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On the Efficiency of Competitive Energy Storage

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ABSTRACT

When energy storage is employed to facilitate large-scale integration of intermittent renewable electricity generation, do competitive bulk power markets continue to provide incentives for efficient investment? This essay adds competitively-supplied storage to a Boiteux-Turvey model of an electric power system with two types of periods. In the most interesting, tractable cases of this dynamic model, all efficient points are long-run competitive equilibria, and the long-run equilibrium value of storage capacity minimizes expected system cost conditional on generation capacities. But the analysis here cannot rule out the existence of inefficient equilibria.

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Wind and solar electricity generation have become more important in recent years, reflecting both declines in their costs and growing public interest, and this trend is generally forecast to continue.[2] These renewable technologies have near-zero marginal costs and are intermittent: their outputs vary over time and are imperfectly predictable. Energy storage can help balance supply and demand in the presence of intermittent generation and can reduce generation cost by adding to demand when system marginal cost is low and adding to supply when it is high.[3] As the cost of energy storage has also declined, its deployment to facilitate the integration of intermittent renewable has attracted considerable attention and support from policy-makers in the U.S. Notably, the U.S. Federal Regulatory Commission (2018) has recently issued Order 841, which is intended to open wholesale energy markets to merchant storage providers.[4] This Order rests on the presumption that existing markets will provide at least approximately optimal incentives for investment in both storage and generation: it does not contemplate the establishment of new markets or new policies. This essay provides a formal exploration of the validity of this presumption.

One might expect the validity of this presumption to follow directly from general theorems that establish the efficiency of competitive equilibria. But in the presence of uncertainty in demand or supply, those theorems require a complete set of contingent claims markets, which are neither present in real power systems nor in simple economic models of such systems.

Alternatively, one might expect the validity of this proposition, at least in simple models, to follow directly from the substantial literature that has built on the work of Boiteux (1960,

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[2] For instance, the U.S. Energy Information Administration's 2019 *Annual Energy Outlook* projects that even without additional supportive policies, wind and solar generation would grow from 9% of total generation in 2018 to 23% in 2050 (US EIA 2019, p. 21). And additional supportive policies are widely expected.

[3] See, for instance, Sisternes et al (2016). Storage can also perform other functions in electric power systems. Depending on the technology employed, storage facilities can provide frequency regulation, deferral of wires investment, and reducing the cost of spinning reserves. For discussions, see Giuletti et al (2018) and U.S. Government Accountability Office (2018), and for a worked example of a storage project that could perform multiple functions, see Sidhu et al (2018). The focus here is exclusively on the use of storage for energy arbitrage.

[4] In addition, at the U.S. federal level, storage facilities that are charged only by solar generators are eligible for a 30% investment tax credit. At the state level, since the promulgation of statutory requirements in 2010, the California Public Utilities Commission has been requiring load-serving entities to procure storage (Petlin et al (2018). Storage targets have also recently been established in New Jersey, New York, Massachusetts, and Oregon, and they are under consideration in other states as well.

1964) and Turvey (1968).[5] That literature assumes constant returns to scale, stochastic and (generally) inelastic demand, and multiple dispatchable generation technologies. If shortages occur, the system is assumed not to collapse, and price is assumed to rise to the value of lost load.[6] Then for essentially any probability distribution of demand, if generators behave competitively, a set of generation capacities such that all suppliers have zero expected profits is both necessary and sufficient for minimization of expected system cost. These results suggest as long as scale economies and other sources of non-convexity are not too important, and prices are not capped below the value of lost load,[7] real energy markets will provide at least approximately optimal incentives for investment in generation.

