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MIT Center for Energy and Environmental Policy Research

Working Paper Series

Competitive Energy Storage and the Duck Curve

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JULY 2020

CEEPR WP 2020-012

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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And The Duck Curve

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ABSTRACT

Power systems with high penetrations of solar generation need to replace solar output when it falls rapidly in the late afternoon – the duck curve problem. Storage is a carbon-free solution to this problem. This essay considers investment in generation and storage to minimize expected cost in a Boiteux-Turvey-style model of an electric power system with alternating daytime periods, with solar generation, and nighttime periods, without it. In the most interesting cases, if energy market prices are uncapped, all expected cost minima are long-run competitive equilibria, and the long-run equilibrium value of storage capacity minimizes expected system cost conditional on generation capacities.

Keywords: electricity, storage, solar, renewable, equilibrium

June 2020

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1. INTRODUCTION

In 2008, modelers at the National Renewable Energy Laboratory predicted, correctly, that increased penetration of residential photovoltaic generation would lead to the regular appearance of hourly patterns of total and net generation in the area served by the California Independent System Operator (CAISO) like the pattern shown in Figure 1 (Denholm et al 2008).[2] A few years later, the hourly pattern of grid generation net of wind and (especially) solar shown there was christened "the duck curve". As solar penetration has increased, the duck's back in mid-day has deepened, and a greater increase in output from other sources has been required in late afternoons, when solar generation drops off and residential load increases (in what can be described as the duck's neck). The traditional solution to this problem would be to build and use more gas turbines or combined cycle plants that can increase output rapidly.

However, building more fossil-fueled generators is inconsistent with the goal in California and elsewhere of reducing carbon dioxide emissions. As the costs of storage, particularly lithium-ion battery storage, have declined, storage has emerged as a potentially attractive, carbon-free alternative way of offsetting diurnal declines in solar generation (Patel (2018)). And, since the promulgation of statutory requirements in 2010, in part to facilitate the integration of solar and other variable renewable generation,[3] the California Public Utilities Commission has been requiring load-serving entities to procure storage (Petlin et al 2018, California Public Utilities Commission n.d.). Storage targets have also recently been established in Massachusetts, Nevada, New Jersey, New York, and Oregon, and they are under consideration in other states.

This state-level reliance on mandates contrasts with an apparent preference at the federal level to rely on competition to drive investment in storage facilities. Figure 2 shows the average hourly day-ahead locational marginal spot prices (LMP) in the CAISO in 2010, 2015, 2016, and

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[2] Figure 1 is from Joskow (2019), p. 314, used by permission of Oxford University Press.

[3] Storage can also perform other functions in electric power systems. Depending on the technology employed, storage facilities can provide frequency regulation, deferral of wires investment, and reducing the cost of spinning reserves. For discussions, see Giuletti et al (2018) and U.S. Government Accountability Office (2018), and for a worked example of a storage project that could perform multiple functions, see Sidhu et al (2018). The focus here is exclusively on the use of storage for energy arbitrage to solve the duck curve problem and related problems posed by the variability of renewable energy resources.

2017, divided by the annual averages of those prices (to control for changes in fuel prices).[4] As solar penetration has increased, intra-day price differences have also increased. The pattern in Figure 2 suggests that with sufficient solar penetration, competitive storage providers could find it profitable to buy at mid-day when prices are low and sell a few hours later as solar generation begins to drop off and prices become high, thus mitigating or perhaps solving the duck curve problem. The U.S. Federal Regulatory Commission (2018) has recently issued Order 841, which is intended to open wholesale energy markets (and other wholesale markets) to merchant storage providers.[5] Similarly, The European Union's Clean Energy Package, most recently modified in 2019, calls for competitive supply of storage (Glowacki 2020) and, in contrast to California's mandate, restricts ownership by distribution system operators. In both the US and the EU, efforts are ongoing to reach agreement on exactly how to define markets and establish tariffs to ensure that storage providers have access to wholesale markets on appropriate terms.[6]

The FERC and EU policies rest on the presumption that energy markets can provide at least approximately optimal incentives for competitive investment in storage as well as generation.[7] This essay explores the validity of this presumption in the context of the duck curve by investigating the properties of a Boiteux (1960, 1964)-Turvey (1968)-style model of an electric power system augmented with the addition of storage.[8] Models in this tradition, and the model developed here, assume constant returns to scale, stochastic and (generally) inelastic demand, and multiple dispatchable generation technologies without significant startup costs or minimum generation levels. If shortages occur, the system is assumed not to collapse, and price is assumed to rise to the value of lost load.[9]

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[4] Figure 2 is from Joskow (2019), p. 314, used by permission of Oxford University Press.
[5] In addition, at the U.S. federal level, storage facilities that are charged only by solar generators are eligible for a 30% investment tax credit.
[6] See, e.g., https://www.dwt.com/blogs/energy--environmental-law-blog/2020/06/federal-energy-storage-regulatory-activity
[7] A number of studies have investigated the profitability of energy arbitrage under various cost assumptions and observed price trajectories; see, e.g. Salles et al (2017) and Giuletti et al (2018). The usual finding is that arbitrage profits do not cover the capital costs of storage facilities *under current price patterns*. This finding sheds no light on the general optimality of the investment incentives that would be provided by energy markets when storage is available and its deployment is profitable at the margin because energy prices are more volatile than at present.
[8] See Dréze (1964) for an elegant exposition of Boiteux' work, originally written in the early 1950s, and see Joskow (1976) for a discussion of closely related later work. Joskow and Tirole (2007) extend this literature substantially.
[9] See Joskow and Tirole (2007) on this assumption, to which I return in Section 5. If energy prices are capped below the value of lost load, as seems to be the case in many real markets, investment incentives for generation are inadequate; see Joskow (2007, 2008) for discussions. "Capacity mechanisms" of various sorts have been added to a number of systems in the US and the EU to deal with this "missing money" problem.

There are a number of ways that storage has been added to models of this sort. In the earliest formal treatments of storage in this context of which I am aware, Gravelle (1968) and Nguyen (1968) consider two-period – peak and off-peak – models and simply assume that an unlimited amount of the quantity being sold can be transferred between adjacent periods at a constant per-unit cost. Several authors, including Steffen and Weber (2013) and Korpås and Botterud (2020) have added storage to timeless Boiteux-Turvey-style models by assuming that power can be purchased whenever the price of energy is low and resold whenever the price is high. This amounts to assuming that energy storage capacity is effectively infinite, since low-price and high-price periods may be far apart in time. Other authors, including Crampes and Trochet (2019), Brown and Reichenberg (2020) and Junge et al (2020) have introduced time explicitly but assumed perfect foresight. While these models yield a number of general results regarding investment in and operation of storage facilities under competition, the perfect foresight assumption is strong and eliminates the precautionary demand for storage. Relaxing that assumption, however, requires explicitly modeling the relevant stochastic processes, as demonstrated by Geske and Green (2019). This limits the generality of results that can be obtained. In perhaps the model closest to the one presented here, Helm and Mier (2018) consider a dynamic model with a constant demand curve and non-stochastic renewable output that follows a regular cyclic trajectory.

To focus on a (necessarily) simplified version of the duck curve problem, the model considered here has alternating periods of two types, labeled daytimes and nighttimes, corresponding roughly to the duck's back and its neck. Renewable generation has positive, stochastic output only in daytime periods. Gas generation, which, for simplicity, stands in for the whole suite of dispatchable generation technologies, is assumed to be available in both daytime and nighttime periods. Short-term storage can be installed at a constant cost per unit of capacity, and storage involves a constant fractional round-trip loss of energy.[10] Demand in both

[10] This simple description of storage costs is reasonable for battery storage of a few hours duration with negligible self-discharge and negligible variable operating and maintenance costs. In the most general case, seven parameters are necessary to describe storage costs, even under constant returns to scale (Junge et al 2020).

days and nights is stochastic, constant within periods, and perfectly inelastic.[11] Section 2 presents these assumptions in more detail and introduces the notation used in what follows.

Under constant returns, competitive generators' operating rules are simple: produce if and only if market price of energy is greater than or equal to marginal cost. In general, optimal charging or discharging of storage under competition depends on the current energy market price, the amount of energy in storage, and expectations regarding future energy prices. In general, it does not seem possible describe the behavior of competitive storage suppliers when storage is not fully discharged in each nighttime period without additional assumptions or (per Geske and Green (2019)) resorting to numerical methods. In the context of the duck curve, however, at least in the near term, imposing the restriction that storage is fully discharged in each nighttime seems reasonable. Doing so leads to three possible regimes relating the marginal cost of gas generation to expected nighttime prices, and Section 3 derives the three corresponding competitive operating rules that storage suppliers would follow. Section 3 also presents a sufficient consistency condition for each rule: if that condition holds and if competitive storage suppliers follow the corresponding operating rule, they will in fact find it optimal to sell all stored energy each nighttime.

Section 4 considers minimization of expected total cost conditional on the algebraically simplest competitive operating rule derived in Section 3. Parallel analyses under the other two regimes are summarized in Appendices A and B. Section 5 discusses some implications of the results of this analysis.

2. Assumptions and Notation

The daytime and nighttime periods in each day are assumed to be of equal length for convenience, and the probability distributions governing demand within daytime and nighttime periods and the output of renewable generation are assumed to be independent. Independence rules out weather-induced correlations, among other things, but it is not clear how to relax this strong assumption and maintain tractability. There are four technologies with constant returns to scale. Their capacities are initially assumed to be determined by a benevolent social planner

[11] Joskow and Tirole (2007) and a number of other papers assume that at least some demand is price-sensitive and consider welfare maximization rather than cost minimization. I assume perfectly inelastic demand here for the sake of simplicity

interested in minimizing expected total cost. After the implications of that assumption have been explored, we consider whether a continuum of risk-neutral, perfectly competitive firms would provide the same capacities in long-run equilibrium:

Gas stands in for all dispatchable fossil and nuclear technologies, has capacity G, per-day unit capacity cost C_G, and per-MWh operating cost c.

