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ABSTRACT

We consider welfare-optimal investment in and operation of electric power systems with constant returns to scale in multiple available generation and storage technologies under perfect foresight. We extend a number of classic results on generation, derive conditions for investment and operations of storage technologies described by seven cost/performance parameters, and develop insights on power systems with multiple storage technologies. Simulation of a deeply decarbonized "Texas-like" power system with two available storage technologies shows both the non-existence of simple "merit-order" rules for storage operation and the value of frequency domain analysis to describe efficient operation. Our analysis points to the critical role of the capital cost of energy storage capacity in influencing efficient storage operation.

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1. Introduction

Driven mainly by concerns about climate change, variable renewable energy (VRE) resources, mainly wind and solar, are becoming increasingly important sources of electricity in many regions. Because the maximum output of VRE generators is variable and imperfectly predictable, however, increased penetration of VRE generation makes it more difficult for power system operators to match supply and demand at every instant. The traditional solution to this problem would be to employ more gas turbines or gas combined-cycle plants, both of which can increase and decrease output rapidly. But building more gas-fired generation is inconsistent with a desire to reduce carbon dioxide emissions.

As the costs of storage, particularly lithium-ion (Li-ion) battery storage, have declined rapidly, storage has emerged as a potentially attractive, carbon-free alternative solution to problems posed by increased VRE penetration (Patel 2018). Policy-makers in the U.S. and the E.U. have accordingly encouraged the deployment of storage. The California Public Utilities Commission has been requiring load-serving entities to procure storage since the promulgation of statutory requirements in 2010 (Petlin et al 2018, California Public Utilities Commission n.d.). As of late 2020, battery storage targets have also been established in Massachusetts, Nevada, New Jersey, New York, and Oregon, and they are under consideration in other states (DSIRE database n.d.). The U.S. Federal Regulatory Commission (2018) has recently issued Order 841, which is intended to open wholesale energy markets (and other wholesale markets) to merchant storage providers.[1] Similarly, The European Union's Clean Energy Package, most recently modified in 2019, calls for competitive supply of storage (Glowacki 2020).

In this essay, we explore what economic theory implies about the general properties of cost-efficient electric power systems in which storage performs energy arbitrage to help balance supply and demand.[2] We start from an investment planning model based on the work of Boiteux

[1] In addition, at the federal level in the U.S., storage facilities that are charged only by solar generators are eligible for up to a 30% investment tax credit.

[2] Storage can also perform other functions in electric power systems. Depending on the technology employed, storage facilities can provide frequency regulation, deferral of wires investment, and reducing the cost of spinning reserves. For discussions, see Giuletti et al (2018), Balducci et al (2018), and U.S. Government Accountability Office (2018). See Sidhu et al (2018) for a worked example of a storage project that could perform multiple functions. The focus here is exclusively on the use of storage for energy arbitrage to solve the problems posed by increasing VRE penetration.

(1960, 1964) and Turvey (1968).[3] In models of this sort, constant returns to scale are generally assumed in generation, i.e., costs are assumed to be linear in the capacities and outputs (up to capacity) of each of several types of dispatchable generators. There are no startup costs or ramping constraints, which that limit thermal generators' ability to change output. There are thus no non-convexities or links between time periods on the supply side. Similarly, demand may vary from period to period, either deterministically or according to a known probability distribution, but the demand function in each period is independent of prices charged in other periods. Thus the multiple time periods in these models are linked only by the generation capacities that are chosen at the outset.

It is important to note that these assumptions are not descriptive of systems in which coal or nuclear generation are important supply sources. Both technologies have significant economies of scale, giving rise to nonconvexities. In addition, coal and nuclear plants take time and incur costs to start up and ramp down[4], which breaks the independence among time periods. Power systems with these characteristics resist general algebraic analysis, and sophisticated numerical optimization tools have been developed to permit explicit multi-period analysis of particular cases.[5]

For modern gas generators and VRE facilities, however, neither lumpiness nor startup or ramp down costs are nearly as important. Boiteux-Turvey-style models are thus reasonable approximations for systems without significant coal or nuclear generation, particularly when modeling hourly dispatch of power systems.[6] There are a number of ways that storage has been added to models of this sort. In the earliest formal treatments of storage in this context of which we are aware, Gravelle (1976) and Nguyen (1976) consider two-period – peak and off-peak – models and simply assume that an unlimited amount of the quantity being sold can be transferred between adjacent periods at a constant per-unit cost. Several authors, including Steffen and

[3] For an early exposition of models of this sort, see Drèze (1964), and for an excellent recent textbook treatment, see Biggar and Hesamzadeh (2014, esp. ch. 9). Following most of this tradition, we neglect the spatial dispersion of real power systems and assume everything happens at a single point.

[4] Although the existing fleet of nuclear power plants are capable of flexible operation within limits, they are more constrained than flexibility of competing grid resources like natural gas power generation and energy storage (U.S. Department of Energy, 2015).

[5] See, for instance, Jenkins and Sepulvada (2017) and Johnston et al. (2019).

[6] Modern combined cycle gas turbines (CCGT) and open cycle gas turbine (OCGT) power plants can ramp up or down 100% of their nameplate capacity within an hour. See for example specifications for GE's 7HA gas turbines (https://www.ge.com/content/dam/gepower-pgdp/global/en_US/documents/product/gas%20turbines/Fact%20Sheet/2017-prod-specs/7ha-power-plants.pdf)

Weber (2013) and Korpås and Botterud (2020) have added storage to Boiteux-Turvey-style models by assuming that power can be purchased whenever the price of energy is low and resold whenever the price is high. This amounts to assuming that the amount of energy that can be stored is effectively infinite, since low-price and high-price periods may be far apart in time. Helm and Mier (2018) consider a dynamic model with a constant demand curve and non-stochastic renewable output that follows a regular cyclic trajectory. Schmalensee (2020) considers a model with stochastic demand and alternating daytime and nighttime periods in which VRE generation is only available in the daytime periods. His emphasis is on the efficiency of the competitive supply of storage.

Here we follow Crampes and Trochet (2019) and Brown and Reichenberg (2020) and consider an explicitly dynamic Boiteux-Turvey-style model with perfect foresight. We follow most of the literature and assume constant returns to scale in storage as well as in generation. We are able to obtain a number of general results regarding investment in and operation of storage facilities under competition and to illustrate the complexity of systems in which multiple storage technologies are optimally deployed. The perfect foresight assumption is of course strong and eliminates the precautionary demand for storage. Relaxing that assumption, however, requires explicitly modeling the relevant stochastic processes, as demonstrated by Geske and Green (2019). The importance of eliminating precautionary demand will depend on the details of those processes, and it is not clear that general results are available.

Our results go beyond the analysis presented by Crampes and Trochet (2019) and Brown and Reichenberg (2020) and prior literature in the following ways. First, we show that problem of maximizing overall social welfare in that model can be decomposed into the problems faced by profit-maximizing, perfectly competitive suppliers of each available technology, even when considering limited energy capacity of energy storage and ramping constraints for dispatchable generation. This provides a new, direct link between welfare- and profit-maximization for linear electric power systems while explicitly considering limited energy capacity of energy storage and ramping constraints for dispatchable generation. Second, we show that merit-order dispatch for thermal generators is not generally optimal when ramping constraints are binding.

Third, we generalize results on optimal investment in and operation of storage by modeling a generalized characterization of storage technologies that uses seven distinct parameters, including independent charging and discharging power capital costs and efficiencies.

We show that all deployed storage technologies break even at equilibrium under constant returns to scale. Fourth, we present an analytical framework that yields insight into efficient configurations and operations of systems employing multiple storage technologies and points to the importance of the relative costs of power capacity and energy storage capacity. Finally, we employ a numerical case study to illustrate the complexity of operating patterns of storage in systems with multiple storage technologies. This exercise supports the insights developed analytically, shows that general analytical results of the "merit-order" variety are likely not available for storage, and demonstrates the value of frequency domain analysis via Fourier Transforms to characterize the cost-efficient operating regimes of each storage technology.

Section 2 presents the (linear) capacity planning model employed, which involves a very general description of storage technologies and ramping constraints on thermal generators, and shows the relations between the problem of maximizing overall social welfare and the problems faced by profit-maximizing, perfectly competitive suppliers of each available technology.

Section 3 considers optimal load-shedding, and operation of and investment in dispatchable and VRE generation. We present brief derivations of a number of known results for the sake of completeness and derive new results on the impacts of ramping constraints.

Section 4 provides generalizations of recent results regarding optimal investment in, profitability of, and operation of individual storage technologies.

Section 5 develops intuition regarding patterns of investment in and operation of storage when it is optimal to employ multiple technologies, and Section 6 provides simulation results that support the intuition developed in Section 5 and demonstrates the value of frequency domain analysis of storage operations.

Section 7 provides some concluding observations.

2. Optima and Equilibria

We formally consider a linear *T*-period model with one dispatchable technology (which we will often refer to as gas), one VRE technology, and a single storage technology. The restriction to a single technology of each type in this section is simply to reduce notational clutter. In later sections we consider systems with multiple technologies of each type when appropriate.

Throughout we abstract from storage's ability to supply frequency regulation and other ancillary services and to defer investment in transmission or distribution systems.

Because our focus is on the supply of electricity, we assume perfectly inelastic demand for simplicity. That is, we assume that demand in period t is equal to the exogenous quantity Q_t, for prices below ω, the value of lost load. Then total welfare, to be maximized, is given by

$$W = \omega\Sigma[Q_t - L_t] - \left[C_G G + C_R R + C_P^A P^A + C_P^D P^D + C_E E + v\Sigma g_t + o^A \Sigma A_t + o^D \Sigma D_t\right], \tag{1}$$

where L_t is the non-negative lost load in period t.[7] Throughout, sums are over t from 1 to T, unless otherwise specified.[8]

We assume constant returns to scale, so that we can work with the aggregate capacities and outputs of all facilities using the same technology.[9] From left to right the Cs in equation (1) are the T-period per-MW capital costs of dispatchable capacity, G, of renewable capacity, R, of charge power capacity of storage, P^A, and of discharge power capacity of storage, P^D, respectively. C_E is the T-period per-unit capital cost of energy storage capacity, E.

For a pumped hydro storage facility, for instance, P^A would be the maximum rate at which water can be pumped into the uphill reservoir, measured by the instantaneous power consumption of the pumping system in MW, P^D would be the maximum rate at which the facility can generate electricity, again in MW, and E would be the capacity of the reservoir in, say, cubic meters. (We discuss alternative definitions of E below.) For convenience we assume that P^A, P^D, and E can be chosen independently, though for some storage technologies this may not be

[7] To allow for price-responsive demand, the first term in (1) would be replaced by $U(Q_t;t)$, with U a concave utility function that is shifted by changes in t, and Q_t is a non-negative choice variable. With this change, for positive values of Q_t condition (9) below would be replaced by a requirement for marginal cost pricing, and nothing else in the analysis would change. Joskow and Tirole (2007) analyze markets with both price-responsive and unresponsive demand and also consider system collapses and inefficient rationing.

[8] With a single investment period, allowing for uncertainty that is resolved after investment would mainly complicate formulas and change the focus of break-even analysis from total net revenue to total expected net revenue. Similarly, allowing for discounting would complicate formulas and change the focus of break-even analysis from total net revenue to total discounted net revenue. The only difference in optimal operation would be in the evolution of the value of stored energy, which is discussed below Proposition 7.