A natural approach to examine the validity of the presumption that underlies Rule 841, which is followed here, is to add storage to a Boiteux-Turvey style model and see if investment incentives are optimal at competitive equilibra. Unfortunately, while competitive generators can be modeled as reacting only to the current price in the energy market, the behavior of competitive storage suppliers must also depend on expectations about future prices. Thus adding competitive storage to a static Boiteux-Turvey style model, in which demand is stochastic, necessarily leads to a dynamic model in which, as this essay demonstrates, efficiency is more difficult to analyze.[8]

The analysis here provides some support for the presumption that energy markets need not be supplemented to support efficient outcomes when storage is deployed. In the most

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[5] See Dréze (1964) for an elegant exposition of Boiteux' work, originally written in the early 1950s, and see Joskow (1976) for a discussion of closely related later work. This older literature generally considered optimal period-specific retail prices that were determined before the realization of stochastic demand, not real-time wholesale pricing as is assumed here and as is descriptive of modern bulk power markets. Joskow and Tirole (2017) extend this literature substantially. In all these models, as here, generators' startup costs and minimum generation levels are neglected. If they are important, the sequence of different demand levels matters, not just their relative frequencies, so that, particularly in the presence of economies of scale, numerical methods must be employed to find optimal generation capacities. See, for instance, Jenkins and Sepulveda (2017).

[6] See Joskow and Tirole (2017) on this assumption. They and a number of other papers assume that at least some demand is price-sensitive and consider welfare maximization rather than cost minimization. I assume perfectly inelastic demand here for the sake of simplicity.

[7] If energy prices are capped below the value of lost load, as seems to be the case in many real markets, investment incentives are inadequate; see Joskow (2007, 2008) for discussions. Markets for capacity have been added in a number of systems to deal with this "missing money" problem. With effectively uncapped energy prices, the Electric Reliability Council of Texas (ERCOT) has for some years relied entirely on energy markets to provide investment incentives.

[8] In a recent working paper, Helm and Mier (2018) consider a system of the sort analyzed here but without uncertainty. They show (Proposition 1) that the first-best solution can be supported as a subsidy-free competitive equilibrium with a Pigouvian tax on the emissions of fossil generators.

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interesting tractable special cases of a Boiteux-Turvey model with storage, all minima of expected system cost correspond to long-run competitive equilibria. It has not been possible to establish the converse, however, so that the possible existence of inefficient competitive equilibria has not been ruled out. On the positive side, conditional on generation capacities, in long-run competitive equilibria storage capacity does minimize expected total cost, so there is no support for the notion that the entry of storage in competitive bulk power markets requires additional markets or policies to provide efficient incentives for investment in storage.

The model considered here has alternating periods of two types, labeled daytimes and nighttimes. Renewable generation with zero short-run marginal cost is only available during the daytime. This model is thus a greatly simplified representation of a system with a large amount of photovoltaic generation, which produces only in the daytime when the sun shines.[9] The focus is thus on arbitrage involving predictable rather than stochastic variation in renewable generation, but, as discussed below, it does not seem that making renewable generation stochastic would fundamentally change the results. Gas generation, which, for simplicity, stands in for the whole suite of fossil-fueled and nuclear technologies, is assumed dispatchable in both daytime and nighttime periods. Demand in both days and nights is stochastic, constant within periods, and perfectly inelastic. Section 1 presents these assumptions in more detail and introduces the notation used in what follows.

Without storage, Section 2 shows that the main result in the Boiteux-Turvey literature is easily established: zero-expected-profit (i.e., long-run) competitive equilibria satisfy the first- and second-order conditions for minimization of expected total cost. All efficient points thus correspond to such equilibria, and all such equilibria are efficient. Appendix A shows that the first-order conditions for expected cost minimization continue to hold at long-run competitive equilibria even if renewable output is stochastic, and the second-order conditions can be expected to hold in most cases. Allowing for stochastic renewable output makes the model more algebraically complex but does not change its fundamental structure. Hence renewable output is treated as non-stochastic succeeding sections.

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[9] The problems faced by real systems of this sort are, of course, more complicated than lack of renewable generation at night; see, for instance, Patel (2018) on the California system.