Renewables have capacity R, per-day unit capacity cost C_R, and zero operating cost. Maximum renewable output is zero during the night and is equal to θR MWh during the day, where θ is a random variable with smooth distribution function $H(\theta)$ on $[\underline{\theta},1]$, with $\underline{\theta} > 0$, and density function $h(\theta)$.[12] It is assumed that renewable generation can be costlessly curtailed whenever daytime demand is less than available renewable supply.

Scarcity operates when load exceeds capacity and there is lost load. Scarcity involves zero capital cost and has per-MWh variable cost v, the value of lost load. The probability of scarcity is assumed to be positive in both periods. (It must be positive in at least one period for gas to recover its capital cost under competition.[13])

Storage has capacity S, per-day unit capacity cost C_S, and round-trip efficiency $\eta < 1$. This paper focuses on the empirically interesting case (for at least some time to come) $S < \underline{\theta}R$. The (stochastic) amount of energy in storage at the end of a daytime period is s.

Efficiency requires that the capital costs all include appropriately discounted disposal costs, including environmental impact costs, and the operating cost of dispatchable generation must include the associated environmental costs. As an accounting convention, the loss of energy due to storage occurs when storage is discharged, not when it is charged.

Cost parameters are assumed to satisfy the following inequalities:

$$C_G < C_R, \quad c < v, \quad and \quad C_R < c. \tag{2.1}$$

The first two of these are familiar: gas has lower capital cost than renewables, and the value of lost load exceeds the incremental cost of gas generation. The third is necessary for gas to be economical. Gas capacity is necessary to meet demand at night when renewable generation is

[12] This follows Llobet and Padilla (2018) and essentially assumes perfect correlation among the outputs of renewable generating facilities. Solar generation is obviously zero at night; the assumption of a positive minimum daytime capacity factor seems reasonable.

[13] For clear discussions, see Joskow (2007, 2008).

not available, so for it to be efficient for any renewable capacity to be installed and used during the day, the total per-MWh cost of renewable generation, C_R, must be less than the incremental cost of gas generation from existing capacity, c. A high value of c is most naturally interpreted as reflecting a substantial price of carbon emissions.

The two periods in each day are as follows:

Daytime load before storage *purchases* (i.e., final demand), L_D, is distributed according to smooth distribution function $F_D(L)$ on $[0,\infty]$ with density $f_D(L)$.

Nighttime load before storage *sales* (i.e., final demand), L_N, is distributed according to smooth distribution function $F_N(X)$ on $[0,\infty]$, with density $f_N(X)$.

I assume that C_S is low enough, η is high enough, and nighttime load is on average high enough relative to daytime load that some energy arbitrage via storage is economic, and I thus concentrate on internal minima of expected system cost.

The values of R, G, and S are initially assumed chosen by a benevolent planner to minimize expected system cost, including the cost of lost load, conditional on competitive behavior by the holders of those assets. At the start of every daytime period, values of L_D and θR are realized. Storage suppliers then decide how much energy to carry into the next nighttime, s, based on the observed daytime market price of energy, P_D, and the expected nighttime price of energy, $\overline{P}_N$. At the start of every nighttime period, the value of s from the previous daytime is revealed, the value of L_N is realized, and the actual nighttime price of energy, P_N, is determined. I first describe minima of expected system cost and then consider their relation to long-run competitive equilibria.

3. Competitive Operating Rules for Storage

As noted above, it seems that to make the analysis of storage operations tractable without additional restrictive assumptions, we must require that storage be fully discharged during each nighttime period, so that each daytime period begins with zero stored energy.[14] Since this model

[14] Let $\Gamma(s)$ be the end-of-daytime value of an incremental unit of stored energy when storage contains s MWh, and let $\Delta(y)$ be the end-of-nighttime value of an incremental unit of stored energy when storage contains y MWh. If storage is fully discharged every nighttime, $\Gamma(s)$ is just η times the expected nighttime price, exactly as assumed below. Without that assumption, however, $\Gamma(s)$ would also reflect the fact that under some nighttime conditions, discussed in the text, it would more profitable not to discharge fully, but rather to carry some energy into the next daytime. The nighttime decision of competitive storage suppliers involves comparing $\Delta(y)$, which would depend on

is an abstract representation of systems, like the CAISO system, in which there is an abundance of solar generation during most daytimes and robust demand in the evening when solar output falls to zero, this seems the most interesting case.

There are two ways competitive storage suppliers could end a nighttime period with positive stored energy. First, they could have begun the period with positive stored energy, s, and encountered perfectly inelastic demand, L_N, that was less than ηs. They would then sell L_N and carry ($s-L_N/\eta$) into the next daytime.[15] The simplest way to rule this out is to assume that the minimum nighttime demand exceeds the maximum supply from storage:

(3.1) $$F_N(L_N) = 0 \textit{ for } L_N \le X, \quad \textit{where } X > \eta S.$$

Since S is endogenous, this is not fully satisfactory. But the notion of a non-trivial, positive minimum nighttime demand is plausible in general and plainly so in the duck curve context.

The second way competitive storage suppliers might end a nighttime period with positive stored energy could occur if the energy market price were c and gas capacity were not exhausted. Depending on expectations regarding daytime prices, they might find it optimal to purchase energy at c and carry it into the next daytime. The marginal value of energy purchased to carry forward in this fashion is clearly non-increasing, so to rule out this behavior it suffices to rule out buying and carrying forward the first marginal unit. As discussed below, the conditions ruling this out, (3.3), (3.4), and (3.6), are specific to each of the three possible operating regimes. Unfortunately, like (3.1), these conditions also involve endogenous variables.

If full nighttime discharge is optimal, a competitive supplier of storage would only add to charge in the daytime if the marginal cost of doing so did not exceed the expected nighttime revenue per unit stored, which is η times the expected nighttime price. Because nighttime demand is perfectly inelastic, the expected nighttime price is just the expected marginal cost of supply:

(3.2a) $$\bar{P}_N(G,s) = cF_N(G+\eta s) + v\left[1 - F_N(G+\eta s)\right].$$

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expected daytime conditions, with the marginal nighttime price of energy. In this general case, it is not apparent how to relate the two value functions involved to the primitives of the model, or even to each other.

[15] Recall the convention that energy losses occur when storage is discharged, not when it is charged.

By assumption (3.1), stored energy alone is never sufficient to meet demand. With probability $F_N(G+\eta s)$, stored energy plus gas *is* sufficient, and there is no shortage. Because increases in gad capacity and stored energy shift the nighttime supply curve to the right, the expected price falls with G and s:

$$\text{(3.2b)} \qquad \partial \overline{P}_N(G,s)/\partial s = \eta[\partial \overline{P}_N(G,s)/\partial G] = -\eta(v-c) f_N(G+\eta s) < 0.$$

In what follows, dependence of $\overline{P}_N$ on G is generally suppressed to reduce notational clutter.

If all stored energy is to be sold at night regardless of price, a risk-neutral competitive storage supplier would store incremental energy during the day if the per-unit cost of doing so exceeded $\eta\overline{P}_N(0)$. From (3.2a), $\overline{P}_N(0)$ is always between c and v, the two possible nighttime prices when there are no sales from storage, but η times this quantity may be above or below c. There are thus three possible daytime regimes, listed in order of increasing analytical complexity:

1. $\eta\overline{P}_N(S) < \eta\overline{P}_N(0) < c.$
2. $c < \eta\overline{P}_N(S) < \eta\overline{P}_N(0).$
3. $\eta\overline{P}_N(S) < c < \eta\overline{P}_N(0).$

It follows from the discussion above that the daytime competitive demand curve for energy from final demand and competitive storage suppliers is vertical at L_D (no demand from storage) for prices above $\eta\overline{P}_N(0)$, declines to (L_D+S) for prices between $\eta\overline{P}_N(0)$ and $\eta\overline{P}_N(S)$, and is vertical at (L_D+S) for lower prices (storage completely filled). This shape is illustrated in Figure 3, where P_D is the daytime energy price, and Q_D is the corresponding quantity demanded. Using this demand curve, we can now describe the daytime market and competitive determination of s under each of these regimes.[16]

<u>Regime 1</u>: $\eta\overline{P}_N(S) < \eta\overline{P}_N(0) < c.$ Under this regime and the assumption of full nighttime discharge, competitive storage suppliers will charge as much as possible as long as the daytime

[16]Each operating rule developed below is optimal for a particular regime, but the operating rule followed by storage suppliers will affect the expected nighttime price function and could in some cases thereby alter the regime. It is not clear how to rule out this sort of inconsistency as a general matter.

market price is less than c, so only zero-marginal-cost output from renewables is stored. Competitive behavior determines the demand from storage, s, gas output, marginal operating cost, and the market price of energy, P_D, as functions of the relationship between final demand, L_D, renewable generation, and the capacities of gas generation and storage. Maintaining the assumption $S < \underline{\theta}R$, this regime implies a competitive operating rule with four daytime cases:

Case	s	Gas Output	Marginal Cost	P_D
(a) $L_D \le \theta R - S$	S	0	0	0
(b) $\theta R - S \le L_D \le \theta R$	$\theta R - L_D$	0	0	$\eta \bar{P}_N(\theta R - L_D)$
(c) $\theta R \le L_D \le \theta R + G$	0	$L_D - \theta R$	c	c
(d) $\theta R + G \le L_D$	0	G	v	v

In case (a) there is sufficient renewable output both to satisfy final demand and to charge storage to capacity without requiring gas generation. In case (b), charging is curtailed as necessary to avoid calling on gas generation and driving price above $\eta \bar{P}_N(0)$. Marginal operating cost remains at zero, but the market price of energy is bid up by competitive storage to the expected value of additional nighttime sales, as illustrated in Figure 4. In case (c) final demand is high enough that gas is necessarily on the margin, and both the marginal operating cost and the energy price are c. No charging occurs. Case (d) involves shortage, and the system marginal cost and price are the value of lost load, so there is, again, no charging.