[9] For gas generation in moderately large systems, constant returns is a good approximation. Some storage technologies may exhibit increasing returns, however: the surface area of a tank rises less than proportionately with its volume, for instance.

possible. For pumped hydro for instance, the same turbine is often used to pump water into the reservoir and to generate electricity when the water is released.[10]

Also in equation (1), v is the (constant) marginal cost of dispatchable generation, g_t is dispatchable generation in period t, o^A is the variable operation and maintenance (O&M) cost per MWh used to charge storage, A_t is MWh used to charge storage in period t, o^D is the variable O&M per MWh discharged, and D_t is MWh discharged from storage in period t. In the context of pumped hydro storage, one can think of o^A and o^D reflecting the marginal wear and tear caused by pumping water into the uphill reservoir and using water from that reservoir to generate electricity, respectively. In the case of battery storage systems, these parameters are best thought of as providing an approximation to the degradation caused by charging and discharging.[11]

We consider maximization of W subject to a set of linear constraints. The Karush-Kuhn-Tucker (KKT) stationary conditions can thus be employed to characterize maxima. The constraints that supply (including lost load) is equal to demand in each period can be written as

$$(2) \qquad L_t + g_t + [\theta_t R - R_t^C] + D_t - A_t - Q_t = 0 \quad (\lambda_t), \quad t = 1,\ldots,T.$$

Here θ_t is the fractional capacity factor of renewables, R_t^C is the non-negative amount of renewable output that is curtailed, all in period t. The KKT multipliers λ_t are the marginal values of energy in period t in the planner's problem and the spot price of electricity under competition. In all that follows, we show the multipliers corresponding to each set of constraints in parentheses after the constraints. Multipliers corresponding to non-negativity constraints are themselves non-negative. The constraints that renewable curtailment not exceed renewable output are

$$(3) \qquad \theta_t R - R_t^c \geq 0 \quad (\vartheta_t), \quad t = 1,\ldots,T.$$

While λ_t can have any sign in this model, one would expect it to be non-negative in most time periods. In more general models (like the simulation model used to generate the results in

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[10] If for technological reasons $P^A = P^D = P$ for some technology, then if some facilities using that technology are charging and others are simultaneously discharging, total charging plus total discharging cannot exceed P. Since, as discussed below, simultaneous charge and discharge for the same technology can occur only in very special cases, this constraint is not explicitly imposed here.

[11] The degradation of Lithium-ion batteries with both time and usage has been much studied; see Gailani et al (2020) for a recent contribution and references to that literature.

Section 6) that enforce minimum stable outputs of thermal generators or costs of startup and shutdown of such generators, the marginal value of energy can be negative. If thermal generation is needed in period t+1 but optimally shut down in period t, for instance, an increase in demand in period t that would enable avoidance of startup costs in period t+1 would lower system cost.[12] The competitive energy price in period t would accordingly be negative. For the sake of generality, we do not constrain λ_t to be non-negative.

Let S_t be the amount of energy in storage at the end of period t, with value at the start of period one (end of period zero) equal to S_0. We impose the constraint that storage not accumulate or dissipate energy over the T periods:

(4) $$S_0 - S_T = 0 \quad (\mu_0).$$

Energy in storage is analytically somewhat similar to the amount of water in a hydroelectric reservoir, except that it is increased by taking energy from the power system rather than by exogenous inflows. The equations of motion for end-of-period energy in storage are

(5) $$\chi S_{t-1} + r^A A_t - (1/r^D) D_t - S_t = 0 \qquad (\mu_t), \quad t = 1,...,T.$$

Here $\chi \leq 1$ is a constant reflecting self-discharge in some storage systems (e.g., evaporation from pumped hydro reservoirs, battery self-discharge) and μ_t is the marginal value of energy in storage at the end of period t. (Note that unlike λ_t, μ_t does not correspond to an observable market price.) Charging and discharging storage involves loss of useful energy. For every MWh used to charge storage, r^A of energy is actually stored. Similarly, in order to discharge one MWh from storage, $(1/r^D)$ of stored energy must be used.[13]

If stored energy is measured by the quantity of water in a reservoir, it is clear that the values of r^A and r^D will depend on whether that quantity is measured in cubic meters or

[12] The idea that increasing demand can sometimes lower system cost is not just a theoretical possibility (Hawai'i Natural Energy Institute 2019). A diesel generator is used to follow load on the Hawaiian island of Moloka'i, population 7,345. In 2015 the local utility found that if it granted all pending applications for rooftop solar generation, the difference between demand and solar output would occasionally fall below the diesel's minimum output level, causing the generator to trip off and the island to black out. To avoid the high cost of blackouts when those applications had been granted, the utility installed a "load bank", a dispatchable resistive load that could be used when necessary to transform electric energy into waste heat.

[13] This model of storage generalizes that of Crampes and Trochet (2019). They assume that $\chi = 1$, $o^A = o^D = 0$, and $r^D = 1$. As the discussion below indicates, this last restriction does not entail a loss of generality). Brown and Reichenberg (2020) assume $o^A = o^D = 0$.

milliliters. Round-trip efficiency, the incremental MWh discharge made possible by an incremental use of one MWh in charging, $r \equiv r^A r^D$, is independent of how E is defined and how S is measured and is strictly less than one in real storage systems.

Because energy is lost in the process of charging and discharging storage, one would expect the value of stored energy to be non-negative. But if the shadow price of energy is negative, as discussed above, so that an increase in supply would raise system costs, it is not implausible that the value of stored energy should also be negative. In addition, as we discuss further below, it can sometimes be optimal to increase demand by simultaneously charging and discharging facilities that use the same technology and thus dissipating energy.[14]

Condition (5) is consistent with the "capacity of the reservoir" definition of E given above, so that if neither r^A nor r^D equals one, S_t is equal neither to the electric energy used to store it nor to the electric energy recoverable from it. The maximum electric energy recoverable from a reservoir of capacity E is $E^d \equiv r^D E.$ which one might call "electrical energy discharge capacity" to distinguish it from the "size of the reservoir" definition that underlies (5). Assuming linearity, the unit cost of energy storage capacity thus defined would be $C_E^d = (1/r^D)C_E$, and, continuing to use d superscripts to denote quantities corresponding to a "maximum energy recoverable" definition of energy storage capacity, condition (5) would become

$$\text{(5')} \qquad \chi S_{t-1}^d + rA_t - D_t - S_t^d = 0 \qquad (\mu_t^d), \quad t = 1,\dots,T$$

To be consistent with the definition of E^d, S_t^d must be the energy deliverable to the system corresponding to that state of charge, so $S_t^d = r^D S_t$, and equation (5') is just equation (5) multiplied through by r^D.

One could similarly re-write the model by instead defining energy storage capacity as the maximum amount of electric energy the system could absorb. Under that definition, one would obtain the equation of motion corresponding to (5) by dividing through by r^A. Then A_t would have a coefficient of one, and D_t would have a coefficient of (1/r). The cost of energy storage capacity, C_E, depends on what definition of capacity is used, as does the per-unit shadow price of

[14] "Simultaneously" in this discrete-time framework means "within the same short period".

energy in storage, μ_t. Both the power cost parameters C_P^A and C_P^D and the flow parameters o^A, o^D, r, and χ are independent of the definition used to characterize energy storage capacity.

Most of the relevant technical literature uses the "size of the reservoir" definition of energy storage capacity, since separate consideration of charge and discharge efficiencies may be of interest in the analysis of the internal operation of storage facilities. From the point of view of a power system, however, all that matters is their product, r, which relates an increment of energy used to charge storage to the incremental energy subsequently available for discharge. We can thus simplify notation without loss of generality by adopting the "maximum energy deliverable" definition of energy storage capacity and using (5') without the d superscripts as the equation of motion of end-of-period energy in storage. A storage technology in this model is thus described by seven parameters: three capital cost parameters (C_P^A, C_P^D, and C_E) and four flow parameters (o^A, o^D, r, and χ).

We assume that limitations on per-period changes in the outputs of dispatchable generators relative to their capacities, so-called ramping constraints, may sometimes be binding:

(6a) $$\beta^U G + g_{t-1} - g_t \geq 0, \quad (\rho_t^U), \quad t = 2,...,T,$$

(6b) $$\beta^D G + g_t - g_{t-1} \geq 0, \quad (\rho_t^D), \quad t = 2,...,T.$$

Here β^U and β^D are exogenous, positive constants strictly less than one. Note that there are no ramping constraints on first-period output. The only formal analysis of a ramping constraint for which we are aware is Biggar and Hesamzadeh (2014, Section 4.9), and they constrain only the absolute increase in generation.

In addition to conditions (2) – (6), the following inequalities must also be satisfied, with R^U and E^U positive constants:

(7a) $$R \geq 0 \quad (R_0), \qquad R^U - R \geq 0 \quad (R_u), \qquad G \geq 0 \quad (G_0).$$

(7b) $$P^A \geq 0 \quad (P_0^A), \quad P^D \geq 0 \quad (P_0^D), \quad E \geq 0 \quad (E_0), \quad E^U - E \geq 0 \quad (E_u).$$

The upper bound constraint on renewable capacity in (7a) can arise if, for instance, some bounded sites are particularly good for wind generation. Similarly, the upper bound constraint on energy storage capacity could reflect, for instance, limits on the size of the uphill reservoir in a pumped hydro system.

In addition, the following inequalities must hold for all t:

(7c) $L_t \geq 0 \quad (\eta_t), \quad R_t^C \geq 0 \quad (r_t^0), \quad g_t \geq 0 \quad (\gamma_t^0), \quad G - g_t \geq 0 \quad (\gamma_t^U).$

(7d) $D_t \geq 0 \quad (\delta_t^0), \quad P^D - D_t \geq 0 \quad (\delta_t^U), \quad A_t \geq 0 \quad (\alpha_t^0), \quad P^A - A_t \geq 0 \quad (\alpha_t^U).$

(7e) $S_t \geq 0 \quad (\varepsilon_t^0), \quad E - S_t \geq 0 \quad (\varepsilon_t^U).$

Note that in (7e), the non-negativity constraint is also enforced for S_0 with a corresponding multiplier, ε_0^0. The planner's problem is to choose capacities (G, R, P^A, P^D, and E) and flow variables ($L_t, g_t, R_t^C, D_t, A_t$, and S_t for $t=1,\dots,T$) to maximize W subject to constraints (2) – (7). The Lagrangian for this problem is the following:

(8)
$$\begin{aligned}\Lambda = {} & \omega\Sigma[Q_t - L_t] - \left[C_G G + C_R R + C_P^A P^A + C_P^D P^D + C_E E + \nu\Sigma g_t + o^A\Sigma A_t + o^D\Sigma D_t\right] \\ & + \Sigma\lambda_t[L_t + g_t + (\theta_t R - R_t^C) + D_t - A_t - Q_t] + \Sigma\vartheta_t[\theta_t R - R_t^c] \\ & + \Sigma\mu_t[\chi S_{t-1} + rA_t - D_t - S_t] + \mu_0[S_0 - S_T] \\ & + \Sigma_{t=2}^T\rho_t^U[\beta^U G + g_{t-1} - g_t] + \Sigma_{t=2}^T\rho_t^D[\beta^D G + g_t - g_{t-1}] + R_0 R + R_u[R^U - R] + G_0 G \\ & + P_0^A P^A + P_0^D P^D + E_0 E + E_u[E^U - E] + \Sigma\eta_t L_t + \Sigma r_t^0 R_t^C + \Sigma\gamma_t^0 g_t + \Sigma\gamma_t^U[G - g_t] \\ & + \Sigma\delta_t^0 D_t + \Sigma\delta_t^U[P^D - D_t] + \Sigma\alpha_t^0 A_t + \Sigma\alpha_t^U[P^A - A_t] + \Sigma\varepsilon_t^0 S_t + \Sigma\varepsilon_t^U[E - S_t].\end{aligned}$$

Rearrangement of terms in this equation yields

(9) $$\Lambda = \Lambda_L(\vec{L};\vec{Q}\vec{\lambda}) + \Lambda_R(R,\overrightarrow{R^C};\vec{\lambda}) + \Lambda_G(G,\vec{g};\vec{\lambda}) + \Lambda_S(P^A,P^D,E,\vec{A},\vec{D};\vec{\lambda}),$$

where an arrow above a variable means the values of that variable from t=1 to t=T, and

(10a) $$\Lambda_L = \Sigma(\lambda_t - \omega)L_t + \Sigma(\omega - \lambda_t)Q_t + \Sigma\eta_t L_t$$