Under constant returns, competitive generators' operating rules are simple: produce if and only if market price is greater than or equal to marginal cost. To establish a benchmark, Section 3 investigates the implications of a comparably simple operating rule for competitive suppliers of storage: charge fully in the daytime when renewable generation is available and discharge fully at night when it is not. The first-order necessary conditions for minimizing expected total cost conditional on this operating rule describe a zero-expected-profit competitive equilibrium. However, those conditions are generally sufficient for cost minimization *only* in the very special case that daytime demand can always be satisfied by renewable plus gas generation so there is zero probability of a daytime shortage. Moreover, because this operating rule calls for charging even under shortage conditions in the daytime and discharging storage completely at night even if demand is less than the available supply from storage, it would not be followed by competitive suppliers of storage.

Section 4 considers operating rules that would be followed by competitive storage suppliers in this model. In general, optimal charging or discharging depends on the current energy market price, the amount of energy in storage, and expectations regarding future energy prices.[10] Even imposing rational expectations, this simple model does not seem to be tractable in the general case in which storage may not be empty at the start of daytime periods. Section 4 derives the three possible operating rules that could be followed by competitive storage suppliers if storage *is* empty at the start each daytime period. These rules depend on the distribution of nighttime prices. Section 4 also presents a sufficient consistency condition for each rule: if that condition holds and if competitive storage suppliers follow the corresponding operating rule, they will in fact find it optimal to sell all stored energy each nighttime.[11]

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[10] There is an extensive literature on the optimal control of energy storage devices. Xu et al (2019) provides a recent contribution and numerous citations to that literature. A good deal of the existing literature on storage assumes perfect foresight and investigates the profitability of arbitrage under various cost assumptions and observed price trajectories; see, e.g. Salles et al (2017) and Giullet et al (2018). The usual finding is that arbitrage profits do not cover the capital costs of storage facilities. While of some interest, this finding sheds essentially no light on the general optimality of the investment incentives provided by energy markets when storage is available and its deployment is profitable at the margin.

[11] There remains a consistency problem, however. Each operating rule developed in Section 4 is optimal under a different set of nighttime price distributions, but the operating rule followed by storage suppliers will affect that distribution. Even in this simple model it is not clear how to enforce consistency between an operating rule for storage and the nighttime price distributions it supports. This paper follows what seems to be the only tractable way forward: I identify operating rules for storage suppliers that are consistent with competitive equilibrium under some price distributions and investigate the relation between cost minimization and zero-expected-profit competitive equilibria conditional on each of those rules being followed by competitive storage suppliers.

Section 5 considers minimization of expected total cost under the algebraically simplest operating rule derived in Section 4. Parallel analyses under the other two operating rules are summarized in Appendices B and C. Section 6 discusses some implications of the results of this analysis.

1. Assumptions and Notation

The daytime and nighttime periods in each day are assumed to be of equal length for convenience, and the probability distributions governing events within daytime and nighttime periods are assumed to be independent. There are four technologies with constant returns to scale supplied by risk-neutral firms:

> **Gas**, which stands in for all dispatchable fossil and nuclear technologies, has capacity G, per-day unit capacity cost g, and per-kwh operating cost c.

> **Renewables**, have capacity R, per-day unit capacity cost r, and zero operating cost. Maximum renewable output is zero during the night and is equal to R for certain in the day, except in Appendix A, where it is assumed to vary stochastically between zero and R. It is assumed that renewable generation can be costlessly curtailed whenever daytime demand is less than available renewable supply.

> **Scarcity**, which operates when load exceeds capacity and there is lost load, has unlimited capacity with zero capital cost and per-kwh variable cost v, the value of lost load. The probability of scarcity is assumed to be positive in both periods. (It must be positive in at least one period for gas to recover its capital cost under competition.[12])

> **Storage,** which is introduced in Section 3, has capacity S, per-day unit capacity cost s, and round-trip efficiency $\sigma < 1$. This paper focuses on the empirically interesting case (for at least some time to come) $S < R$. In Sections 4 and 5, when storage is present, the stochastic amount of energy in storage at the end of a daytime period is Z.

As an accounting convention, the loss of energy due to storage occurs when storage is discharged, not when it is charged.

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[12] For clear discussions, see Joskow (2007, 2008).