Given the operating rule just described, suppose a competitive storage supplier considers storing a unit of energy at night at a cost of c for sale the next day. For this to be unprofitable, c must exceed η times the expected value of P_D the next day. Using the upper bound on the energy price in case (b), c, a sufficient condition for storage at night at a cost of c to be unprofitable is

$$(3.3) \qquad \int_{\underline{\theta}}^{1} \eta\left\{c\left[F_D(\theta R + G) - F_D(\theta R - S)\right] + v\left[1 - F_D(\theta R + G)\right]\right\} h(\theta) d\theta < c.$$

If (3.1) and (3.3) are satisfied, storage will be empty at the start of every daytime period under competition, as assumed. Since $\eta < 1$, (3.3) will be satisfied unless the probability of a daytime shortage, the expected value of $[1 - F_D(\theta R + G)]$, is sufficiently high.

Regime 2: $c < \eta \bar{P}_N(S) < \eta \bar{P}_N(0)$. There are, again, four cases under this configuration of costs and expected prices:

Case	s	Gas Output	Marginal Cost	P_D
(a) $L_D \le \theta R - S$	S	0	0	0
(b) $\theta R - S \le L_D \le \theta R + G - S$	S	$L_D - (\theta R - S)$	c	c
(c) $\theta R + G - S \le L_D \le \theta R + G$	$\theta R + G - L_D$	G	c	$\eta \bar{P}_N(\theta R + G - L_D)$
(d) $\theta R + G \le L_D$	0	G	v	v

In cases (a) and (b), there is sufficient capacity both to satisfy final demand and to charge storage fully without driving the energy price above c. The marginal generating cost is zero in case (a), and in case (b) it is c because gas is on the margin. In case (c), storage cannot be fully charged without driving the energy price to v. As in case (b) under regime 1, competitive storage suppliers bid the energy price above marginal cost, c, as illustrated in Figure 5. Finally, in case (d), there is an unavoidable shortage, marginal cost and the energy price are the value of lost load, v, and storage demand is reduced to zero.

Using the upper bound on price in case (c), v, the reasoning that let to condition (3.3) yields a sufficient condition that rules out the profitability of purchasing energy at night at a price of c and reselling the next day under regime 2:

$$(3.4) \qquad \int_{\underline{\theta}}^{1} \eta \left\{ c\left[F_D(\theta R + G - S) - F_D(\theta R - S)\right] + v\left[1 - F_D(\theta R + G - S)\right] \right\} h(\theta) d\theta < c.$$

As before, the probability of a daytime shortage must not be too high.

Regime 3: $\eta \bar{P}_N(S) < c < \eta \bar{P}_N(0)$. The competitive operating rule implied by this regime is a bit more complicated than those under Regimes 1 and 2:

Case	s	Gas Output	Marginal Cost	P_D
(a) $L_D \le \theta R - S$	S	0	0	0
(b) $\theta R - S \le L_D \le \theta R - \tilde{s}$	$\theta R - L_D$	0	0	$\eta \bar{P}_N(\theta R - L_D)$
(c) $\theta R - \tilde{s} \le L_D \le \theta R + G - \tilde{s}$	$\tilde{s}$	$L_D - (\theta R - \tilde{s})$	c	$\eta \bar{P}_N(\tilde{s})$
(d) $\theta R + G - \tilde{s} \le L_D \le \theta R + G$	$\theta R + G - L_D$	G	c	$\eta \bar{P}_N(\theta R + G - L_D)$
(e) $\theta R + G \le L_D$	0	G	v	v

Here $\tilde{s}$ is implicitly defined by

(3.5a) $$c = \eta \bar{P}_N(G, \tilde{s}).$$

Increases in *G* reduce the expected nighttime price and thus reduce the amount of energy it is profitable to store for nighttime sale when its cost is *c*:

(3.5b) $$\frac{\partial \tilde{s}(G,c)}{\partial G} = -\frac{1}{\eta} < 0.$$

This follows from equation (3.2b).

In case (a), final demand plus *S* is less than renewable output, so storage is fully charged, and the energy price is zero. In case (b), as in case (b) under regime 1, total demand at a price of zero, (L_D+S), exceeds renewable output. Marginal operating cost remains at zero, but the market price of energy is bid up by competitive storage suppliers to the expected value of incremental nighttime sales. This resembles the situation illustrated in Figure 4, except that $\eta \bar{P}_N(0)$ exceeds *c* here. In case (c), illustrated by Figure 6, gas is on the margin, and storage is limited to $\tilde{s} < S$ because additional storage would have negative expected profit. In case (d), which resembles case (c) under Rule 2, which is illustrated by Figure 5, marginal operating cost is *c*, but price is driven above *c* by competition among storage suppliers. Finally, in case (e), final demand exceeds renewable output plus gas capacity, there is a shortage, and storage is not charged. Note that storage capacity is fully utilized only in case (a).

Using upper bounds on energy prices in cases (b) and (d) and the reasoning that led to conditions (3.3) and (3.4), a sufficient condition for purchasing energy at night at a cost of *c* and storing it for resale the next day to be unprofitable is

(3.6) $$\int_{\underline{\theta}}^{1} \eta \left\{ c\left[F_D(\theta R + G - \tilde{s}) - F_D(\theta R - S) \right] + v\left[1 - F_D(\theta R + G - \tilde{s}) \right] \right\} h(\theta) d\theta < c.$$

Because this condition employs two upper bounds on prices, it is likely considerably stronger than necessary.

4. Optima and Equilibria Under Regime 1

This section analyzes minimization of expected total cost conditional on competitive storage operation under Regime 1. We first evaluate expected system cost as a function of *R*, *G*, and *S* and then compare the conditions for minimization of that function with conditions for long-run competitive equilibrium.

This analysis is algebraically simpler than but parallel to the analyses conditional on Regimes 2 and 3, which are sketched in the Appendix. Equation numbering in the Appendix tracks the numbering in this section to facilitate comparisons. The results in the Appendix are compared with those obtained here in Section 5.

It follows from the four cases under regime 1 that the expected amount of energy in storage at the end of a daytime (and the start of a nighttime) is

$$E(s)=\int_{\underline{\theta}}^{1}\left\{F_D(\theta R-S)S+\int_{\theta R-S}^{\theta R}(\theta R-L)f_D(L)dL\right\}h(\theta)d\theta. \tag{4.1a}$$

Differentiation shows that a small unit increase in R increases the expected value of s by less than expected generation per unit of renewable capacity:

$$\partial E(s)/\partial R=\int_{\underline{\theta}}^{1}\theta[F_D(\theta R)-F_D(\theta R-S)]h(\theta)d\theta<\int_{\underline{\theta}}^{1}\theta h(\theta)d\theta. \tag{4.1b}$$

In this regime, an increase in R only increases charging when L_D is between θR and (θR-S). Similarly, increasing storage capacity by a small unit increases the expected amount stored by less than one unit:

$$\partial E(s)/\partial S=\int_{\underline{\theta}}^{1}F_D(\theta R-S)h(\theta)d\theta<1. \tag{4.1c}$$

An increase in S has no impact on charging for final demand above (θR–S), but there is an increase in the amount stored at lower levels of final demand, when storage is charged fully.

It follows from the discussion of regime 1 above that expected *daytime* operating cost is given by

$$\Lambda(R,G)\equiv\int_{\underline{\theta}}^{1}\left\{\begin{aligned}&c\int_{\theta R}^{\theta R+G}[L-\theta R]f_D(L)dL+cG\left[1-F_D\left(\theta R+G\right)\right]\\&\qquad+v\int_{\theta R+G}^{\infty}[L-(\theta R+G)]f_D(L)dL\end{aligned}\right\}h(\theta)d\theta. \tag{4.2a}$$

Expected daytime operating cost is independent of S under this regime because storage demand never causes gas to turn on or induces a shortage. Increases in either R or G reduce expected daytime operating cost:

(4.2b) $$\partial\Lambda/\partial R = -\int_{\underline{\theta}}^{1} \theta\left\{c\left[F_D(\theta R+G)-F_D(\theta R)\right]+v\left[1-F_D(\theta R+G)\right]\right\}h(\theta)d\theta < 0,$$

(4.2c) $$\partial\Lambda / \partial G = -(v-c)\int_{\underline{\theta}}^{1}[1-F_D(\theta R+G)]h(\theta)d\theta < 0.$$

To get expected *nighttime* operating cost, it is simplest first to evaluate expected cost conditional on s and then take the expectation over s. If ηs MWh are sold from storage at night, expected nighttime operating cost is given by

(4.3a) $$\begin{aligned}\omega(G,s) \equiv\, & c\int_{\eta s}^{\eta s+G}[L-\eta s]f_N(L)\,dL + cG\left[1-F_N(G+\eta s)\right]\\ & + v\int_{\eta s+G}^{\infty}[L-(\eta s+G)]f_N(L)\,dL.\end{aligned}$$

When nighttime final demand, L_N, is between ηs and $\eta s+G$, it can be met by gas generation and sales from storage. Higher levels of demand lead to shortages, gas generation runs at capacity, and the system marginal cost rises to the value of lost load, v.[17]

Differentiating (4.3a) yields the impacts of increasing G and s on conditional expected cost:

(4.3b) $$\partial\omega/\partial G = -(v-c)\left[1-F_N(G+\eta s)\right] < 0,$$

(4.3c) $$\partial\omega/\partial s = -\eta\left\{cF_N(G+\eta s)+v\left[1-F_N(G+\eta s)\right]\right\} = -\eta\bar{P}_N(s) < 0.$$

The right-hand side of (4.3b) is minus the expected nighttime net earnings of a unit of gas capacity, conditional on G and s. The right-hand-side of (4.3c) is minus η times the expected nighttime marginal cost, again conditional on G and s. Since nighttime demand is perfectly inelastic, it is also minus η times the (conditional) expected nighttime market price of energy, which is the net revenue from a unit of charge in storage at the start of a nighttime period.