(10b) $$\Lambda_R = \Sigma(\lambda_t + \vartheta_t)(\theta_t R - R_t^C) - C_R R + R_0 R + R_u[R^U - R] + \Sigma r_t^0 R_t^C,$$

(10c)
$$\begin{aligned}\Lambda_G = {} & \Sigma(\lambda_t - \nu)g_t - C_G G + \Sigma_{t=2}^T\rho_t^U[\beta^U G + g_{t-1} - g_t] + \Sigma_{t=2}^T\rho_t^D[\beta^D G + g_t - g_{t-1}] \\ & + G_0 G + \Sigma\gamma_t^0 g_t + \Sigma\gamma_t^U[G - g_t],\end{aligned}$$

$$\Lambda_S = \Sigma\lambda_t(D_t - A_t) - \left[C_P^A P^A + C_P^D P^D + C_E E + o^A \Sigma A_t + o^D \Sigma D_t\right]$$
$$+ \Sigma\mu_t[\chi S_{t-1} + rA_t - D_t - S_t] + \mu_0[S_0 - S_T]$$
(10d)
$$+ P_0^A P^A + P_0^D P^D + E_0 E + E_u[E^U - E] + \Sigma\delta_t^0 D_t + \Sigma\delta_t^U[P^D - D_t]$$
$$+ \Sigma\alpha_t^0 A_t + \Sigma\alpha_t^U[P^A - A_t] + \sum_{t=0}^{T}\varepsilon_t^0 S_t + \Sigma\varepsilon_t^U[E - S_t].$$

By inspection, each of expressions (10a)-(10d) is the Lagrangian for the problem of choosing associated stock and flow variables to maximize the profit of a particular technology (including loss of load) subject to the inequality constraints relevant to that technology, treating the λ_t as exogenous. This is exactly the problem that would be solved by a perfectly competitive industry supplying that technology and treating energy prices as given. We have thus established

> **Proposition 1: Equilibria and Optima.** Under constant returns to scale, the necessary conditions for maximizing social welfare are identical to the necessary conditions for maximizing the profits of competitive industries supplying each of the available technologies

In what follows we consider operation of and (except for loss of load) investment in each of the available technologies in turn. We will refer to a point at which all the KKT conditions for constrained welfare maximization are satisfied as "an optimum," understanding that such a point is also a constrained maximum of technology-specific profits under competition with the λ_t as energy prices. Under competition and constant returns to scale, one might expect that the suppliers of each technology would just break even at an optimum. We verify this expectation below.

The KKT necessary conditions for constrained maxima include both that the derivatives of the corresponding Lagrangian with respect to each decision variable be zero and the complementary slackness conditions corresponding to the inequality constraints in (3), (6) and (7). These require that the products of the non-negative multipliers and the corresponding constrained quantities be zero. Thus, for instance, at the optimum $R^0R = 0$, so that if $R > 0$, then $R^0 = 0$, and if $R^0 > 0$, then $R = 0$.[15]

[15] Purely to simplify the presentation, we will not deal explicitly with knife-edge cases in which both the multiplier and the constrained quantity are zero.

3. Generation and Load

This brief section provides a reasonably complete presentation of general results relating to investment in and operation of renewable and dispatchable generation, as well as the conditions for loss of load. Differentiating (10a), at an optimum, we must have

(11) $$\partial\Lambda_L / \partial L_t = \lambda_t - \omega + \eta_t = 0.$$

From (7c), if lost load is positive, $\eta_t = 0$. Then condition (11) implies that $\lambda_t = \omega$, the value of lost load, establishing

Proposition 2: Lost Load. At an optimum, if lost load is positive in any period, the energy price equals the value of lost load, ω.

Differentiating (10b), at an optimum, the following first-order conditions related to VRE technologies must be satisfied:

(12a) $$\partial\Lambda_R \big/ \partial R_t^C = -(\lambda_t + \vartheta_t) + r_t^0 = 0, \quad t = 1,\ldots,T,$$

(12b) $$\partial\Lambda_R / \partial R = \Sigma(\lambda_t + \vartheta_t)\theta_t - C_R + R_0 - R_u = 0.$$

Condition (12a) establishes that if curtailment is positive in some period, so $r_t^0 = 0$, then $(\lambda_t + \vartheta_t)$ must equal zero. If curtailment is partial, so that $R_t^c < \theta_t R$, then $\vartheta_t = 0$, and the energy price must also be zero. If VRE output is completely curtailed, so (3) is binding and $\vartheta_t > 0$, the energy price must be negative.

If there is no curtailment in period t, $\vartheta_t = 0$, and revenue per unit of VRE capacity is just $\lambda_t\theta_t$. If there is curtailment, $(\lambda_t + \vartheta_t) = 0$, and revenue is zero either because the energy price is zero or because VRE output is completely curtailed. Thus the first two terms on the right of (12b) are VRE generator profit per unit of capacity. Note first that at most one of R_u and R_0 can be positive. If both are zero, so is total per-unit profit, consistent with competitive investment behavior. If the lower-bound constraint on investment binds, so that $R_0 > 0$ and the socially optimal investment is zero, it follows that the derivative of profit with respect to capacity is negative at zero capacity, so that profit would be reduced below zero if capacity were increased above zero. Finally, if the upper-bound constraint binds and $R_u > 0$, profit is positive in competitive equilibrium and at a social optimum. As noted above, a binding upper-bound constraint most plausibly reflect the limited size of a particularly good site for wind or solar generation, in which case the value of the multiplier on that constraint corresponds to the rental value of the corresponding site. We have thus established.

> **Proposition 3: VRE**. At an optimum, (a) since the marginal cost of VRE supply is zero, VRE generation is curtailed only when the energy price is non-positive, (b) VRE generation is completely curtailed only when the energy price is negative, (c) any VRE technology for which investment is positive earns a positive profit if and only the upper-bound constraint on capacity is binding, otherwise its profit is zero.

We now turn to dispatchable generation. From (10c), in addition to complementary slackness conditions on G, the g_t, and the percentage increases in gas generation, the following must hold at an optimum:

(13a)
$$\partial\Lambda_G / \partial g_t = 0, \quad t = 1 \ldots T, \quad or$$
$$\begin{aligned} &\lambda_1 - v = (\gamma_1^U - \gamma_1^0) - (\rho_2^U - \rho_2^D), && t = 1, \\ &\lambda_t - v = (\gamma_t^U - \gamma_t^0) + (\rho_t^U - \rho_t^D) - (\rho_{t+1}^U - \rho_{t+1}^D), && t = 2,\ldots,T-1, \\ &\lambda_T - v = (\gamma_T^U - \gamma_T^0) + (\rho_T^U - \rho_T^D) && t = T. \end{aligned}$$

(13b)
$$\partial\Lambda_G / \partial G = -C_G + \Sigma_{t=2}^T \rho_t^U \beta^U + \Sigma_{t=2}^T \rho_t^D \beta^D + G_0 + \Sigma \gamma_t^U = 0.$$

To understand condition (13a), note from (6) that beginning in period 1, incremental dispatchable generation in period t serves to relax the upward ramping constraint in period $t+1$ and tighten the downward ramping constraint in that period, but generation in period T has neither effect.

Suppose that for some dispatchable generation technology, ramping constraints are either absent or never binding and that positive capacity is optimal. Then condition (13a) becomes

(13a')
$$\lambda_t - v = \gamma_t^U - \gamma_t^0, \quad t = 1,\ldots,T.$$

Whenever this technology has positive output, $\gamma_t^0 = 0,$ and if generation is at capacity, $\gamma_t^U > 0,$ and we have established

> **Proposition 4: Operation of Dispatchable Generation Without Ramping Constraints**. At an optimum, for any dispatchable generation technology for which optimal capacity is positive, if ramping constraints are absent or never binding, then in

any period (a) generation is positive only if the market price of energy is greater than or equal to marginal cost, (b) if the inequality is strict, generation is at capacity, and (c) if two dispatchable technologies have positive capacities and different marginal costs, if the one with the higher marginal cost has positive generation, so does the one with lower marginal cost.

Parts (a) and (c) describe classic merit-order dispatch, in which plants with lower marginal costs are dispatched before those with higher marginal costs.

Now consider ramping constraints by examining condition (13a). If only period *t*'s *upward* ramping constraint binds ($\rho_{t+1}^U = 0, \rho_{t+1}^D = 0, \rho_t^D = 0, \rho_t^U > 0$), clearly g_t is positive, so that $\gamma_t^0 = 0,$ and price is strictly greater than marginal cost. If only period *t*'s *downward* ramping constraint is binding ($\rho_{t+1}^U = 0, \rho_{t+1}^D = 0, \rho_t^D > 0, \rho_t^U = 0$), clearly g_t is less than capacity, so that $\gamma_t^U = 0,$ and price is strictly less than marginal cost. On the other hand, if only the next period's *upward* ramping constraint binds ($\rho_{t+1}^U > 0, \rho_{t+1}^D = 0, \rho_t^D = 0, \rho_t^U = 0$), it must be that g_t is less than capacity and $\gamma_t^U = 0$. In this case, the marginal benefit from current generation exceeds the energy price, and generation may be positive even if marginal cost exceeds that price. Finally, if only the next period's downward ramping constraint binds ($\rho_{t+1}^U = 0, \rho_{t+1}^D > 0, \rho_t^D = 0, \rho_t^U = 0$), it must be that g_t is positive and $\gamma_t^0 = 0,$ so price must strictly exceed marginal cost.

Proposition 6: Operation of Dispatchable Generation with Ramping Constraints.[16] At an optimum, for any gas generation technology for which capacity is positive, then in any period (a) if only the current period's upward (downward) ramping constraint is binding, the energy price is strictly greater than (less than) marginal cost, (b) if only the next period's upward ramping constraint is binding, positive generation may be optimal even when the energy price is less than marginal cost, (c) if only the next period's downward ramping constraint is binding, price is strictly greater than marginal cost and

[16] Biggar and Hesamzadeh (2014, Section 4.9) provide a formal discussion of operation with ramping constraints in this basic setup. They consider a constraint on the absolute increase in generation, independent of the level of capacity, and they derive versions of (a) and (b) of Proposition 5.

(d) if ramping constraints are sometimes binding, merit-order dispatch may not always be optimal.

Part (d) follows because (b) implies that in some period a high-*v* technology may generate at a price below its marginal cost if only its next-period upward ramping constraint is binding.

Even though merit-order dispatch is not always optimal in the presence of ramping constraints, one would expect the predictions of Proposition 4 to hold most of the time. As we discuss in Section 5, one would expect a similar (though less general) set of predictions regarding the optimal dispatch of different storage technologies to hold most of the time, and we present simulation evidence using Texas data to support that expectation.

Appendix A contains a proof that any dispatchable generation with positive capacity at an optimum breaks even, even with ramping constraints:

Proposition 6: Investment in Dispatchable Generation.[17] At an optimum, any dispatchable generation technology for which investment is positive earns zero profit.

4. Storage: General Results

Proceeding as above, in addition to the complementary slackness conditions corresponding to the storage-related inequality constraints in (7), the necessary conditions for operating and investing in any storage technology to maximize welfare or for a competitive equilibrium in storage supply are

(14a) $$\partial \Lambda_S / \partial D_t = \lambda_t - o^D - \mu_t + \delta_t^0 - \delta_t^U = 0.$$

(14b) $$\partial \Lambda_S / \partial A_t = -\lambda_t - o^A + r\mu_t + \alpha_t^0 - \alpha_t^U = 0.$$

(14c) $$\begin{aligned} \partial \Lambda_S / \partial S_t &= \mu_0 + \chi\mu_1 + \varepsilon_0^0, && t = 0, \\ &= -\mu_t + \varepsilon_t^0 - \varepsilon_t^U + \chi\mu_{t+1} = 0, && t = 1, \ldots, T-1, \\ &= -\mu_T + \varepsilon_T^0 - \varepsilon_T^U - \mu_0 = 0, && t = T. \end{aligned}$$

[17] We have not seen a zero-profit proof for dispatchable generation with ramping constraints elsewhere. Without those constraints, the zero profit result seems to have first been asserted, but not proven, in Crew and Kleindorfer (1986, Section 3.3). For recent zero-profit proofs for dispatchable generation without ramping constraints in a linear model of the sort considered here, see Biggar and Hesamzadeh (2014, chs. 9-10) for a timeless model and Brown and Reichenberg (2020, Section 6.1) for a dynamic model with perfect foresight.

