0.170	0.100
30	1.960	0.222	0.240	0.191	0.115
40	1.523	0.252	0.272	0.205	0.125
50	1.208	0.278	0.298	0.214	0.131
60	0.969	0.301	0.321	0.220	0.135
70	0.783	0.321	0.342	0.223	0.138
80	0.635	0.340	0.360	0.224	0.138
90	0.514	0.356	0.377	0.227	0.137
100	0.415	0.371	0.392	0.219	0.135

SM-2 Budget \$ Trillions vs. Damage/ $\beta = \delta T^2$ and Four Control Mechanism Shares (Low Cost Case)					
Budget \$ Trillions	D(B)/ $\beta = \delta T(2100)$	$\varphi$ Emission Reduction	$\phi$ Emission Removal	$\lambda$ Geo engineering	$\chi$ Adaptation
0.0	9.000	0.000	0.000	0.000	0.000
0.5	7.145	0.042	0.054	0.030	0.014
1.0	6.403	0.061	0.081	0.040	0.018
1.5	5.843	0.077	0.104	0.046	0.021
2.0	5.376	0.091	0.125	0.050	0.023
2.5	4.968	0.104	0.145	0.053	0.024
3.0	4.601	0.115	0.164	0.054	0.024
4.0	3.951	0.136	0.201	0.054	0.024
5.0	3.379	0.154	0.237	0.051	0.023
6.0	2.863	0.169	0.273	0.047	0.021
7.0	2.390	0.182	0.307	0.041	0.019
8.0	1.956	0.191	0.342	0.036	0.017
9.0	1.557	0.199	0.376	0.031	0.014
10.0	1.193	0.204	0.410	0.026	0.012
11.0	0.868	0.208	0.444	0.021	0.010
12.0	0.584	0.210	0.478	0.016	0.008
13.0	0.347	0.210	0.512	0.011	0.006
14.0	0.164	0.209	0.547	0.007	0.004
15.0	0.045	0.207	0.582	0.004	0.002

Concentration time trajectory:

$$c_{\varphi,\phi}(t) = c_0(t_0) + \int_{t_0}^t d\tau(1 - \varphi(\tau))q(\tau) - \phi(t)c_0(\tau_0)$$

This assumes the removal strategy is remove a fraction  $\phi(t)$  of the initial concentration :

If  $q(t)$  is constant with time then the for solutions with  $\varphi$  and  $\phi$  will be constant with time:

$$\frac{c_{\varphi,\phi}(t)}{c_0(t_0)} = 1 + q(1 - \varphi)(t - t_0)/c_0(t_0) - \phi.$$

The resulting temperature change

$$\delta T(H) = \delta T_0(H) + \varepsilon \ln \left[ \frac{1 + q(1 - \varphi)(H - t_0)/c_0(t) - \phi}{1 + q(1 - \varphi)(H - t_0)/c_0(t_0)} \right]$$

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