[17] Recall that by assumption (3.1), L_N is never less than ηs.

Taking the expectation of ω over s, using the description of competitive operating behavior under Regime 1 in Section 3, yields unconditional expected nighttime operating cost:

$$\text{(4.4a)} \qquad \Omega(R,G,S) \equiv \int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} F_D(\theta R - S)\omega(G,S) + \int_{\theta R - S}^{\theta R} \omega(G,\theta R - L) f_D(L)\,dL \\ + \left[1 - F_D(\theta R)\right]\omega(G,0) \end{array} \right\} h(\theta)d\theta.$$

Differentiating this expression shows that increases in renewables capacity reduce expected nighttime operating cost by increasing expected storage:

$$\text{(4.4b)} \qquad \begin{aligned} \partial\Omega / \partial R &= \int_{\underline{\theta}}^{1} \left[\int_{\theta R - S}^{\theta R} \frac{\partial \omega(G,\theta R - L)}{\partial R} f_D(L)dL \right] h(\theta)d\theta \\ &= -\int_{\underline{\theta}}^{1} \theta\eta \left\{ \int_{\theta R - S}^{\theta R} \overline{P}_N(G,\theta R - L)] f_D(L)dL \right\} h(\theta)d\theta. \end{aligned}$$

An increase in renewable capacity increases the expected end-of-daytime value of s, per equation (4.1b), thus lowering expected nighttime cost by increasing the expected nighttime supply from storage. The second equality in (4.4b) follows from (4.3c).

Increases in gas capacity reduce expected nighttime operating costs by reducing the probability of a nighttime shortage:

$$\text{(4.4c)} \qquad \begin{aligned} \partial\Omega / \partial G &= \int_{\underline{\theta}}^{1} \left\{ F_D(\theta R - S)\frac{\partial \omega(G,S)}{\partial G} + \int_{\theta R - S}^{\theta R} \frac{\partial \omega(G,\theta R - L)}{\partial G} f_D(L)dL + \left[1 - F(\theta R)\right]\frac{\partial \omega(G,0)}{\partial G} \right\} h(\theta)d\theta \\ &= -(v-c)\int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} F_D(\theta R - S)[1 - F_N(G + \eta S)] + \int_{\theta R - S}^{\theta R} [1 - F_N(G + \eta(\theta R - L))] f_D(L)dL \\ + [1 - F_D(\theta R)][1 - F_N(G)] \end{array} \right\} h(\theta)d\theta \\ &= -(v-c)\int_{\underline{\theta}}^{1} \Pr\left(P_N = v \middle| \theta\right) h(\theta)d\theta. \end{aligned}$$

The expression in in curly brackets in the second line is the expected probability, conditional on θ, of a nighttime shortage, the probability that L_N exceeds the supply from storage plus the supply from gas generation operating at capacity. To see this, note that such a shortage can arise in three possible ways. The first term in that expression reflects the fact that with probability $F_D(\theta R–S)$, storage is fully charged, and the conditional probability of a nighttime shortage is then $[1–F_N(G+\eta S)]$. At the other extreme, the third term reflects the fact that with probability [1–

$F_D(\theta R)$], daytime load exceeds θR so that gas must be used, the marginal operating cost is c, no charging occurs, and s=0. Conditional on no supply from storage, the probability of a nighttime shortage is just $[1–F_N(G)]$. Intermediate values of L_D, which imply $0 \leq s=\theta R–L_D \leq S$ from the discussion of competitive operation under regime 1, have density $f_D(L_D)$ and conditional nighttime shortage probability $\{1–F_N[G+\eta(\theta R–L)]\}$.

Finally, increases in storage capacity reduce expected nighttime operating cost by increasing expected nighttime sales from storage:

(4.4d) $$\partial\Omega / \partial S = \int_{\underline{\theta}}^{1} \left\{ F_D(\theta R - S) \frac{\partial \omega(G,S)}{\partial S} \right\} h(\theta) d\theta = -\eta \overline{P}_N(S) \int_{\underline{\theta}}^{1} F_D(\theta R - S) h(\theta) d\theta.$$

From (4.1c), the expected increase in s from a small unit increase in S is the expected value of $F_D(\theta R–S)$, and that increase occurs when the daytime load is such that generation is being fully charged. Since storage is only charged when the marginal cost of electricity is zero, there is no associated daytime cost increase. The reduction in expected nighttime operating cost is just expected nighttime marginal cost when storage is fully charged, $\overline{P}_N(S)$, times the increase in expected energy sales from storage, the expectation of $\eta F_D(\theta R–S)$.

Using (4.2a) and (4.4a), expected system total capacity plus operating cost as a function of R, G, and S can be written as

(4.5) $$E(TC) = C_R R + C_G G + C_S S + \Lambda(R,G) + \Omega(R,G,S).$$

Drawing on the analyses of Λ and Ω above, the first-order conditions for minimizing this quantity are given by:

(4.6a) $$\partial E(TC)/\partial R = C_R - \int_{\underline{\theta}}^{1} \theta \left\{ \begin{array}{l} \left(c[F(\theta R+G) - F(\theta R)] + v[1 - F(\theta R + G)] \right) \\ + \left(\eta \int_{\theta R - S}^{\theta R} \overline{P}_N(G, \theta R - L) f_D(L) dL \right) \end{array} \right\} h(\theta) d\theta = 0,$$

(4.6b) $$\partial E(TC)/\partial G = C_G - (v - c) \int_{\underline{\theta}}^{1} \left\{ [1 - F_D(\theta R + G)] + \Pr(P_N = v | \theta) \right\} h(\theta) d\theta = 0,$$

(4.6c) $$\partial E(TC)/\partial S = C_S - \eta \overline{P}_N(G,S) \int_{\underline{\theta}}^{1} F_D(\theta R - S) h(\theta) d\theta = 0.$$

Condition (4.6a) reflects the fact that renewable suppliers earn revenue whenever the daytime market price of energy is positive. With probability [$F_D(\theta R+G)$-$F(\theta R)$], that price is c, and with probability [1-$F(\theta R+G)$], that price is v. The discussion of regime 1 in Section 3 establishes that when L_D is between $\theta R–S$ and θR, the market price of energy is $\eta \bar{P}_N(G, \theta R - L)$. The integrand on the right of (4.5b) is thus the expectation of output per unit of renewable capacity, θ, times the expectation (over daytime and nighttime final demands) of revenue per unit of output, conditional on θ. That product is expected revenue per unit of capacity conditional on θ, and the integral is the unconditional expected revenue per unit of renewable capacity. Condition (4.6a) thus says that the cost of a unit of renewable capacity must equal the corresponding expected per-unit operating revenue. This necessary condition for R to minimize total system cost is a zero-expected-profit condition that must hold in long-run competitive equilibrium.

Gas generators earn revenue in excess of marginal operating cost in both daytime and nighttime only when conditions of shortage prevail. The integrand in (4.6b) is just per-unit net revenues conditional on shortage, times the sum of the daytime and nighttime probabilities of shortage conditions, conditional on θ. The integral is thus the unconditional sum of the two shortage probabilities. Condition (4.6b) requires that capital cost equal expected net operating revenue, another zero-expected-profit condition for minimization of total system cost that must hold in long-run competitive equilibrium.

Condition (4.6c) is also a zero-expected-profit condition. When L_D is below (θR-S) storage is fully charged. The integral in (4.5d) is the unconditional probability of that event, and it is multiplied by expected nighttime revenue per unit of capacity when storage is fully charged. Storage is also partially charged when L_D is between (θR-S) and θR. But the price paid for stored energy in that case is exactly equal to expected revenue of subsequent nighttime sales, so there is no effect on expected profit

Conditions (4.6) thus establish that all minima of expected total cost can be supported as equilibria of markets with continua of infinitesimal, price-taking suppliers of generation and storage. To prove that all competitive equilibria are minima of expected total cost, it would be necessary to show that the Hessian corresponding to conditions (4.6). I have been unable to do this, but I have been able to sign the diagonal elements of that matrix:

$$(4.7\text{a})\qquad \partial^2 E(TC)\big/\partial R^2 = \int_{\underline{\theta}}^{1} \theta \left\{ \begin{array}{l} \theta f(\theta R)[c-\eta \bar{P}_N(G,0)] + (v-c)\theta f_D(\theta R + G) \\ +\eta\theta f_D(\theta R - S)\bar{P}_N(G,S)] \\ -\eta \displaystyle\int_{\theta R - S}^{\theta R} \frac{\partial \bar{P}_N(G,\theta R - L)}{\partial R} f(L)dL \end{array} \right\} h(\theta)d\theta > 0,$$

$$(4.7\text{b})\qquad \partial^2 E(TC)/\partial G^2 = (v-c)\left\{ \begin{array}{l} f_D(\theta R + G) + F_D(\theta R - S) f_N(G+\mu S) \\ + \displaystyle\int_{\theta R - S}^{\theta R} f_N[G+\eta(R-L)]f(L)dL + [1-F_D(\theta R)]f_N(G) \end{array} \right\} > 0.$$

$$(4.7\text{c})\qquad \begin{aligned} \partial^2 E(TC)/\partial S^2 &= \eta \bar{P}_N(G,S)\int_{\underline{\theta}}^{1} f_D(\theta R - S)h(\theta)d\theta \\ &\quad + \eta^2 (v-c) f_N(G+\eta S)\int_{\underline{\theta}}^{1} F_D(\theta R - S)h(\theta)d\theta > 0. \end{aligned}$$

The first term on the right in (4.7a) is positive by the definition of Regime 1, and the integral in the last term is negative by (3.2b). It follows from equations (4.7) that conditional on the values of any two of the three stock variables, *R*, *G*, and *S*, the long-run competitive equilibrium value of the third minimizes expected total cost.