(14d) $\partial\Lambda_G / \partial P^A = -C_P^A + P_0^A + \Sigma\alpha_t^U = 0.$

(14e) $\partial\Lambda_S / \partial P^D = -C_P^D + P_0^D + \Sigma\delta_t^U = 0.$

(14f) $\partial\Lambda_S / \partial E = -C_E + E_0 - E_u + \Sigma\varepsilon_t^U = 0.$

To understand (14c), note from (5') that S_t relaxes the storage constraint in period t+1, while S_T has no such effect. In considering (14a) – (14c), it is important to remember that, unlike the λ_t, which correspond to observable energy prices, the μ_t are technology-specific (and, as noted above, dependent on the unit of measure of E and S) and unobservable.

Inspection of (14a) and (14b) and complementary slackness conditions (7d) imply that if storage is discharging (charging) then $\delta_t^0 = 0$ ($\alpha_t^0 = 0$) and if the charging (discharging) rate is below capacity, then $\delta_t^U = 0$ ($\alpha_t^U = 0$) else $\delta_t^U > 0$ ($\alpha_t^U > 0$). This serves to establish

Proposition 7: Operation of Storage.[18] At an optimum, in every period (a) if $\lambda_t \geq o^D + \mu_t$, storage is discharging, (b) it is discharging at capacity if the inequality is strict, (c) if $(\lambda_t + o^A)(1/r) \leq \mu_t$, storage is charging, (d) it is charging at capacity if the inequality is strict, and (e) otherwise, it is idle.

These are simple arbitrage conditions. Using the "maximum energy deliverable" definition of capacity, (a) reflects the fact that energy in storage can be delivered to the grid at a marginal cost of $(\mu_t + o^D)$. If the value of energy is at least equal to that cost, discharge may be optimal. Similarly, the marginal cost per MWh used to charge storage is $(\lambda_t + o^A)$, and it takes (1/r) MWh from the grid to increase energy in storage by one MWh.

We noted above (footnote 8) that it may sometimes be optimal to charge some facilities using a particular storage technology while simultaneously discharging other facilities using the same technology. In this case $\delta_t^0 = \alpha_t^0 = 0$, and both δ_t^U and α_t^U are non-negative. Conditions (14a) and (14b) then imply

[18] This is a generalization of Proposition 1 in Crampes and Trochet (2019) to allow for more general storage technologies. They begin with the problem of maximizing the profit of a price-taking storage supplier and do not embed it in the problem of welfare maximization as we do here. In addition, they do not allow for variable O&M.

(15a) $$\lambda_t = o^D + \mu_t + \delta_t^U \geq o^D + \mu_t,$$

(15b) $$\lambda_t = -o^A + r\mu_t - \alpha_t^U \leq -o^A + r\mu_t.$$

Combining and re-arranging (15a) and (15b) yields a necessary condition for simultaneous charge and discharge to be optimal:

(15c) $$\mu_t \leq -(o^A + o^D)/(1-r).$$

If μ_t is positive, this condition cannot be satisfied. Condition (15a) then implies that λ_t is also positive, and system cost cannot be reduced by increasing demand.

If $(o^A + o^D) = 0,$ condition (15c) can be satisfied with $\mu_t \leq 0.$ If $\mu_t = 0,$ conditions (15a) and (15b) require $\lambda_t = 0.$ If condition (15c) is satisfied with $\mu_t < 0,$which is the only way it can be satisfied if variable O&M cost is positive, condition (15b) requires $\lambda_t < 0.$Summarizing this discussion, we have

Proposition 8: Simultaneous Charge and Discharge. If for any type of storage in any period (a) if $\mu_t > 0,$ simultaneous charge and discharge is not optimal, (b) if $(o^A + o^D) = 0,$simultaneous charge and discharge may be optimal if $\mu_t = \lambda_t = 0$ or if both quantities are negative, and (c) if $(o^A + o^D) > 0,$ simultaneous charge and discharge may be optimal only if $\mu_t < 0$ and $\lambda_t < 0.$

If two or more types of storage are optimally employed, the numerical analysis discussed in Section 6 has revealed that it is occasionally optimal to charge units of one type while discharging units of another type even if the conditions of Proposition 8 are not satisfied for either type.

Inspection of condition (14c) immediately establishes

Proposition 9: Value of Stored Energy. At an optimum for any storage technology, for $t = 1,\ldots,T$-1, (a) when storage is neither full nor empty, $\mu_{t+1} = (1/\chi)\mu_t \geq \mu_t,$ (b) if storage is full μ increases more rapidly, and (c) if storage is empty μ decreases if $\chi = 1$ but may increase if $\chi < 1$.

Crampes and Trochet (2019) note that when $\chi = 1$, the behavior of the μ_t when storage is neither empty nor full is consistent with Hotelling's (1931) rule: under competition and perfect foresight, the value of a durable asset must rise at the rate of interest, which is zero here. When $\chi < 1$, so the physical quantity of S declines when storage is neither empty nor full, the per-unit shadow value μ increases so keep the aggregate value of S constant.[19]

Appendix A provides the proof of

> **Proposition 9: Investment in Storage.**[20] Any storage technology for which optimal capacity is positive earns a positive profit only if the upper bound constraint on energy storage capacity is binding. Otherwise, profit is zero.

5. Multiple Storage Technologies: General Analysis

In Section 3, we considered situations in which it was optimal to have both baseload (e.g. combined-cycle gas plant) and peaker (e.g. simple cycle gas plant) gas generation capacity. In the absence of ramping constraints, it was easy to establish in this multi-period framework the classic result that peaking gas plants, which have higher variable cost, are used only when demand is particularly high and baseload capacity is fully utilized. We also showed, however that the intertemporal linkages that follow from ramping constraints add a level of operational complexity and destroy the universal validity of that classic result.

Because storage technologies with constant returns to scale are characterized in the most general case in this model by the values of seven parameters ($C_P^A, C_P^D, C_E, o^A, o^D, r$, and χ), it is not as simple to compare storage technologies as to compare constant-returns generation technologies that are completely described by their levels of per-unit fixed and variable cost. Moreover, one might expect the intertemporal linkages inherent in storage operation to invalidate any general rules as to which storage technologies would be used under what conditions. A natural, if informal, division is between short-term storage, in which intervals of charging and of discharging are close in time and long-term storage, in

[19] Suppose B equals one plus the positive rate of interest. Since μ_t is the period-t value of stored energy, it needs to be discounted by B^{-t} to obtain the value as of period zero. Equation (12c) then implies that when S is away from its bounds, $\mu_{t+1}=B(1/\chi)\mu_t$. The current-period value of stored energy rises at the rate of interest when χ=1 (the discounted value as of period zero is constant) and more rapidly when χ<1.

[20] The only prior zero-profit proof for storage of which we are aware is in Section 9.2 of Brown and Reichenberg (2020). As noted above, they assume $o^A = o^D = 0$.

which energy remains in storage for longer periods before it is discharged. An example of short-term storage would be charging batteries in mid-day using excess solar generation and then discharging them when the sun goes down. In contrast, some have argued that it could be valuable to have long-term storage that would enable energy provided by solar generators in the summer to be used to make up for lower solar output in the winter.

In the rest of this section we first present a simple cost analysis that suggests which sorts of storage technologies would be more suitable for short-term storage and which would be more suitable for long-term storage. We then use the KKT conditions developed above to provide further support for this suggestion, which is further substantiated via numerical experiments in Section 6.

It is useful to begin by considering a symmetric charge-discharge cycle for a storage facility with no variable O&M cost and no self-discharge. Suppose the facility is charged for a time t^A at average power p and then discharged at the same average power for a time t^D until the original state of charge is reached. Continuing to use the "maximum electric energy recoverable" definition of capacity, the total amount of energy stored in this cycle, e, is just pt^D. Letting $Z = 1/r$, a measure of round-trip *in*efficiency, the total amount of energy taken from the grid during the charging phase is $eZ = pt^A$. The total length of this cycle, t^t, is thus given by

$$t^t = t^A + t^D = \frac{eZ}{p} + \frac{e}{p} = \frac{e(1+Z)}{p}. \tag{16}$$

Longer cycles involve higher ratios of energy stored to average power employed in charging and discharging. This suggests that technologies with low ratios of energy storage capacity cost to charge and discharge power capacity cost are best suited to providing long-duration storage, all else (including Z) equal.

To refine this suggestion, it is necessary to consider a specific charge/discharge cycle. As we discuss further below, if it is optimal to employ a particular storage technology, it will be optimal for that technology to be fully charged during some periods. Similarly, if it were not fully discharged during some (other) periods, costs could have been saved by reducing energy storage capacity.[21] Thus the longest charge-discharge cycle any particular storage facility could experience would be from full discharge to full charge and back again. It is instructive to examine how the average cost of delivered power associated with such a maximal cycle depends on the cycle length and various cost parameters.

Putting aside the cost of energy to charge a particular storage installation and the revenue from discharging and selling energy from storage, the total capital and operating cost of such a maximal cycle is given by

(17) $$TC = t^t[c_P^A P^A + c_P^D P^D + c_E E] + o^A \Sigma A_t + o^D \Sigma D_t,$$

where t^t is the total time the cycle takes in hours, and the *c'*s are per-hour costs of the various capacities. For concreteness, we assume that the facility is initially fully discharged, then charges at power P^A until it is fully charged, then completes the cycle by discharging at power P^D until it is fully discharged. To simplify formulas, let $k = P^D/P^A$, where k is a positive constant. For some technologies k is fixed (e.g., $k = 1$ for electrochemical storage), while for others it is an outcome of an optimization that we suppress here. In addition, it is convenient to define $x \equiv 1 - \chi$, the rate of self-discharge.

Let $t^A(x)$ be the time taken to charge storage fully as a function of the self-discharge rate, let $t^D(x)$ similarly be the time taken to discharge storage completely, so that the total time for a charge/discharge cycle, $t^t(x)$, is just the sum of t^A and t^D. The total amounts of energy delivered to and taken from the grid in one cycle are just $t^D P^D$ and $t^A P^A$, respectively. Dividing equation (17) by total energy delivered to the grid, $t^D P^D$, yields the average cost per MWh:

(18a) $$AC(x) = \left[1 + \left(\frac{t^A}{t^D}\right)\right]\left[c_P + c_E \frac{E}{P^D}\right] + \left[\left(\frac{t^A}{t^D}\right)\frac{o^A}{k} + o^D\right],$$ where

(18b) $$c_P \equiv (c_P^A / k) + c_P^D.$$

When $x = 0$, so there is no self-discharge, $t^A = E/rP^A = ZkE/P^D$, $t^D = E/P^D$, and equation (18a) becomes

[21] As a practical matter, storage facilities may be degraded by either being fully charged or fully discharged, so that the normal range of operation is somewhat smaller than the technical level of capacity would indicate.

(19) $$AC(0) = (1 + Zk)c_P + t^t c_E + (Zo^A + o^D).$$

This equation implies that for values of x sufficiently close to zero, if a storage unit is continuously charged at maximum charging power capacity until it is fully charged and then discharged at maximum discharge power until it is empty, the average cost per discharged MWh over that cycle has three components. The first reflects the average capital cost of power charge and discharge capacity. Low round-trip efficiency (high Z) in effect raises power capacity cost, because each unit of power capacity is less effective at producing deliverable energy. The third component measures effective round-trip O&M cost. Low round-trip efficiency (high Z) increases round-trip O&M cost because more energy must be taken from the grid for each MWh later returned to it. The second component is the only one that depends on the total duration of the charge/discharge cycles, t^t. The derivative of overall cost with respect to duration here is exactly equal to the per-period energy storage capacity cost (c_E).