Cost parameters are assumed to satisfy the following inequalities:

(1.1) $$g < r, \quad c < v, \quad and \quad r < c.$$

The first two of these are familiar: gas has lower capital cost than renewables, and the value of lost load exceeds the incremental cost of gas generation. The third is necessary for gas to be economical. Gas capacity is necessary to meet demand at night when renewable generation is not available, so for it to be efficient for any renewable capacity to be installed and used during the day, the total per-kwh cost of renewable generation, r, must be less than the incremental cost of gas generation from existing capacity, c. A high value of c is most naturally interpreted as reflecting a substantial price of carbon emissions.

The two periods in each day are as follows:

Daytime: load before storage *purchases* (native load), L, is distributed according to smooth distribution function $F(L)$ on $[0,\infty]$ with density $f(L)$.

Nighttime: Load before storage *sales* (native load), X, is distributed according to smooth distribution function $H(X)$ on $[0,\infty]$, with density $h(X)$.

I assume that s is low enough, σ is high enough, and nighttime load is on average high enough relative to daytime load that some arbitrage via storage is generally economic, and I thus concentrate on internal minima of expected system cost.

2. The System Without Storage

Assuming that renewable output during the day is always equal to R, storage is not available, and the energy market is competitive, so that generation technologies (including scarcity) are used in order of increasing marginal cost, expected total (daily) cost is given by

$$E(TC) = rR + gG + c\int_{R}^{R+G}(L-R)f(L)dL + cG[1-F(R+G)] + v\int_{R+G}^{\infty}[L-(R+G)]f(L)dL$$

(2.1) $$+ c\int_{0}^{G}Xh(X)dX + cG[1-H(G)] + v\int_{G}^{\infty}(X-G)h(X)dx.$$

The first two terms on the right of (2.1) are the system's capital cost. The third term is the expected cost of gas generation during the day when demand exceeds renewable capacity but is less than renewable plus gas capacity. The fourth term is the expected cost of running gas at

capacity during a daytime shortage, and the final term in the first line is the expected cost of lost load when there is a shortage during the day. The three terms in the second line are the nighttime equivalents of the last three terms in the first line, with zero renewable output.

Differentiating equation (2.1) yields the first-order condition for renewable capacity:

(2.2a) $\qquad \partial E\left(TC\right)/\partial R = r - \left\{c\left[F(R+G) - F\left(R\right)\right] + v\left[1 - F(R+G)\right]\right\} = 0.$

Here $F(R)$ is the probability that renewables alone can meet daytime demand, and $F(R+G)$ is the probability that gas plus renewables can meet demand. So $[F(R+G)-F(R)]$ is the probability that the load is high enough that gas is dispatched, demand is met, and gas sets the energy market price at c under competition. Similarly, $[1-F(R+G)]$ is the probability that there is unmet demand, so under competition the energy price is the value of lost load, v. Thus the quantity in curly brackets in equation (2.2a) is the expected revenue per unit of renewable capacity, and equation (2.2a) is a zero-expected-profit condition for renewable capacity.

Differentiating equation (2.1) again yields the first-order condition for gas capacity:

(2.2b) $\qquad \partial E\left(TC\right)/\partial G = g - \left(v-c\right)\left\{\left[1 - F\left(R+G\right)\right] + \left[1 - H\left(G\right)\right]\right\} = 0.$

Under energy market competition, the revenue to gas generators exceeds marginal operating cost only during shortages when the price of energy rises to v. The second term in equation (4b) is the difference between the scarcity/shortage price and gas's per-unit operating cost, times the probability of scarcity during the day plus the probability of scarcity at night. So equation (2.2b) says that gas's expected net revenue under competition would equal its capital cost, and it is thus a zero-expected-profit condition for gas capacity.