5. Concluding Remarks

The results in Appendices A and B that assume Regimes 2 and 3, respectively, exactly parallel those above. In all three possible regimes in this model, the first-order necessary conditions for minimization of expected total cost imply that competitive suppliers of renewable generation, gas generation, and storage earn zero expected profit. Thus, all minima of expected system cost can be supported as zero-expected-profit long-run competitive equilibria. Moreover, all own second partial derivatives are positive, so that conditional on the values of any two of the three stock variables, *R*, *G*, and *S*, the long-run competitive equilibrium value of the third minimizes expected total cost.

Under all three regimes, however, it has not been possible to show that the relevant Hessian is always positive definite. Thus, I cannot rule out the possibility that for some parameter values and probability distributions there exist zero-expected-profit competitive equilibria that do not correspond to minima of expected total cost. Given that this model is otherwise well-behaved and that the diagonal elements of the relevant Hessian are always positive, if such inefficient equilibria actually exist, it seems likely that they do so only in odd and unusual cases.

If energy prices are not capped below the value of lost load, Boiteux (1960, 1964)-Turvey (1968)-style models indicate that revenue from competition in energy markets leads to the economically efficient supply of generation capacity. The results here provide the same sort of support for reliance on the competitive supply of storage, at least in the context of the duck curve problem. These results thus provide support for the preference in the EU and at the federal level in the US for storage to be determined by market competition.

In most energy markets in the US and the EU, however, prices are capped below reasonable estimates of the value of lost load,[18] and those caps are occasionally binding. Boiteux-Turvey-style models then imply that revenues from sales in energy markets will provide inadequate incentives for investment in generation.[19] And in most electricity markets in the US and the EU, "capacity mechanisms" have been developed to supplement energy market revenues. These mechanisms typically involve a determination by a regulator or system operator of the level of generation capacity necessary for an acceptable level of reliability along with a capacity market or other mechanism for compensating suppliers of that capacity.

Just as caps on wholesale energy prices reduce incentives for investment in generation, it follows from the Boiteux-Turvey-style analysis here that caps on wholesale energy prices will lead to inadequate incentives for investment in storage for energy arbitrage. It seems unlikely that this theoretical finding is what has motivated the US states that have adopted quantitative storage targets, but it does seem likely that some analog to "capacity mechanisms" may come to be felt to be necessary to supplement energy arbitrage revenues to increase the supply of storage. "Capacity mechanisms" use reliability to determine the appropriate level of generation capacity; it is not clear how the appropriate level of storage capacity of various sorts would sensibly be determined.

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[18] The only exception of which I am aware is the ERCOT market, which serves most of the state of Texas.
[19] See Joskow (2007, 2008), and Joskow and Tirole (2007).

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Appendix A: Optima and Equilibria in Regime 2

It follows from the discussion of Regime 2 in Section 3 that expected charge at the end of a daytime period is given by

(A.1a) $$E(s) = \int_{\underline{\theta}}^{1} \left\{ F_D(\theta R + G - S)S + \int_{\theta R+G-S}^{\theta R+G} (\theta R + G - L) f_D(L) dL \right\} h(\theta) d\theta.$$

Increases in either *R* or *G* increase the expected value of *Z*:

(A.1b) $$\partial E(s)/\partial R = \int_{\underline{\theta}}^{1} \theta \left[F_D(\theta R + G) - F_D(\theta R + G - S) \right] h(\theta) d\theta < \int_{\underline{\theta}}^{1} \theta h(\theta) d\theta,$$

(A.1c) $$\partial E(s)/\partial G = \int_{\underline{\theta}}^{1} \left[F_D(\theta R + G) - F_D(\theta R + G - S) \right] h(\theta) d\theta < 1,$$

(A.1d) $$\partial E(s)/\partial S = \int_{\underline{\theta}}^{1} F_D(\theta R + G - S) h(\theta) d\theta < 1.$$

Expected *daytime* operating cost follows from competitive storage behavior in Regime 2:

(A.2a) $$\Lambda\left(R, G, S\right) \equiv \int_{\underline{\theta}}^{1} \left\{ \begin{matrix} c \int\limits_{\theta R - S}^{\theta R + G - S} \left[L - (\theta R - S) \right] f_D(L) dL + cG\left[1 - F_D\left(\theta R + G - S \right) \right] \\ + v \int\limits_{\theta R + G}^{\infty} \left[L - (\theta R + G) \right] f_D(L) dL. \end{matrix} \right\} h(\theta) d\theta.$$

In contrast to Regime 1, daytime operating cost here depends on *S* because demand from storage may cause gas to turn on. The derivatives of the daytime cost function are the following:

(A.2b) $$\partial \Lambda / \partial R = -\int_{\underline{\theta}}^{1} \theta \left\{ c[F_D(\theta R + G - S) - F_D(\theta R - S)] + v[1 - F_D(\theta R + G)] \right\} h(\theta) d\theta < 0,$$

(A.2c) $$\partial \Lambda / \partial G = \int_{\underline{\theta}}^{1} \left\{ c[1 - F_D(\theta R + G - S)] - v[1 - F_D(\theta R + G)] \right\} h(\theta) d\theta,$$

(A.2d) $$\partial \Lambda / \partial S = \int_{\underline{\theta}}^{1} \left\{ c\left[F(\theta R + G - S) - F(\theta R - S) \right] \right\} h(\theta) d\theta > 0.$$

If ηs MWh are sold from storage, expected *nighttime* operating cost is given by equation (4.3a) in the text:

$$\text{(A.3)}\qquad \begin{aligned} \omega(G,s) \equiv\ & c\int_{\eta s}^{\eta s+G} [L-\eta s] f_N(L)\,dL + cG\left[1-F_N(G+\eta s)\right] \\ & + v\int_{\eta s+G}^{\infty} [L-(\eta s+G)] f_N(L)\,dL. \end{aligned}$$

Equations (4.3b) and (4.3c) are thus also valid under Regime 2.

Taking the expectation over s from the description of competitive behavior under Regime 2 in Section 3 yields unconditional expected nighttime operating cost:

$$\text{(A.4a)}\quad \Omega(R,G,S) \equiv \int_{\underline{\theta}}^{1} \left\{ \begin{gathered} F_D(\theta R+G-S)\,\omega(G,S) + \int_{\theta R+G-S}^{\theta R+G} \omega(G,\theta R+G-L) f_D(L)\,dL \\ +\left[1-F_D(\theta R+G)\right]\omega(G,0) \end{gathered} \right\} h(\theta)\,d\theta.$$

The derivatives of this function parallel equations (4.4b) – (4.4d) in the text:

$$\text{(A.4b)}\qquad \begin{aligned} \partial\Omega/\partial R &= \int_{\underline{\theta}}^{1} \left[\int_{\theta R+G-S}^{\theta R+G} \frac{\partial \omega(G,\theta R+G-L)}{\partial R} f_D(L)\,dL \right] h(\theta)\,d\theta \\ &= -\int_{\underline{\theta}}^{1} \theta \left\{ \int_{\theta R+G-S}^{\theta R+G} \eta \overline{P}_N(G,\theta R+G-L)] f_D(L)\,dL \right\} h(\theta)\,d\theta, \end{aligned}$$

$$\text{(A.4c)}\qquad \begin{aligned} \partial\Omega/\partial G &= \int_{\underline{\theta}}^{1} \left\{ \begin{gathered} F_D(\theta R+G-S)\frac{\partial \omega(G,S)}{\partial G} + \left[1-F(\theta R+G)\right]\frac{\partial \omega(G,0)}{\partial G} \\ +\int_{\theta R+G-S}^{\theta R+G} \left[\frac{\partial \omega(G,\theta R+G-L)}{\partial G} + \frac{\partial \omega(G,\theta R+G-L)}{\partial s} \right] f_D(L)\,dL \end{gathered} \right\} h(\theta)\,d\theta \\ &= -(v-c)\int_{\underline{\theta}}^{1} \left\{ \begin{gathered} F_D(\theta R+G-S)[1-F_N(G+\eta S)] + [1-F_D(\theta R+G)][1-F_N(G)] \\ +\int_{\theta R+G-S}^{\theta R+G} [1-F_N(G+\eta(\theta R+G-L))] f_D(L)\,dL \end{gathered} \right\} h(\theta)\,d\theta \\ &\qquad -\int_{\underline{\theta}}^{1} \left\{ \int_{\theta R+G-S}^{\theta R+G} \overline{\eta P}_N(\theta R+G-L) f_D(L)\,dL \right\} h(\theta)\,d\theta \\ &= -(v-c)\int_{\underline{\theta}}^{1} \Pr(P_N = v|\theta) h(\theta)\,d\theta - \int_{\underline{\theta}}^{1} \left\{ \int_{\theta R+G-S}^{\theta R+G} \overline{\eta P}_N(\theta R+G-L) f_D(L)\,dL \right\} h(\theta)\,d\theta, \end{aligned}$$

(A.4d) $\partial\Omega / \partial S = \int_{\underline{\theta}}^{1} \left\{ F_D(\theta R + G - S) \frac{\partial \omega(G,S)}{\partial S} \right\} h(\theta) d\theta = -\eta \overline{P}_N(S) \int_{\underline{\theta}}^{1} F_D(\theta R + G - S) h(\theta) d\theta.$

Aside from the general substitution of ($\theta R+G$) for θR, the main difference between equations (A.4) and equations (4.4) in the text is the $\overline{P}_N$ integral (A.4c) that does not appear in (4.4c). That term stems from case (c), in which competitive suppliers bid the price of energy above c, thus enabling gas generators to more than cover their marginal cost. (See Figure 5.)