If self-discharge is positive, it takes longer to charge the storage fully because energy is lost during the charging process through self-discharge, and it takes less time to completely empty the storage for the same reason. It follows that more energy is taken from the grid during charging, and less energy is delivered to the grid during the discharge phase of the cycle. Appendix A evaluates the charge and discharge times for positive values of x and obtains an approximate value of AC for small but non-zero values of x, equation (A.9). AC is increasing in x for x near zero, as one would expect, but the main implications of equation (19) are preserved.

Consider two storage technologies, 1 and 2, with technology 1 having higher ratio of energy storage capacity costs to average power capacity costs compared to technology 2. For the same flow cost parameters (Z, o^A, o^D), then the average cost per discharged MWh of technology 2 can only be equal to average cost per discharged MWh of technology 1 so long as the duration of charge/discharge cycles of technology 2 is greater than the duration of cycles for technology 1. This is a (very) rough analog to the usage implications of dispatchable generators with different levels of fixed and variable costs.

In the case of gas generation, it is a familiar result that if it is optimal to have positive capacities of two different technologies, the one with the higher variable cost must have lower fixed costs or it would have been dominated and not part of an efficient mix. In the case of storage, one expects that if it is optimal to have positive capacities of two storage technologies, the one with the lower cost of storage capacity must have higher charging/discharging costs. Equation (19) suggests that the technology with the lower energy storage capacity cost will tend to be used for longer duration storage, generally involving in effect higher values of t^t, than the one with the higher energy storage cost.[22]This suggestion has implications for the focus of R&D efforts concerned with long-term storage.

Additional support for this suggestion can be derived from the KKT necessary conditions, equations (14), and the relevant complementary slackness conditions. If a particular energy storage technology is deployed then by complementary slackness conditions applied to storage related constraints in (7), $E_0 = 0$, and, if the upper bound on energy storage capacity constraint is not binding, $E_u = 0$. Condition (14f) then reduces to

(20) $$C_E = \Sigma \varepsilon_t^U$$

Additionally, by complementary slackness conditions in (7) related to storage energy capacity, $\varepsilon_t^U = 0$ for all periods when energy in storage is below the installed storage capacity. Thus the summation on the right hand side of equation (20) can be reduced to periods when energy storage is at capacity. Letting the set of such periods be *F*, we have

(21) $$C_E = \sum_{t \in F} \varepsilon_t^U$$

The right hand side of (21) can be written terms of the stored value of energy using condition (14c), with the understanding that $\varepsilon_t^0 = 0 \; \forall t \in F$. For simplicity, we assume that storage is not full at the end of the last period.[23] This leads to

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[22] Crampes and Trochet (2019, section 3.2) provide a less formal discussion that reaches the same general conclusion.

[23] If storage is full at the end of the last period, then $C_E = \sum_{t \in F \setminus \{T\}} \left(\chi \mu_{t+1} - \mu_t \right) + \chi \mu_1 - \mu_T$ based on applying condition 14c and the fact that $\varepsilon_0^0 = 0$ (because of (4) and given that storage is full at period T). This is equivalent to $C_E = \sum_{t \in F} \left(\chi \mu_{t+1} - \mu_t \right)$ with the condition that period T+1 is identical to period 1.

(22) $$C_E = \sum_{t \in F} (\chi \mu_{t+1} - \mu_t)$$

If two storage technologies, 1 and 2, are deployed with non-zero energy storage capacities such that $C_E^1 > C_E^2$, equation (22) implies

(23) $$\sum_{t \in F^1} \left(\chi^1 \mu_{t+1}^1 - \mu_t^1 \right) > \sum_{t \in F^2} \left(\chi^2 \mu_{t+1}^2 - \mu_t^2 \right)$$

Here the superscripts 1 and 2 correspond to technologies 1 and 2, respectively. In Appendix A we demonstrate that for $r = 1$, all of the terms in parentheses in (23) are bounded above by $(\chi \lambda_{t+1} - \lambda_t)$. For $r < 1$, the bounds depend on the charge/discharge patterns for energy storage in periods t and t+1.

The most natural way for condition (23) to be satisfied is for storage technology 1 with higher capital costs of energy storage to spend more periods fully charged than storage technology 2. This is consistent with storage technology 1 following something like the fast-cycling pattern seen for Li-ion storage in the numerical results from the optimization model discussed in Section 6.

The analysis in this section can only be suggestive. Real storage technologies generally have different round-trip efficiencies and self-discharge rates, and O&M costs may not be negligible. For arbitrary time-paths of renewable generation and load, the optimal pattern of charging and discharging will never be as regular as the maximal cycles analyzed above. Similarly, in the analysis immediately above, there is no guarantee that the two technologies will be fully charged under comparable conditions. To shed more light on how different storage technologies are optimally employed together in practice, we turn to a numerical optimization exercise.

6. Multiple Storage Technologies: Simulation

To illustrate optimal investment in and operation of a power system with multiple storage technologies, we simulated a simplified representation of a future “Texas-like” grid under greenfield conditions and different combinations of low-carbon emissions constraints and storage technology availability scenarios.[24] This model was developed as part of the MIT Energy Initiative's *Future of Storage* study. As in earlier sections, the ability of storage to provide ancillary services and to enable deferral of investment in transmission and distribution systems was not modeled. We employed a capacity expansion model, GenX,[25] to determine the optimal generation and energy storage investments needed to meet exogenous demand over time, while satisfying various grid operation constraints, resource availability limits, and other policy/environmental constraints at an hourly temporal resolution. GenX implements the optimization problem described in section 2, while adhering to various additional technology-specific constraints, such as linearized representation of unit commitment (with startup costs) and minimum up/down time constraints of thermal generators, VRE resource availability limits and other imposed policy/environmental constraints. Notably, the model considers a high temporal resolution, in this case seven years of grid operations with hourly time steps, which allows for assessing the role for both short-duration and long-duration storage technologies. Main model features are listed in Table 1, while data sources, assumptions on capital costs and technological parameters used for generation and storage technologies are reported in Tables B.1-B.3 in Appendix B. The assumptions employed here for illustrative purposes may differ from those finally adopted in the *Future of Storage* study.

We consider Lithium-ion batteries and power-to-hydrogen-to-power ("Li-ion" and "H_2" for short) as the available storage technologies, with the estimated energy storage capacity cost much lower for H_2 than for Li-ion (Table B.3). We focus our numerical analysis on scenarios with stringent carbon emission intensity constraints in which storage is important: 10 grams and 1gram of CO_2 per kWh[26]. Model-optimal investment results for the two emissions constraint scenarios are summarized in Table 2.

Not surprisingly, increasing the stringency of carbon emissions constraints leads to increased roles for VRE generation and for storage technologies and a reduced role for thermal

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[24] We used data for the Electric Reliability Council of Texas (ERCOT), which operates the wholesale market that meets roughly 90% of demand in Texas.

[25] Jenkins and Sepulvada (2017). See Mallapragada et al. (2020) for detailed discussion of most current storage operational constraints in GenX.

[26] Without a carbon emissions constraint, the least-cost model solution yields an emissions intensity of 82.9 gCO_2/kWh and a system average electricity cost of \$39.5/MWh.

generation. Notably, going from the looser to the tighter emissions constraint leads to a 5-fold increase in the optimal energy storage capacity of H_2. Overall system average electricity cost increases by 12% as the CO_2 emissions constraint is tightened from 10 gCO_2/kWh to 1 gCO_2/kWh. Table 2 highlights that overall lost load is generally small compared to total demand, owing to the relatively high value of lost load (i.e., the maximum wholesale price) assumed in the scenarios ($50,000/MWh, see Table 1).

Figure 1 illustrates how the stored energy for the two technologies changes over time for the two CO_2 emissions constraint cases during three illustrative months of operation.[27] Storage operation is not described by regular charge-discharge cycles. On the contrary, the pattern of operation changes over time and between emission constraints, and most charge-discharge cycles are not complete. In the 1g CO_2/kWh case, H_2 mainly (but not exclusively) displays long-term storage behavior, while in the 10g CO_2/kWh cases H_2 cycles more frequently. In both cases, the frequency of storage discharge varies from month to month. Li-ion, on the other hand is primarily used for shorter cycles across both CO_2 emissions constraint cases, but it cycles more frequently in some periods than in others.

As noted above, the model setup does allow for the possibility of simultaneous charging and discharging of each storage technology as a way to avoid the startup and shutdown costs associated with thermal generators (which would otherwise lead to negative prices). That said, the assumptions about variable operating costs for storage and startup costs for thermal generation in the runs reported here result in no instances of simultaneous charge and discharge of each storage technology.

The numerical results also indicate that it may be optimal to simultaneously charge one storage technology and discharge the other, with such instances occurring 1.2% and 0.4% of the time for the 10 gCO_2/kWh and 1gCO_2/kWh cases, respectively.[28] In most of these instances, Li-ion is discharging and H_2 is charging. This behavior involves a loss of energy, but it must be that stored energy is sufficiently more valuable on the margin in H_2 than in Li-ion to make up for the loss, perhaps because stored energy will be required for a longer period in the future than Li-ion's limited energy capacity can handle.

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[27] Dowling et al (2020) present similar graphical depictions of the evolution of stored energy.

[28] This behavior has also been observed by Dowling et al (2020).

In both emission intensity cases, Table 1 highlights that Li-ion spends more hours per year in a fully charged state compared to H_2, which is consistent with the discussion of equation (19) above: the technology with the higher energy capital cost spends more periods in a fully charged state. Overall, these numerical observations of the optimal use of these technologies are broadly consistent with the intuition developed in Section 5, which suggested that the lower energy capital cost storage technology generally is deployed for longer-duration charge-discharge cycles.

Since the metric of equivalent discharges/year does not fully capture and reveal the complex nature of the operation of these systems, we use frequency domain analysis of the state of charge to produce a quantitative picture of the dominant modes of storage operation.[29] First, we applied the Fast Fourier Transform (FFT) to each state-of-charge time series. Next, we selected frequency bands of interest that contribute to different cycling periods and computed each band's contribution to the signal root mean square (RMS) value. As noted before, the cycling of storage technologies is complex and therefore it is not only described by single frequency components. Consequently, the frequency bands analysis allows us to aggregate multiple frequency components in a simple metric that provides an instructive summary of the relative importance of different modes of storage operation. We present results using the following indicative frequency bands:

- 0 to 12 cycles/year: Long-term or seasonal cycling
- 12 to 52 cycles/year: Intra-month cycling
- 52 to 365 cycles/year: Intra-week cycles
- Above 365 cycles/year: Intra-day cycles.

Table 3 displays the results of this analysis, using the full seven years of simulated data. It indicates that whereas in the 10g CO_2/kWh case H_2 storage mainly oscillates at monthly frequencies, in the 1g CO_2/kWh case seasonal oscillations become more relevant, and the contribution from the weekly and monthly cycling bands decreases. With a very tight emissions constraint, natural gas cannot be used to provide energy for appreciable seasonal storage, and H_2

[29] Fourier analysis has been addressed in an extensive literature. Brigham (1988, chapters 1 and 2) describes the FFT in a succinct way and shows the ubiquitous use of the method in different fields. In the context of storage integration in power systems, Victoria et al. (2019) have previously used Fourier-spectra analysis to illustrate the different operational behavior of storage under various carbon emissions constraint scenarios.

is the cheaper long-term storage technology as compared to Li-ion. Similarly, Li-ion displays a change in operation towards lower frequency cycling (less daily and weekly cycling) as the carbon constraint is tightened.

Finally, it is instructive to examine how the presence or absence of one storage technology influences the operating pattern of the other storage technology. Table 4 summarizes the model investment results for the scenario in which Li-ion is the only available energy storage technology, with all else equal. Comparing Tables 2 and 4 indicates that when H_2 is not available, VRE capacity is increased along with gas generation capacity, as well as Li-ion power and energy storage capacity, most noticeably in the tightest emissions constraint scenario (1 gCO_2/kWh). Total storage capacity is decreased substantially, however, since the relatively cheap storage provided by H_2 is not available.