To see if long-run energy market equilibrium is sufficient as well as necessary for optimal capacity choice, consider the Hessian for this problem, where the first row and column relate to R, and the second row and column relate to G:

(2.3) $\qquad \begin{pmatrix} cf(R)+a & a \\ a & (v-c)h(G)+a \end{pmatrix},$
$\qquad \qquad where \quad a \equiv (v-c)f(R+G) > 0.$

The diagonal elements of this matrix are positive, so the matrix is positive definite if its determinant is positive. That determinant is given by

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$$(2.4) \quad \begin{aligned} &\left[a+cf\left(R\right)\right]\left[a+(v-c)h(G)\right]-\left[a\right]^2 \\ &=\left[a\right]\left[(v-c)h(G)\right]+\left[cf\left(R\right)\right]\left[a\right]+\left[cf\left(R\right)\right]\left[(v-c)h(G)\right]. \end{aligned}$$

Since all the quantities in square brackets in the second line are positive, so is the determinant, the Hessian (2.3) is positive definite, and long-run competitive equilibria always signal minima of expected total cost..

Appendix A considers a generalization of this model to allow renewable output to be stochastic. In that model, the first-order conditions for expected cost minimization are again zero-expected-profit conditions for long-run competitive equilibrium. The second-order conditions are more complicated. Both the second own-partial derivatives are always positive, and the determinant of the Hessian is positive if F is uniform. While that determinant cannot be signed in general, the discussion in Appendix A indicates that its structure suggests that is positive except in very unusual cases, so that social optima in capacities generally coincide with competitive equilibria.

3. A Simple Operating Rule for Storage

As a starting point for the analysis of the impact of storage, this section considers the simplest plausible operating rule for storage: charge fully every daytime, when zero-marginal-cost renewable generation is available, and sell all stored energy every nighttime, when it is not.

Daytime demand thus equals $(L+S)$ regardless of price; nighttime demand equals X regardless of price; daytime competitive supply is a stair-step with jumps at R and G; and nighttime competitive supply is a stair-step with jumps at σS and $(G+\sigma S)$. Taking expectations over L and X, expected total cost under competitive operations is given by

$$(3.1) \quad \begin{aligned} E\left(TC\right) =& rR+gG+sS+c\int_{R-S}^{R+G-S}\left[L-(R-S)\right]f(L)dL+cG\left[1-F(R+G-S)\right] \\ &+v\int_{R+G-S}^{\infty}\left[L-(R+G-S)\right]f(L)dL+c\int_{\sigma S}^{G+\sigma S}\left[X-\sigma S\right]h(X)dX \\ &+cG\left[1-H\left(G+\sigma S\right)\right]+v\int_{G+\sigma S}^{\infty}\left[X-(G+\sigma S)\right]h(X)dx. \end{aligned}$$

The first three terms in (3.1) are capital costs as in (2.1), and the next three terms equal the corresponding terms in (2.1) with $(R–S)$, the net supply from storage available to meet native load, replacing R there. The last three terms are expected nighttime costs. Storage is on the

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margin with an effective marginal cost of zero if the load is less than σS, gas is on the margin if the load is between σS and $(G+\sigma S)$, and loss-of-load occurs (and the competitive market price is v) when load exceeds $(G+\sigma S)$.

Differentiating equation (3.1) yields the first-order conditions for minimizing expected total cost:

(3.2a) $\quad \partial E(TC)/\partial R = r - \left\{ c\left[F(R+G-S)-F(R-S)\right] + v\left[1-F(R+G-S)\right]\right\} = 0,$

(3.2b) $\quad \partial E(TC)/\partial G = g - (v-c)\left\{\left[1-F(R+G-S)\right]+\left[1-H(G+\sigma S)\right]\right\} = 0,\ and$

(3.2c)
$$\partial E(TC)/\partial S = s + \left\{ c\left[F(R+G-S)-F(R-S)\right] + v\left[1-F(R+G-S)\right]\right\}$$
$$- \sigma\left\{ c\left[H(G+\sigma S)-H(\sigma S)\right] + v\left[1-H(G+\sigma S)\right]\right\} = 0.$$

Equations (3.2a) and (3.2b) correspond exactly to equations (2.2), with net load distributions shifted by the (deterministic) operations of storage, and they are zero-expected-profit conditions as before. Equation (3.2c) says that the capital cost per unit of storage capacity, plus the average daytime price, which is the expected per-unit cost of charging, must equal σ times the expected nighttime price, which is the expected per-unit revenue from nighttime sales. Equation (3.2c) is thus also a zero-expected-profit condition. Equations (3.2) accordingly describe a long-run competitive equilibrium so that all minima of expected total cost correspond to such equilibria.