Expected total cost is given by equation (4.5a) in the text, modified to reflect the fact that S affects daytime operating costs under Regime 2:

(A.5) $$E(TC) = rR + gG + sS + \Lambda(R,G,S) + \Omega(G,S).$$

The first-order conditions for minimizing this quantity are given by

(A.6a) $\partial E(TC) / \partial R = C_R - \int_{\underline{\theta}}^{1} \theta \left\{ \begin{array}{l} c[F(\theta R + G - S) - F(\theta R - S)] + v[1 - F(\theta R + G)] \\ + \int_{\theta R + G - S}^{\theta R + G} \eta \overline{P}_N(G, \theta R + G - L)] f_D(L) dL \end{array} \right\} h(\theta) d\theta = 0,$

(A.6b) $\partial E(TC) / \partial G = C_G - \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} (v - c)\left\{ [1 - F(\theta R + G)] + \Pr(P_N = v | \theta) \right\} \\ + \int_{\theta R + G - S}^{\theta R + G} [\eta \overline{P}_N(G, \theta R + G - L) - c] f_D(L) dL \end{array} \right\} h(\theta) d\theta = 0,$

(A.6c) $\partial E(TC) / \partial S = C_S + \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} c[F(\theta R + G - S) - F(\theta R - S)] \\ -[\eta \overline{P}_N(S) F_D(\theta R + G - S)] \end{array} \right\} h(\theta) d\theta = 0.$

These are, again, zero-expected-profit conditions. Condition (A.6a) compares capital cost per unit of renewable capacity to expected revenue per incremental unit of capacity when the market price is c (in case b), when it is v (in case d), and in case (c) when competition suppliers raises the market price above marginal generation cost. Similarly, condition (A.6b) reflects the fact that in case (c) competition among storage suppliers raises the market price above c, so that gas suppliers earn positive operating profits. Finally, condition (A.6c) says that per-unit capital cost of storage plus expected incremental daytime charging cost in case (b) must equal expected incremental nighttime revenue. Charging cost in case (a) is zero. Expected charging cost in case (c), which does not appear in (A.6c), is exactly offset by expected nighttime revenues in that case, which also do not appear.

As in the case of Regime 1, I have been unable to prove that the Hessian corresponding to equations (A.6) is always positive definite. The diagonal elements of that matrix are the following:

(A.7a) $$\partial^2 E(TC)/\partial R^2 = \int_{\underline{\theta}}^{1} \theta \left\{ \begin{array}{c} \theta f_D(\theta R + G - S)[\eta \overline{P}_N(S) - c] \\ +\theta f_D(\theta R + G)[v - \eta \overline{P}_N(0)] + c\theta f_D(\theta R - S) \\ -\int_{\theta R+G-S}^{\theta R+G} \eta \frac{\partial \overline{P}_N(G, \theta R + G - L)}{\partial R} f_D(L)dL \end{array} \right\} h(\theta)d\theta > 0.$$

(A.7b) $$\partial^2 E(TC)/\partial G^2 = (v - c)\int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} F_D(\theta R + G - S) f_N(G + \eta S) \\ +[1 - F_D(\theta R + G)] f_N(G) \\ +(1+\eta)^2 \int_{\theta R+G-S}^{\theta R+G} f_N[G + \eta(\theta R + G - L)] f_D(L)dL \end{array} \right\} h(\theta)d\theta > 0,$$

(A.7c) $$\partial^2 E(TC)/\partial S^2 = \int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} f_D(\theta R + G - S)[\eta \overline{P}_N(S) - c] + cf_D(\theta R - S) \\ -F_D(\theta R + G - S)\eta \frac{\partial \overline{P}_N(G, S)}{\partial S} \end{array} \right\} h(\theta)d\theta > 0.$$

The first and second terms on the right of (A.7a) and the first term on the right of (A.7c) are positive by the definition of Regime 2. Equation (3.2b) implies that $\overline{P}_N(G, s)$ is a decreasing function of its second argument. As under Regime 1, these conditions imply that given values of any two of *R*, *G*, and *S*, the competitively determined value of the third variable minimizes expected total social cost.

Appendix B: Optima and Equilibria in Regime 3

The discussion of Regime 3 in the text implies the following equation for expected end-of-daytime charge:

$$\text{(B.1a)}\quad E(s)=\int_{\underline{\theta}}^{1}\left\{\begin{array}{l} F_D(\theta R-S)S+\int_{\theta R-S}^{\theta R-\tilde{s}}(\theta R-L)f_D(L)dL \\ +\tilde{s}[F_D(\theta R+G-\tilde{s})-F_D(\theta R-\tilde{s})] \\ +\int_{\theta R+G-\tilde{s}}^{\theta R+G}(\theta R+G-L)f_D(L)dL \end{array}\right\}h(\theta)d\theta.$$

Differentiation yields

$$\text{(B.1b)}\qquad \partial E(s)/\partial R=\int_{\underline{\theta}}^{1}\theta\left\{\begin{array}{l}[F_D(\theta R-\tilde{s})-F_D(\theta R-S)] \\ +[F_D(\theta R+G)-F_D(\theta R+G-\tilde{s})]\end{array}\right\}h(\theta)d\theta<\int_{\underline{\theta}}^{1}\theta h(\theta)d\theta,$$

$$\text{(B.1c)}\qquad \partial E(s)/\partial G=\int_{\underline{\theta}}^{1}\left\{\begin{array}{l}\frac{\partial\tilde{s}}{\partial G}[F_D(\theta R+G-\tilde{s})-F_D(\theta R-\tilde{s})] \\ +[F_D(\theta R+G)-F_D(\theta R+G-\tilde{s})]\end{array}\right\}h(\theta)d\theta,$$

$$\text{(B.1d)}\qquad \partial E(s)/\partial S=\int_{\underline{\theta}}^{1}F_D(\theta R-S)h(\theta)d\theta<1.$$

Increasing R shifts the $s(L_D)$ curve to the right, and s is increased when L_D is in two distinct intervals. Increasing G decreases $\tilde{s}$ from (3.5b), shifting the curve down for L_D between $(\theta R-\tilde{s})$ and $(\theta R+G-\tilde{s})$ and shifting it to the right beyond $(\theta R+G-\tilde{s})$. Thus, in contrast to Regime 1 (under which $\partial E(s)/\partial G=0$) and Regime 2 (in which $\partial E(s)/\partial G>0$), here the impact of increases in G on s cannot be signed in general. Increasing S just shifts the $s(L_D)$ curve up to the left of $(\theta R-S)$.

Expected daytime operating cost under Regime 3 is given by

$$\text{(B.2a)}\qquad \Lambda(R,G)=\int_{\underline{\theta}}^{1}\left\{\begin{array}{l}c\int_{\theta R-\tilde{s}}^{\theta R+G-\tilde{s}}[L-(\theta R-\tilde{s})]f_D(L)dL+cG[1-F_D(\theta R+G-\tilde{s})] \\ \qquad +v\int_{\theta R+G}^{\infty}[L-(\theta R+G)]f_D(L)dL\end{array}\right\}h(\theta)d\theta.$$

Note that S does not affect daytime operating cost under this regime, since changes in S only affect charging in case (a), when marginal operating cost is zero. Differentiation yields

(B.2b) $$\partial\Lambda/\partial R = -\int_{\underline{\theta}}^{1}\theta\left\{c[F_D(\theta R+G-\tilde{s})-F_D(\theta R-\tilde{s})]+v[1-F_D(\theta R+G)]\right\}h(\theta)d\theta,$$

(B.2c) $$\partial\Lambda/\partial G = \int_{\underline{\theta}}^{1}\left\{\begin{matrix} c[1-F_D(\theta R+G-\tilde{s})]+c\dfrac{\partial\tilde{s}}{\partial G}[F_D(\theta R+G-\tilde{s})-F_D(\theta R-\tilde{s})] \\ -v[1-F_D(\theta R+G)]\end{matrix}\right\}h(\theta)d\theta.$$

If ηs MWh are sold from storage, conditional expected nighttime operating cost, $\omega(G,s)$, is given by equation (4.3a) in the text, as under Regimes 1 and 2:

(B.3) $$\begin{aligned}\omega(G,s) \equiv c\int_{\eta s}^{\eta s+G}[L-\eta s]f_N(L)dL + cG\left[1-F_N(G+\eta s)\right] \\ +v\int_{\eta s+G}^{\infty}[L-(\eta s+G)]f_N(L)dL.\end{aligned}$$

Equations (4.3b) and (4.3c) are accordingly also valid under this regime.