The unavailability of H_2 results in a negligibly higher average electricity cost under the looser emissions constraint and a 5.2% higher average cost under the tighter constraint. Table 5 shows that when Li-ion is operating as the only storage technology, its total contribution in weekly, seasonal and monthly frequency bands are larger (87% vs. 81%) for the tightest emissions constraint 1g CO_2/kWh cases respectively[30]. This change in operating behavior is consistent with the fact that (per Tables 2 and 4) Li-ion spends fewer periods fully charged when H_2 is not available to supply longer-term storage. This comparison indicates that when a new storage technology (H_2 here) becomes economic, the efficient operating pattern of the pre-existing technology (Li-ion here) is likely to change.

7. Concluding Observations

In the classic Boiteux (1960, 1964)-Turvey (1968) framework for describing investment and operations of electric power systems, there are no links between supply or demand conditions in different periods. In order to permit an analysis of energy storage in which energy storage capacity has positive costs, we modified that framework to allow for sequences of periods linked by the operation of storage facilities (and, possibly, ramping constraints on thermal generators), with no restrictions on period-to-period changes in demand or in the output of VRE generators.

[30] We don't see this trend in the case of the 10g CO_2/kWh emissions constraint scenario, in part due to the availability of CCGT-CCS. At 1 gCO_2/kWh emissions scenario, the emissions intensity of CCGT-CCS limits its adoption and therefore there is greater reliance on VRE generation and storage to meet system demand.

Making the standard assumption that energy prices are allowed to rise to the value of lost load in shortage conditions,[31] the classic results for generation hold in this setting. At a welfare optimum or competitive equilibrium, all thermal and renewable generation technologies employed just break even, and the classic merit-order results for thermal generation hold. (Though the latter results are modified when ramping constraints bind.)

Our analysis reveals the greater complexity of efficient investment in and operation of storage facilities. In general, even under constant returns to scale as assumed here, storage technologies are described by the values of seven cost and performance parameters. Like reservoir hydroelectric facilities, optimal energy storage discharge depends on expectations about future demand and supply conditions, encapsulated in the shadow value of stored energy. Unlike reservoir hydro facilities, charging energy storage facilities (including pumped hydro facilities) is a decision, not something determined by nature, and the choice of storage capacity is generally less constrained than the choice of reservoir capacity.

We have nonetheless proven that all storage technologies employed just break even at a social optimum. Since social optima and competitive equilibria coincide in this model, this break-even result provides some support for general reliance on markets to drive investments in energy storage. We have also shown how optimal storage operation depends on the shadow value of stored energy, though that unobservable shadow value depends on conditions in future periods. It is not possible to establish fully general results regarding investment in and operation of multiple storage technologies, however; there is no simple merit-order analog even under perfect foresight.

We have shown that if it is optimal to employ multiple storage technologies, the ones with the lowest capital cost of energy storage capacity are generally best suited to providing long-term storage.[32] But we have also shown by example that storage technologies optimally play multiple roles in grid operations, providing charge-discharge cycles of various durations. Our simulation exercises show that when multiple storage technologies are employed, frequency domain analysis is useful for characterizing the relative importance of the different cycle

[31] It should be noted that this assumption, while standard, is not always descriptive of the behavior of system operators in the U.S.

[32] If it is also optimal to employ a storage technology with a higher energy storage capacity cost, that technology must be superior on some other dimension. In our simulation exercise, Table B3 reveals that Li-Ion has lower charge and discharge power capacity costs, as well as higher round-trip efficiency, than H_2.

durations that each provides and that these relative weights depend on the mix of generation and storage technologies employed.

We see three important directions for future work. First, as noted above, we have assumed that the market price of energy can rise to the value of lost load under shortage conditions, and in our simulation exercises non-served energy events do sometimes occur. In the model analyzed here the quantity ω, which we have called the value of lost load, simply serves as an exogenous cap on the price of energy. If, as in many organized markets, the cap on energy prices is set below the true value of lost load, the competitive market will exhibit a "missing money" problem (Joskow 2008): the equilibrium level of reliability provided will be too low because it will reflect the price cap and not the true value of lost load. This means that non-served energy events will be more important than would be socially optimal.

In systems dominated by dispatchable generation, non-served energy events generally occur at demand peaks, and the prescription for solving the missing money problem has been to provide incentives for investment in generation capacity to bring the capacity level to approximately that implied by the true value of lost load. Capacity mechanisms intended to implement that prescription have been controversial and have been frequently re-designed, however. It is less clear how to solve the missing money problem in principle when VRE generation is important, so that troughs in supply may be more important than peaks in demand, and the availability of VRE generation is weather-dependent. Storage poses even more difficult problems. The ability of storage to relieve system stress depends on its state of charge, which depends on prior operator decisions. It seems plausible that the second-best response to energy price caps set below the true value of lost load involves subsidies to investment in storage, but this has not been proven. Moreover, even if such subsidies are second-best optimal, they surely vary with the characteristics of storage technologies in ways that are not yet understood.

Second, our use of frequency domain analysis here to describe the optimal operation of storage systems seems to us likely to have merely scratched the surface of what that approach can contribute. While no simple merit order result for storage operations exists, even under perfect foresight, examining how the power spectra of alternative storage technologies respond to changes in cost parameters and system conditions may yield broadly useful insights.

Finally, there is clearly a need for computational models that can be used to optimize the operation of real storage systems under realistic stochastic processes of demand and VRE generation, with realistically imperfect foresight. Those models seem likely broadly to resemble the complex stochastic models that have been constructed for reservoir hydro systems,[33] but, as noted above, the storage optimization problem involves deciding on both charging and discharging and is thus more complex than the reservoir hydro problem. The recent paper by Geske and Green (2019) may be an important first step in this direction.

Appendix A.

This appendix contains the proofs of Propositions 6 and 9, provides an approximate generalization of equation (16) in Section 5 for small, positive self-discharge rates, and provides bounds on changes in the value of stored energy when storage is fully charged.

Proof of Proposition 6.

The profit of any particular gas (i.e., dispatchable) generation technology is given by

$$\begin{aligned}\Pi_G &\equiv \Sigma(\lambda_t - v)g_t - C_G G \\ &= \Sigma\gamma_t^U g_t - \Sigma\gamma_t^0 g_t + \Sigma_{t=2}^T \rho_t^U g_t - \Sigma_{t=2}^T \rho_t^D g_t - \Sigma_{t=2}^T \rho_t^U g_{t-1} + \Sigma_{t=2}^T \rho_t^D g_{t-1} - C_G G \\ &= G\Sigma\gamma_t^U + \Sigma_{t=2}^T \rho_t^U g_t - \Sigma_{t=2}^T \rho_t^D g_t - \Sigma_{t=2}^T \rho_t^U g_{t-1} + \Sigma_{t=2}^T \rho_t^D g_{t-1} - C_G G.\end{aligned} \tag{A.1}$$

Condition (13a) was used to substitute for $(\lambda_t - v)g_t$, and the complementary slackness conditions $\gamma_t^0 g_t = 0$ and $\gamma_t^U [G - g_t] = 0$ were employed. Multiplying condition (13b) by G and substituting for $(G\Sigma\gamma_t^U - C_G G)$ in (A.1) and using the complementary slackness conditions corresponding to (6) yields

$$\begin{aligned}\Pi_G &= -\Sigma_{t=2}^T \rho_t^U g_{t-1} + \Sigma_{t=2}^T \rho_t^U g_t + \Sigma_{t=2}^T \rho_t^D g_{t-1} - \Sigma_{t=2}^T \rho_t^D g_t \\ &\quad - G_0 G - G\beta^U \Sigma_{t=2}^T \rho_t^U - G\beta^D \Sigma_{t=2}^T \rho_t^D \\ &= -\Sigma_{t=2}^T \rho_t^U g_{t-1} + \Sigma_{t=2}^T \rho_t^U g_t + \Sigma_{t=2}^T \rho_t^D g_{t-1} - \Sigma_{t=2}^T \rho_t^D g_t \\ &\quad - G_0 G + \Sigma_{t=2}^T \rho_t^U g_{t-1} - \Sigma_{t=2}^T \rho_t^U g_t + \Sigma_{t=2}^T \rho_t^D g_t - \Sigma_{t=2}^T \rho_t^D g_{t-1} \\ &= -G_0 G\end{aligned} \tag{A.2}$$

[33] See DeLadurantaye et al (2009) and the sizeable literature there cited.

Thus, as for renewables, any gas generation technology for which the optimal capacity is positive, so $G_0 = 0$, exactly breaks even. If the lower-bound constraint is binding, it follows that the derivative of profit with respect to capacity at zero capacity is negative, so that increasing capacity above zero would reduce profit below zero.

Proof of Proposition 9.

The profit of any particular storage technology is given by

(A.3)
$$\begin{aligned}\Pi_S &= \Sigma\lambda_t(D_t - A_t) - C_P^A P^A - C_P^D P^D - C_E E - o^A \Sigma A_t - o^D \Sigma D_t \\ &= \Sigma(\lambda_t - o^D)D_t - \Sigma(\lambda_t + o^A)A_t - C_P^A P^A - C_P^D P^D - C_E E.\end{aligned}$$

Substituting for λ_t from equation (14a) and using the complementary slackness conditions involving the D_t from (7) yields

(A.4a)
$$\Sigma(\lambda_t - o^D)D_t = \Sigma\mu_t D_t - \Sigma\delta_t^0 D_t + \Sigma\delta_t^U D_t = \Sigma\mu_t D + P^D \Sigma\delta_t^U.$$

Similarly, using equation (14b) and the complementary slackness conditions involving the A_t from (7) yields

(A.4b)
$$\Sigma(\lambda_t + o^A)A_t = r\Sigma\mu_t A_t + \Sigma\alpha_t^0 A_t - \Sigma\alpha_t^U A_t = r\Sigma\mu_t A_t - P^A \Sigma\alpha_t^U.$$

Conditions (14d) – (14f) imply

(A.4c)
$$P^A \Sigma\alpha_t^U = C_P^A P^A - P_0^A P^A,$$

(A.4d)
$$P^D \Sigma\delta_t^U = C_P^D P^D - P_0^D P^D,$$

(A.4e)
$$C_E E = E_0 E - E_U E + E\Sigma\varepsilon_t^U.$$

Substituting equations (A.4) into equation (A.3) and using the equation of motion (5') yields

(A.5)
$$\begin{aligned}\Pi_S &= \Sigma\mu_t[D_t - rA_t] - [P_0^D P^D + P_0^A P^A + E_0 E - E_U E + E\Sigma\varepsilon_t^U] \\ &= \Sigma\mu_t[\chi S_{t-1} - S_t] - [P_0^A P^A + P_0^D P^D + E_0 E - E_U E] - E\Sigma\varepsilon_t^U.\end{aligned}$$

Condition (14c) and the complementary slackness conditions involving the S_t from (7) yield

(A.6a)
$$\Sigma\mu_t S_t = \Sigma S_t \varepsilon_t^0 - \Sigma S_t \varepsilon_t^U + \Sigma_{t=2}^T \chi S_{t-1}\mu_t - \mu_0 S_T = -E\Sigma\varepsilon_t^U + \Sigma_{t=2}^T \chi S_{t-1}\mu_t - \mu_0 S_T,$$

(A.6b) $\Sigma\mu_t[\chi S_{t-1} - S_t] = \chi\mu_1 S_0 + \Sigma_{t=2}^{T}\chi\mu_t S_{t-1} - \Sigma\mu_t S_t$

Substituting $\Sigma\mu_t S$ from (A.6a) into (A.6b) yields

(A.6c)
$$\begin{aligned}\Sigma\mu_t[\chi S_{t-1} - S_t] &= \chi\mu_1 S_0 + \Sigma_{t=2}^{T}\chi\mu_t S_{t-1} + E\Sigma\varepsilon_t^U - \Sigma_{t=2}^{T}\chi S_{t-1}\mu_t + \mu_0 S_T \\ &= \chi\mu_1 S_0 + E\Sigma\varepsilon_t^U + \mu_0 S_T \\ &= -\varepsilon_0^0 S_0 - \mu_0 S_0 + E\Sigma\varepsilon_t^U + \mu_0 S_T \\ &= E\Sigma\varepsilon_t^U\end{aligned}$$

The first and last term of (A.6c) cancel out because of (4), the first part of condition (14c) and the complementary slackness condition related to (7e). Substituting the above value of $\Sigma\mu_t[\chi S_{t-1} - S_t]$ into equation (A.5) then yields

(A.7)
$$\Pi_S = E_U E - [P_0^A P^A + P_0^D P^D + E_0 E].$$

The first term on the right of (A.7) is positive if and only if *E* is positive and the upper bound constraint on energy storage capacity is binding. Considering the bracketed terms, a binding non-negativity constraint implies that the derivative of profits with respect to the constrained variable is negative at zero, so that raising that variable above zero would lower profit. All three capacity variables must be strictly positive for the optimal capacity of the corresponding technology to be positive. If they are, the expression in brackets is zero, and we have established the Proposition.