To see whether the converse is true, whether all long-run competitive equilibria correspond to minima of total cost, it is necessary to investigate the Hessian for this minimization problem. Differentiating equations (3.2) and letting the first row and column correspond to R, the second to G, and the third to S, that Hessian is

(3.3)
$$\begin{pmatrix} cf(R-S)+A & A & -[A+cf(R-S)] \\ A & A+B & A+\sigma B \\ -[A+cf(R-S)] & A+\sigma B & A+cf(R-S)+\sigma^2\left[B+ch(\sigma S)\right] \end{pmatrix},$$
$$where\ A \equiv (v-c)f(R+G-S) \geq 0, \quad and\ B \equiv (v-c)h(G+\sigma S) > 0.$$

All terms here, including the own second partials on the diagonal, are positive except for $\partial^2 E(TC)/\partial R\partial S$. Note that $A = 0$ if the probability of a daytime shortage is zero.

The 2x2 minors corresponding to pairs of the three supply technologies are also positive for all native load distributions:

$$Det(R,G) = AB + (A+B)cf(R-S) > 0,$$

(3.4)
$$Det(G,S) = (A+B)[cf(R-S) + \sigma^2 ch(\sigma S)] + AB(1-\sigma)^2 > 0,$$

$$Det(R,S) = \sigma^2[cf(R-S) + A][B + ch(\sigma S)] > 0.$$

For the Hessian in (3.3) to be positive definite, its determinant must also be positive. But it seems impossible to sign that determinant in general, despite the results in (3.4), even with strong additional assumptions. Even if native load is uniformly distributed over the same range in both daytime and nighttime periods, for instance, the determinant of (3.3) can have either sign.

In the very special case of a zero probability of daytime shortages, $A=0$, and it is easy to show that the determinant of (3.3) is equal to $\sigma^2 c^2 B f(R-S)h(S)$. As this quantity is positive, it follows that a zero-expected-profit competitive equilibrium is both necessary and sufficient for capacity choices that minimize expected total cost.

Unfortunately, as noted in the introduction, the simple operating rule considered in this section would not be followed under competition except in this very special case.[13] This rule calls for charging storage fully even if there is a shortage in the daytime. The next nighttime price can be no higher than the scarcity price, v, however, and some energy is lost in charging and discharging. If the daytime price is v when storage is charged, and the stored energy is sold at night, there would be a certain operating loss on that round-trip transaction. Thus, no competitive storage supplier would ever charge during a shortage.

Moreover, this operating rule calls for full discharge of storage every night, regardless of the level of demand. Suppose nighttime demand, X, is less than σS, the energy that could be sold from storage. Competition among storage suppliers would cause X to be sold at a price of zero, and this operating rule would require that the excess be somehow disposed of with no cost or revenue consequences. However, the expected value of instead carrying a unit of energy into the next daytime is at least the expected daytime price, which is positive. In this case competitive storage suppliers would find it more profitable simply to carry any unsold excess into the next daytime period.

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[13] I am indebted to Jean Tirole for pointing this out early in my work on this problem.

4. Competitive Operating Rules for Storage

If both the amount of stored energy at the end of each nighttime and the amount at the end of each daytime could both be positive, both would be random variables because both native loads are stochastic. One can easily derive operating rules, like those below, for competitive storage operations in both daytime and nighttime periods that depend on end-of-period values of stored energy.[14] In this general case, however, it is not obvious how to relate the two value functions involved to the primitives of the model, or even to each other. This general case seems intractable. It is plausible that competitive equilibria exist in this general case, but it is not at all clear their relation to minima of expected cost might be characterized.

I focus in what follows on the tractable special case that seems of most interest: energy stored during a daytime day is discharged completely in the next nighttime, so that each daytime period begins with empty energy storage. Since the basic function of storage in this model is to move energy from the daytime, when some energy is available at zero marginal cost, to the nighttime, when all generated energy has positive marginal cost, this seems a particularly interesting special case.