Taking the expectation of ω over s, using the characterization of competitive behavior under this regime in Section 3, yields the unconditional expectation of nighttime operating costs:

(B.4a) $$\Omega(R,G,S) = \int_{\underline{\theta}}^{1}\left\{\begin{matrix} F_D(\theta R-S)\omega(G,S)+\displaystyle\int_{\theta R-S}^{\theta R-\tilde{s}}\omega(G,\theta R-L)f_D(L)dL \\ +[F_D(\theta R+G-\tilde{s})-F_D(\theta R-\tilde{s})]\omega(G,\tilde{s}) \\ +\displaystyle\int_{\theta R+G-\tilde{s}}^{\theta R+G}\omega(G,\theta R+G-L)f_D(L)dL+[1-F_D(\theta R+G)]\omega(G,0)\end{matrix}\right\}h(\theta)d\theta.$$

Differentiation of this expression yields

(B.4b) $$\begin{aligned}\partial\Omega/\partial R &= \int_{\underline{\theta}}^{1}\left\{\int_{\theta R-S}^{\theta R-\tilde{s}}\frac{\partial\omega(G,\theta R-L)}{\partial R}f_D(L)dL+\int_{\theta R+G-\tilde{s}}^{\theta R+G}\frac{\partial\omega(G,\theta R+G-L)}{\partial R}f_D(L)dL\right\}h(\theta)d\theta \\ &= -\eta\theta\int_{\underline{\theta}}^{1}\left\{\int_{\theta R-S}^{\theta R-\tilde{s}}\overline{P}_N(\theta R-L)f_D(L)dL+\int_{\theta R+G-\tilde{s}}^{\theta R+G}\overline{P}_N(\theta R+G-L)f_D(L)dL\right\}h(\theta)d\theta,\end{aligned}$$

(B.4c) $$\partial\Omega/\partial G = -\int_{\underline{\theta}}^{1}\left\{\begin{matrix}(v-c)\Pr(P_N=v|\theta)+c\dfrac{\partial\tilde{s}}{\partial G}[F_D(\theta R+G-\tilde{s})-F_D(\theta R-\tilde{s})] \\ +\eta\displaystyle\int_{\theta R+G-\tilde{s}}^{\theta R+G}\overline{P}_N(\theta R+G-L)f_D(L)dL\end{matrix}\right\}h(\theta)d\theta.$$

(B.4d) $\partial\Omega / \partial S = -\int_{\underline{\theta}}^{1} F_D(\theta R - S)\eta \bar{P}_N(G,S)h(\theta)d\theta.$

Expected total cost is again given by a slight modification of equation (4.5a) in the text:

(B.5) $$E\left(TC\right) = rR + gG + sS + \Lambda\left(R,G\right) + \Omega(R,G,S).$$

Differentiation yields the first-order necessary conditions for a minimum of expected total cost

(B.6a) $$\partial E(TC) / \partial R = C_R - \int_{\underline{\theta}}^{1} \theta \left\{ \begin{array}{c} c[F_D(\theta R + G - \tilde{s}) - F_D(\theta R - \tilde{s})] - v[1 - F_D(\theta R + G)] \\ + \eta \int_{\theta R - S}^{\theta R - \tilde{s}} \bar{P}_N(\theta R - L)\, f_D(L)dL \\ + \eta \int_{\theta R + G - \tilde{s}}^{\theta R + G} \bar{P}_N(\theta R + G - L)\, f_D(L)dL \end{array} \right\} h(\theta)d\theta = 0$$

(B.6b) $$\partial E(TC) / \partial G = C_G - \int_{\underline{\theta}}^{1} \left\{ \begin{array}{l} (v-c)\{\Pr(P_N = v|\theta) + [1 - F_D(\theta R + G)]\} \\ + \int_{\theta R + G - \tilde{s}}^{\theta R + G} [\eta \bar{P}_N(G, \theta R + G - L) - c] f_D(L)dL \end{array} \right\} h(\theta)d\theta,$$

(B.6c) $$\partial E(TC) / \partial S = C_S - \int_{\underline{\theta}}^{1} F_D(\theta R - S)\eta \bar{P}_N(G,S)h(\theta)d\theta = 0.$$

These conditions once again imply zero expected profits for each technology. Condition (B.6a) compares unit capital cost for renewable generation with the sum of expected revenue per unit of capacity in cases (b)-(e). Similarly, condition (B.6b) compares unit capital cost for gas generation with the sum of expected net revenues above variable cost in cases (d) and (e) and in nighttime shortage conditions. (Comparing (B.2c) and (B.4c) reveals that the change in $\tilde{s}$ induced by a marginal increase in G has equal and opposite effects on expected gas generation costs in daytime and nighttime periods.) Finally, (B.6c) compares unit capital cost of storage with the marginal expected revenue from the increased charging in case (a) that a unit increase in storage capacity would induce. As in the other regimes, payments by storage suppliers above marginal generation costs, in cases (b) and (d) here, show up as revenues for renewable and gas generators but not as costs for storage suppliers, since those payments exactly equal the expected nighttime revenue from sales of the incremental stored energy.

As in the cases of Regimes 1 and 2, I have been unable to prove that the Hessian corresponding to equations (B.6) is always positive definite. The diagonal elements of that matrix are the following:

(B.7a) $$\partial^2 E(TC)/\partial R^2 = \int_{\underline{\theta}}^{1} \theta^2 \left\{ \begin{array}{l} \eta \tilde{P}_N(S) f_D(\theta R - S) + [v - \eta \overline{P}_N(0)] f_D(\theta R + G)] \\ +\eta^2 (v-c)\theta \int_{R-S}^{\theta R - \tilde{s}} f_N[G + \eta(\theta R - L)] f_D(L) dL \\ +\eta^2 (v-c) \int_{\theta R + G - \tilde{s}}^{\theta R + G} f_N[G + \eta(\theta R + G - L) f_D(L) dL \end{array} \right\} h(\theta) d\theta > 0,$$

(B.7b) $$\partial^2 E(TC)/\partial G^2 = (v-c) \int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} F_D(\theta R - S) f_N(G + \eta S) \\ + \int_{\theta R - S}^{\theta R - \tilde{s}} f_N[G + \eta(\theta R - L)] f_D(L) dL \\ + f_N(G + \eta \tilde{s})[F_D(\theta R + G - \tilde{s}) - F_D(\theta R - \tilde{s})] \\ +(1+\eta)^2 \int_{\theta R + G - \tilde{s}}^{\theta R + G} f_N[G + \eta(\theta R + G - L)] f_D(L) dL \\ + f_N(G)[1 - F_D(\theta R + G)] \end{array} \right\} h(\theta) d\theta$$

$$+ \int_{\underline{\theta}}^{1} [v - \eta \overline{P}_N(0)] f_D(\theta R + G) h(\theta) d\theta > 0,$$

(B.7c) $$\partial^2 E(TC)/\partial S^2 = \int_{\underline{\theta}}^{1} \left\{ \begin{array}{c} f_D(\theta R - S) \eta \overline{P}_N(S) \\ +F_D(\theta R - S)(v-c)\eta^2 f_N(G + \eta S) \end{array} \right\} h(\theta) d\theta > 0.$$

Equations (3.5) were used in the derivation of (B.7a) and (B.7b). Equation (B.7cc) demonstrates that as under Operating Rules 1 and 2, in long-run competitive equilibrium, the value of *S* minimizes expected total cost conditional on the values of *R* and *G*.