Positive Self-Discharge.

For convenience, we analyze the evolution of the state of charge in continuous time. During the charging phase of the cycle, with charging occurring at maximum charging power capacity, P^A

$$dS/dt = rP^A - xS(t).$$

The solution with S(0) = 0 is

$$S(t) = \frac{rP^A}{x}(1 - e^{-xt}).$$

Setting $S(t^A) = E$, solving, and using the first-order Taylor series expansion for the logarithm yields a convenient expression for the length of a charging phase as a function of the self-discharge rate, x:

$$\text{(A.8a)} \qquad t^A(x) = -\frac{1}{x}\ln\left(1 - \frac{xE}{rP^A}\right) = -\frac{1}{x}\left(-\frac{xE}{rP^A} - \frac{1}{2}\left(\frac{xE}{rP^A}\right)^2 - \cdots\right).$$

Because of self-discharge, t^A is increasing in x. An exactly parallel development with $S(0) = E$ yields the length of a discharging phase:

$$\text{(A.8b)} \qquad t^D(x) = \frac{1}{x}\ln\left(1 + \frac{xE}{P^D}\right) = \frac{1}{x}\left(\frac{xE}{P^D} - \frac{1}{2}\left(\frac{xE}{P^D}\right)^2 + \cdots\right).$$

From the Taylor series, it follows that

$$\text{(A.9)} \qquad \left.\frac{\partial t^A}{\partial x}\right|_{x=0} = \left(\frac{1}{2}\right)\left(\frac{E}{rP^A}\right)^2 > 0, \text{ and } \left.\frac{\partial t^D}{\partial x}\right|_{x=0} = -\left(\frac{1}{2}\right)\left(\frac{E}{P^D}\right)^2 < 0.$$

Combining these and the expressions for $t^A(0)$ and $t^D(0)$ from the text yields

$$\text{(A.10)} \qquad \left.\frac{\partial(t^A / t^D)}{\partial x}\right|_{x=0} = \frac{kZt^D(0)(1+kZ)}{2} > 0.$$

For values of x near zero, we have

$$\text{(A.11)} \qquad \begin{aligned} AC(x) &\cong AC(0) + x\left[\partial AC/\partial x\right]\Big|_{x=0} \\ &= c_P(1+Zk)\left(1 + \frac{t^D(0)Zk}{2}x\right) + c_E\left(t^t(0) + \frac{[t^D(0)]^2 Zk(1+Zk)}{2}x\right) \\ &\quad + o^A Z\left(1 + \frac{t^D(0)(1+Zk)}{2}x\right) + o^D. \end{aligned}$$

Bounds on Changes in the Value of Stored Energy when Storage is Fully Charged.

When a storage facility is fully charged at the end of period t, one can exclude the possibility of storage discharging in period t (because it is never optimal to both charge and discharge storage in the same period) and storage charging in period t+1 (because storage is fully charged in period t). This leaves us with four possible state transitions for storage in period t and t+1:

(a) Charging in period t and discharging in period t+1,

(b) Charging period in t and idle in period t+1,

(c) Idle in period t and discharging in period t+1,

(d) Idle in period t and idle in period t+1.

For simplicity, assume the storage technology considered has zero variable O&M costs, so $o^A = o^D = 0$. The development that follows uses Proposition 7, so that with zero variable O&M costs, storage is optimally charging in period *t* if $\lambda_t \le r\mu_t$, discharging if $\lambda_t \ge \mu_t$, and idle otherwise.

In transition (a), charging in period *t* implies $-\mu_t \le -\lambda_t / r$, and discharging in period *t+1* implies $\mu_{t+1} \le \lambda_{t+1}$. Combining these conditions yields

(A.12a) $$\chi\mu_{t+1} - \mu_t \le \chi\lambda_{t+1} - \frac{\lambda_t}{r}$$

In transition (b), charging in period t again implies $-\mu_t \le -\lambda_t / r$, while storage idle in period *t+1* implies $r\mu_{t+1} \le \lambda_{t+1} \le \mu_{t+1} / r \Rightarrow \mu_{t+1} \le \lambda_{t+1} / r$. Combining these conditions as above yields

(A.10b) $$\chi\mu_{t+1} - \mu_t \le \frac{\chi\lambda_{t+1} - \lambda_t}{r}.$$

Proceeding similarly, upper bounds on ($\chi\mu_{t+1} - \mu_t$) can be obtained for transition (c), condition (A.10c), and transition (d), condition (A.10d):

(A.12c) $$\chi\mu_{t+1} - \mu_t \le \chi\lambda_{t+1} - \lambda_t$$

(A.12d) $$\chi\mu_{t+1} - \mu_t \le \frac{\chi\lambda_{t+1}}{r} - \lambda_t$$

Thus, in all cases, the difference in stored value of energy between period t and t+1 for a given storage technology is bounded above by the change in energy prices between those periods, scaled by storage discharge, charge efficiencies and self-discharge factor. Note that as r→1, the right-hand sides of conditions (A.12) all converge to $(\chi\lambda_{t+1} - \lambda_t)$.

Appendix B.

This appendix contains additional details on the data inputs used for the capacity expansion model implementation for the Texas case study discussed in Section 6.

TABLE B.1 Model Data Sources

Data	Source
VRE Resource	Wind: NREL WIND Toolkit (Wind Integration National Dataset Toolkit n.d.), PV: National Solar Radiation Databaset (NSRDB n.d.)
Load Vector	NREL Electrification Futures Study (2050, High Electrification, moderate technology advancement) (NREL, 2018) Peak demand 2050: 151 GW
Discount rate	4.3%

TABLE B.2 Cost and Performance Assumptions for Generation Technologies

Parameter	Unit	CCGT	CCGT - CCS	OCGT	PV (utility-scale)	Onshore Wind
Overnight cost	$ / kW	817	1,797	816	725	1,085
FOM cost[34]	$ / kW-y	10.6	33.6	12.2	11.1	34.6
VOM cost[35]	$ / MWh	3	7	7	0	0.1
Heat Rate	MMBTU/MWh	6.2	7.5	9.1	-	-
Minimum up time	hours	4	4	4	-	-
Minimum down time	hours	1	3	1	-	-
Minimum stable output level	% of peak capacity	0.33	0.4	0.25	-	-

Except for H_2 storage, the cost and performance assumptions for various generation technologies and Li-ion storage are taken from the 2019 edition of the NREL annual technology baseline (NREL, 2019). Cost and performance assumptions for H_2 are based on a synthesis of literature on the topic and represent typical values for an electrolyzer (charging) coupled with pressurized tank-based hydrogen storage and combined cycle gas turbine for power generation (discharging).

TABLE B.3 Cost and Performance Assumptions for Storage Technologies.

Parameter	Unit	Hydrogen	Li-Ion
CAPEX (Power - Discharge)	$ / kW	1,159	244
CAPEX (Power - Charge)	$ / kW	479	-
CAPEX (Storage)	$ / kWh	7	125
FOM (Discharge)	$ /kW-y	11	6.1
FOM (Charge)	$ / kW-y	20.3	-
FOM (Storage)	$ / kWh-y	0.07	3.1
VOM (discharge)	$ / MWh	2.19	1

[34] FOM: Fixed Operation and Maintenance, annual costs per MW of capacity.
[35] VOM: Variable Operation and Maintenance, cost per MWh of generation, charge, or discharge.

VOM (charge)	$ / MWh	1	1
Efficiency (Discharge)	%	62%	92%
Efficiency (Charge)	%	77%	92%
Efficiency (Roundtrip)	%	48%	85%
Self-discharge rate	%	0%	0%
Capital recovery period	Years	20	20

References

Balducci, Patrick J. , M. Jan E. Alam, Trevor D. Hardy, and Di Wu (2018). "Assigning Value to Energy Storage Systems at Multiple Points in an Electrical Grid," Energy & Environmental Science, 11: 1926-1944.

Biggar, Darryl R., and Mohammad Reza Hesamzadeh (2014). *The Economics of Electricity Markets.* Chichester, West Sussex, UK: Wiley.

Boiteux, Marcel (1960). "Peak Load Pricing," *Journal of Business*, 33(2): 157-179.

__________ (1964). "The Choice of Plant and Equipment for the Production of Electric Energy." In J. Nelson (Ed.), *Marginal Cost Pricing in Practic*e. Englewood Cliffs, N.J.: Prentice-Hall.

Brigham, E. Oran (1988). The Fast Fourier Transform and its Applications. Prentice-Hall.

Brown, Tom, and Lina Reichenberg (2020). Decreasing Market Value of Renewables is a Result of Policy, Not Variability," working paper, available at https://arxiv.org/abs/2002.05209.

California Public Utilities Commission (n.d.), *Energy Storage*, available at https://www.cpuc.ca.gov/General.aspx?id=3462

Crampes, Claude, and Jean-Michel Trochet (2019). "Economics of Stationary Electricity Storage with Various Charge and Discharge Durations." *Journal of Energy Storage*, 24(August): article 100746.

Crew, Michael A., and Paul A. Kleindorfer (1986). The Economics of Public Utility Regulation. Cambridge, MA: MIT Press.

De Ladurantaye, Daniel, et al (2009). "Optimizing profits from hydroelectricity production". *Computers & Operations Research*, 36: 499-529

Dowling, Jacqueline A., et al (2020). "Role of Long-Duration Energy Storage in Variable Renewable Electricity Systems". *Joule*, *4*(9): 1907-1928.

Drèze, Jacques H. (1964). "Some Postwar Contributions of French Economists to Theory and Public Policy," *American Economic Review*, 54(2): 1-64.

DSIRE database (n.d.), *Summary Maps,* available at https://programs.dsireusa.org/system/program/maps

Gailani, Ahmed, Maher Al-Greer, Michael Short, Tracey Crosbie, and Nashwan Dawood (2020). "Lifetime Degradation Cost Analysis for Li-Ion Batteries in Capacity Markets using Accurate Physics-Based Models," *energies*, 13 (11): 2816. https://doi.org/10.3390/en13112816.

Geske, Joachim, and Richard Green (2020). "Optimal Storage, Investment and Management under Uncertainty: It is Costly to Avoid Outages!" *Energy Journal*, 41(2): 1-27.

Giulietti, Monica, Luigi Grossi, Elisa Trujillo Baute, and Michael Waterson (2018). "Analyzing the Potential Economic Value of Energy Storage," *Energy Journal*, 39(S1):101-122.

Glowacki, Michael (2020), *European Union Electricity Market Glossary: Energy Storage*, available at https://www.emissions-euets.com/internal-electricity-market-glossary/1385-energy-storage

Gravelle, Hugh S.E. (1976). "The Peak Load Problem with Feasible Storage." *Economic Journal*, 86(June): 256-277.