Figure 1: **CAISO Total and Net Generation, 7 February 2018**

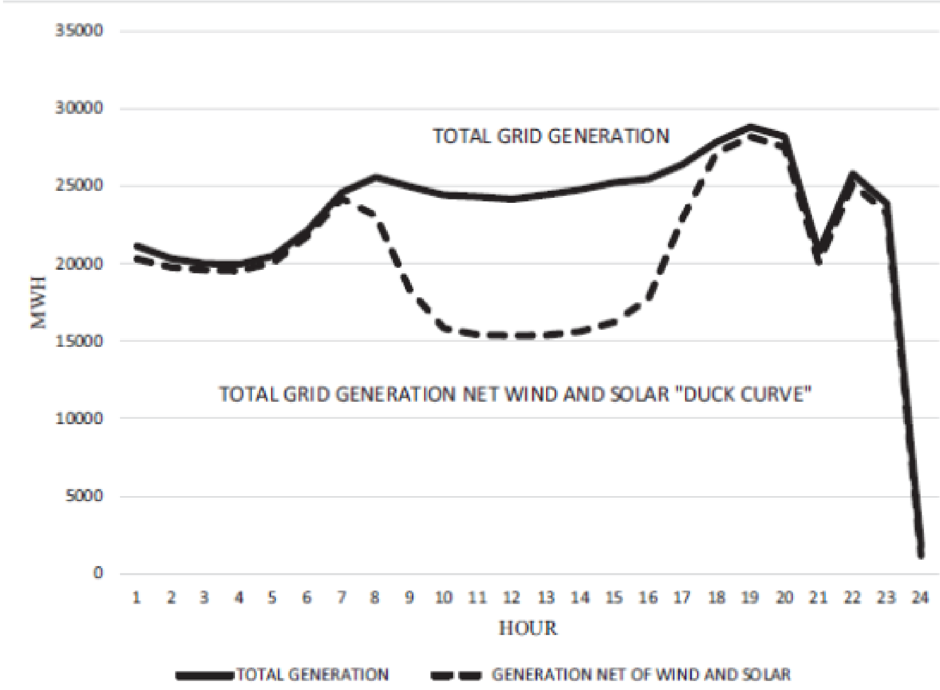


Figure 2: **CAISO average hourly LMP/average annual LMP by year**

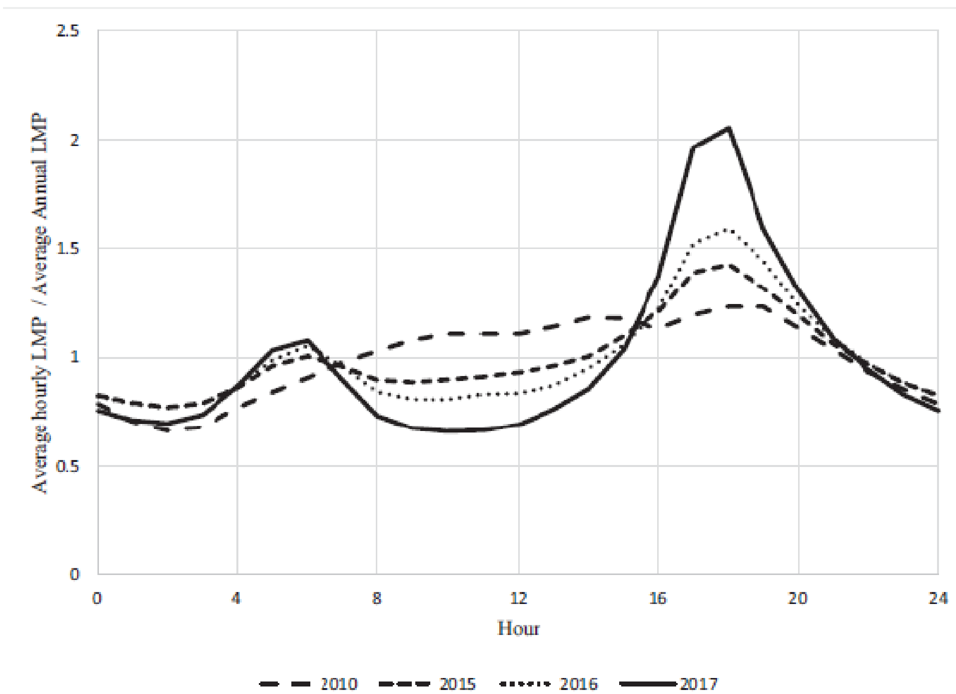


Figure 3: **The Daytime Competitive Demand for Energy**

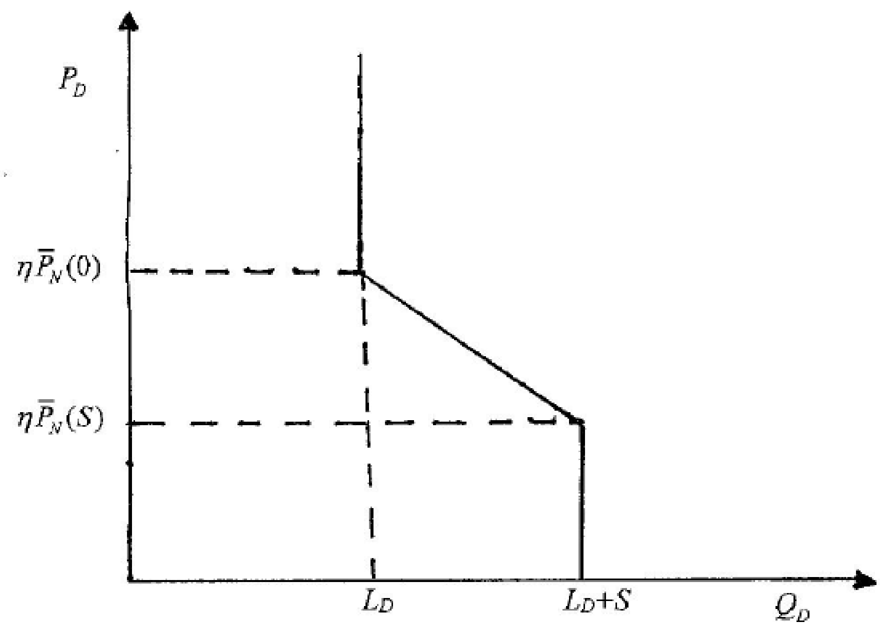


Figure 4: **Equilibrium in Regime 1, Case (b)**

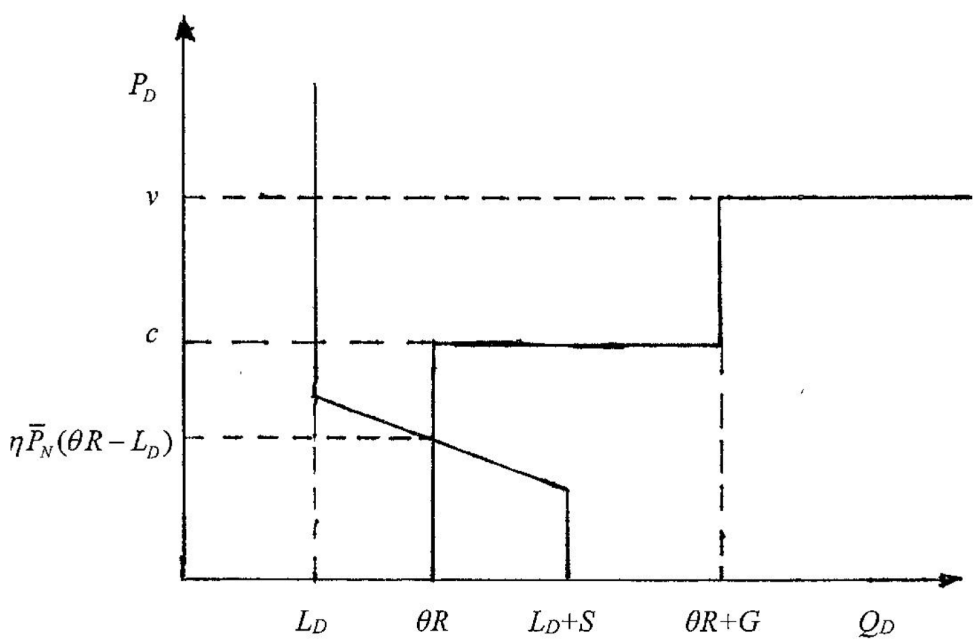


Figure 5: **Equilibrium in Regime 2, Case (c)**

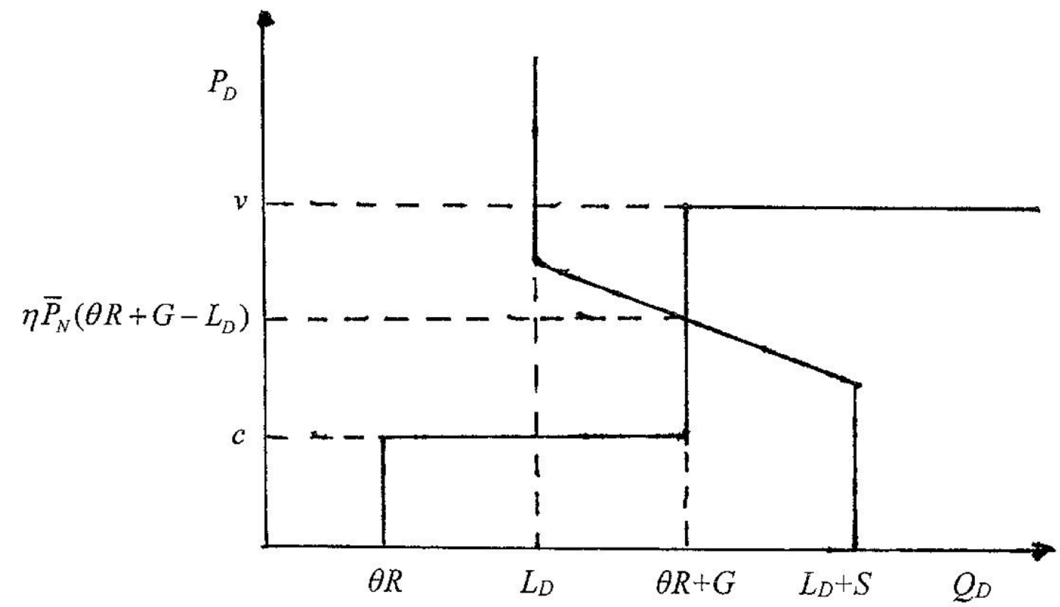
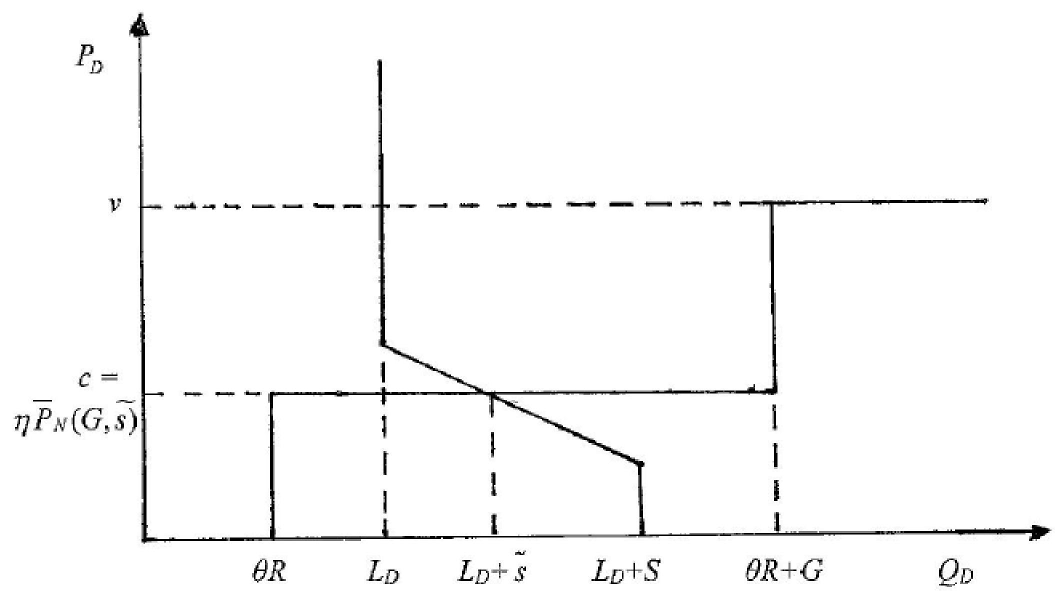


Figure 6: **Equilibrium in Regime 3, Case (c)**



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