Hawai'i Natural Energy Institute (2019), *Dynamic Load Bank: A Practical Solution to Enable Very High Penetration Rooftop PV*, Press Release, University of Hawai'i, 12 July, accessed 31 October 2020 at https://www.hnei.hawaii.edu/wp-content/uploads/Molokai-Dynamic-Load-Bank.pdf.

Helm, Carsten, and Mathias Mier (2018). "Subsidising Renewables but Taxing Storage? Second-Best Policies with Imperfect Carbon Pricing," Oldenberg Discussion Papers in Economics, No.V-413-18, University of Oldenberg, Department of Economics.

Hotelling, Harold (1931). "The Economics of Exhaustible Resources." *Journal of Political Economy*, 39(2): 137-175.

Jenkins, Jesse D., and Nestor A. Sepulveda (2017). "Enhanced Decision Support for a Changing Electricity Landscape: The GenX Configurable Electricity Resource Capacity Expansion Model," Working Paper, MIT Energy Initiative, November 27.

Johnston, Josiah, Rodrigo Henriquez-Auba, Benjamin Mauluenda, and Matthias Fripp (2019), "Switch 2.0: A modern platform for planning high-renewable power systems", *SoftwareX*, 10: 100251.

Joskow, Paul L. (1976). "Contributions to the Theory of Marginal Cost Pricing," *Bell Journal of Economics*, 7(1): 197-206.

__________ (2008). "Capacity Payments in Imperfect Electricity Markets: Need and Design," *Utilities Policy*, 16(3): 159-170.

__________ (2019). "Challenges for Wholesale Electricity Markets with Intermittent Generation at Scale: The U.S. Experience," *Oxford Review of Economic Policy*, 39(2): 292-331.

__________, and J. Tirole (2007). "Reliability and Competitive Electricity Markets." *Rand Journal of Economics*, 38(1): 60-84.

Korpås, Magnus, and Audun Botterud (2020). "Optimality Conditions and Cost Recovery in Electricity Markets with Variable Renewable Energy and Energy Storage." Working Paper 2020-05, Center for Energy and Environmental Policy Research, Massachusetts Institute of Technology, March.

Mallapragada, Dharik S., Sepulveda, Nestor A., and Jenkins, Jesse D. (2020). "Long-run system value of battery energy storage in future grids with increasing wind and solar generation" *Applied Energy,* 275: 115390.

Nguyen, D.T. (1976). "The Problem of Peak Loads and Inventories." *Bell Journal of Economics*, 7(1): 242-248.

NREL (2019), *Annual Technology Baseline*, available at https://atb.nrel.gov/electricity/2019/

NREL (2018), *Electrification Futures Study Load Profiles*, available at https://data.nrel.gov/submissions/126

NSRDB (n.d) National Solar Radiation Database, available at https://nsrdb.nrel.gov/

Patel, Purvin N. (2018). "Developments in Energy Storage Could Spell the End of the Duck Curve," *Power*, June 1. Available at https://www.powermag.com/developments-in-energy-storage-could-spell-the-end-of-the-duck-curve/?printmode=1

Petlin, Gabe, Rachel McMahon, and Kari Smith (2018). "Energy Storage Market Survey and Recommendations." Commissioner Briefing, California Public Utilities Commission, October 24. Available at http://www.cpuc.ca.gov/uploadedFiles/CPUC_Public_Website/Test_Calendar/CPUC%20Energy%20Storage%20Market%20Survey%20and%20Recommendations%2010.22.18(1).pdf

Schmalensee, Richard (2020). "Competitive Energy Storage and the Duck Curve." MIT Center for Energy and Environmental Policy Research, Working Paper WP-2020-012.

Sidhu, Arjan S., Michael G. Polllitt, and Karim L. Anaya (2018). "A Social Cost Benefit Analysis of Grid-Scale Electrical Energy Storage Projects." *Applied Energy*, 212: 881-894.

Steffen, Bjarne, and Christoph Weber (2013). "Efficient Storage Capacity in Power Systems with Thermal and Renewable Generation." *Energy Economics*, 36: 556-567.

Turvey, Ralph (1968). *Optimal Pricing and Investment in Electricity Supply: An Essay in Applied Welfare Economics*. Cambridge, MA: MIT Press.

U.S. Department of Energy (2015). *Quadrennial Technology Review 2015: Hybrid Nuclear-Renewable Energy Systems*. Available at https://www.energy.gov/sites/prod/files/2016/06/f32/QTR2015-4K-Hybrid-Nuclear-Renewable-Energy-Systems.pdf

U.S. Federal Energy Regulatory Commission (2018). *Order 841: Electric Storage Participation in Markets Operated by Regional Transmission Organizations and Independent System Operators*. Issued February 15. Available at https://www.ferc.gov/whats-new/comm-meet/2018/021518/E-1.pdf

U.S. Government Accountability Office (2018). *Energy Storage: Information on Challenges to Deployment for Electricity Grid Operations and Efforts to Address Them*. Report GAO-18-402 (May).

Victoria, Marta, Zhu, Kun, Brown, Tom, Andresen, Gorm B., and Greiner, Martin (2019), "The role of storage technologies throughout the decarbonisation of the sector-coupled European energy system." *Energy Conversion and Management*, 201: 111977.

Wind Integration National Dataset Toolkit (n.d.) available at https://www.nrel.gov/grid/wind-toolkit.html

FIGURE 1. OPTIMAL VARIATIONS IN DISPATCHABLE STORED ENERGY OVER MONTHS 1-4 OF THE SIMULATION

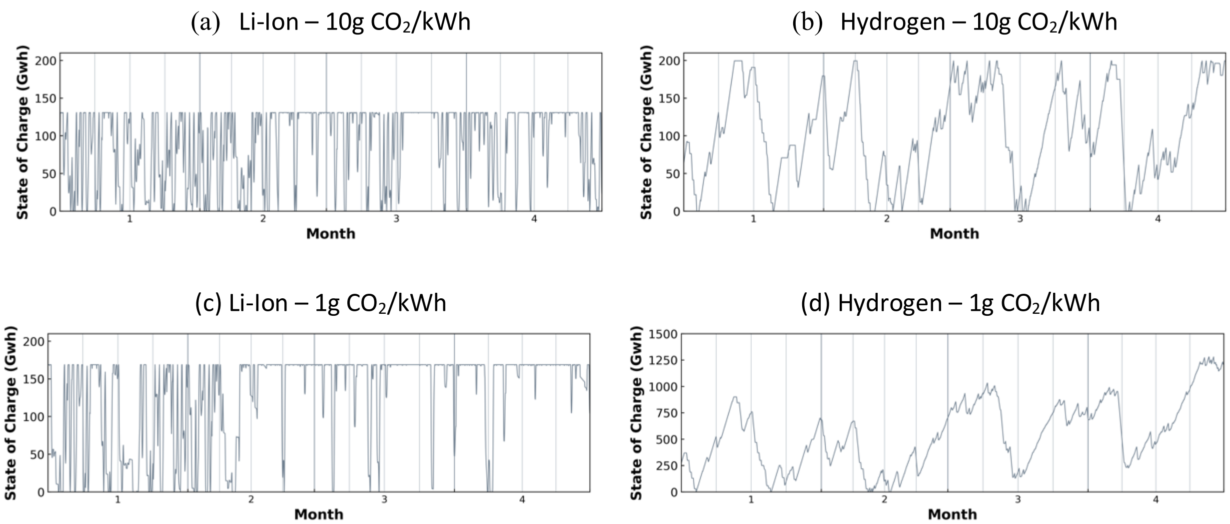

TABLE 1 Model Main Features

Feature	Description
Available dispatchable generation	Combined cycle gas turbine (CCGT); Combined cycle gas turbine with Carbon Capture and Storage (CCGT-CCS); open cycle gas turbine (OCGT)
Variable renewables	Onshore Wind and utility-scale PV, with 7 resource bins per technology used to characterize different types of wind and PV sites. Each resource bin has a unique hourly capacity factor profile. Interconnection cost is added to the baseline VRE capital cost, and maximum capacity in MW.
Available storage technologies	Li-ion ($P^A = P^D$) ; power to hydrogen to power ("H_2") ($P^A \neq P^D$)
Demand	Peak demand = 151 GW, Annual demand = 715 TWh; value of lost load = $50,000/MWh
Spatial resolution	Single zone, no transmission constraints
Temporal resolution	2007-2013 weather years (61,314 hours)
Carbon emission constraints	Two constraints: 10 and 1 gCO_2 / kWh
Thermal plant operating constraints	Linearized unit commitment with ramping constraints and minimum up and down time constraints

TABLE 2 Model results

Result	Technology	Emissions Constraint	
		10 gCO_2/kWh	**1 gCO_2/kWh**
Installed Power Capacity (MW)[36]	CCGT	33.9	9.8
	CCGT-CCS	18.4	23.6
	OCGT	3.1	0.0
	PV	103.9	128.3
	Wind	121.9	136.8
	H_2 (discharge)	5.5	20.6
	H_2 (charge)	3.6	12.4
	Li-ion	38.8	42.8
	Total	329.0	374.3
Installed Energy Storage Capacity (GWh)	H_2	199.5	1279.9
	Li-ion	130.9	168.8
	Total	330.4	1448.8
Average Discharged Energy (TWh/year)[37]	H_2	7.5	25.2
	Li-ion	28.9	22.3
Equivalent discharges / year[38]	H_2	37.5	19.7

[36] In the case of hydrogen, installed power capacity has two components, one each for charging and discharging power.
[37] Average annual energy discharged is calculated as total energy discharged over the seven-year period divided by seven.
[38] Equivalent discharges per year for each storage technology is calculated as the ratio of average annual energy discharged divided by maximum electrical energy recoverable measure of storage capacity.

	Li-ion	220.5	132.3
Number of periods when storage is fully charged (hours/year)	H_2	232.0	68.0
	Li-ion	2942.0	3911.0
Average system electricity cost ($/MWh)	-	45.4	50.7
Average load shedding per year (GWh/year)	-	0.1	0.1

TABLE 3 Relative RMS contribution of different frequency bands to state of charge variation

Mode of operation	10 gCO_2/kWh		1 gCO_2/kWh	
	Li-ion	H_2	Li-ion	H_2
Daily	31.4%	0.5%	18.2%	0.2%
Weekly	37.8%	13.0%	32.1%	3.5%
Monthly	13.6%	52.9%	15.5%	28.4%
Seasonal	17.2%	33.6%	34.2%	67.9%

TABLE 4 Model results – System with Li-ion as the only storage technology

Result	Technology	Emissions Constraint	
		10 gCO_2/kWh	1 gCO_2/kWh
Installed Power Capacity (MW)	CCGT	33.6	12.3
	CCGT-CCS	22.0	32.0
	OCGT	5.5	0.0
	PV	102.9	130.3
	Wind	122.0	163.1
	Li-ion	38.6	52.6
	Total	324.5	390.4
Installed Energy Storage Capacity (GWh)[39]	Li-ion	129.1	249.5
Average Discharged Energy (TWh/year)	Li-ion	29.8	25.3
Equivalent discharges / year	Li-ion	231.0	101.5
Number of periods fully charged (hours/year)	Li-ion	3110.0	5397.0
Average system electricity cost ($/MWh)	-	45.5	53.3
Average Load Shedding per year (GWh/year)	-	0.1	0.0

[39] Energy storage capacity is based on the maximum electrical energy recoverable definition discussed above.

TABLE 5 Frequency RMS analysis for Li-Ion – Comparison between scenarios

Mode of operation of Li-ion	Li-ion + H_2 scenario		Li-ion only scenario	
	10 gCO_2/kWh	1 gCO_2/kWh	10 gCO_2/kWh	1 gCO_2/kWh
Daily	31.4%	18.2%	33.3%	13.1%
Weekly	37.8%	32.1%	39.8%	44.0%
Monthly	13.6%	15.5%	13.4%	26.0%
Seasonal	17.2%	34.2%	13.5%	16.9%

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