There are two ways competitive storage suppliers could end a nighttime period with positive stored energy. First, they could have begun the period with positive stored energy, Z, and encountered perfectly inelastic demand, X, that was less than σZ. As noted at the end of Section 3, they would then sell σZ and carry $(Z-X/\sigma)$ into the next daytime.[15] The simplest way to rule this out is to assume that the minimum nighttime demand exceeds the maximum supply from storage:

(4.1) $$H(X) = 0 \ for \ X \le X_{min}, \quad where \ X_{min} > \sigma S.$$

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[14] Let $\Gamma(Z)$ be the end-of-daytime value of an incremental unit of stored energy when storage contains Z kwh, and let $\Theta(Y)$ be the end-of-nighttime value of an incremental unit of stored energy when storage contains Y kwh. If storage is fully discharged every nighttime, $\Gamma(Z)$ is just σ times the expected nighttime price, and the development in the rest of this section involves comparisons of the incremental cost of charging with its incremental value. In general, however, $\Gamma(Z)$ would also reflect the fact that under some nighttime conditions, discussed in the text, it would more profitable not to discharge fully, but rather to carry some energy into the next daytime. The nighttime decision of competitive storage suppliers involves comparing $\Theta(Y)$ with the marginal nighttime price of energy. I suspect that if one is willing to make the strong assumption that there is only one type of period, so that the Γ and Θ functions are identical, it may be possible to make progress without additional assumptions.

[15] Recall the convention that energy losses occur when storage is discharged, not when it is charged.

Since *S* is endogenous, this is not fully satisfactory. But at least the notion of a positive minimum demand is plausible.

The second way competitive storage suppliers might end a nighttime period with positive stored energy could occur if the energy market price were *c* and gas capacity were not exhausted. Depending on expectations regarding future prices, they might find it optimal to purchase energy at *c* and carry it into the next daytime. The marginal value of energy purchased to carry forward in this fashion is clearly non-increasing, so to rule out this behavior it suffices to rule out buying and carrying forward the first marginal unit. As discussed below, the conditions ruling this out, (4.3), (4.4), and (4.6), are specific to each of the three possible competitive operating rules. Unfortunately, like (4.1), these conditions involve endogenous variables.

Under operating rules that call for full nighttime discharge, a competitive supplier of storage would only add to charge in the daytime if the marginal cost of doing so did not exceed the expected nighttime revenue per unit stored, which is σ times the expected nighttime price. Because nighttime demand is perfectly inelastic, the expected nighttime price is just the expected marginal cost of supply:

(4.2a) $$\bar{P}_N(G,Z) \equiv E_X[P_N(G,Z)] = cH(G+\sigma Z) + v\left[1 - H(G+\sigma Z)\right]$$

By assumption (4.1), stored energy alone is never sufficient to meet demand. With probability $H(G+\sigma Z)$, stored energy plus gas *is* sufficient, and there is no shortage. Because increases in stored energy shift the nighttime supply curve to the right, the expected price falls with *Z*:

(4.2b) $$\partial \bar{P}_N(G,Z)]/\partial Z = -\sigma(v-c)h(G+\sigma Z) < 0.$$

In what follows, dependence of $\bar{P}_N$ on *G* is generally suppressed to reduce notational clutter.

If all stored energy is to be sold at night regardless of price, a risk-neutral competitive storage supplier would not add to storage during the day if the per-unit cost of doing so exceeded $\sigma \bar{P}_N(0)$. From (4.2a), $\bar{P}_N(0)$ is always between *c* and *v*, the two possible nighttime prices when there are no sales from storage, but σ times this quantity may be above or below *c*. At the other extreme, if the daytime price of energy is less than or equal to $\sigma \bar{P}_N(S)$, a positive quantity, it is profitable to charge storage up to capacity.

It follows from this discussion that the daytime competitive demand curve for energy is vertical at L for prices above $\sigma \overline{P}_N(0)$, declines to $(L+S)$ for prices between $\sigma \overline{P}_N(0)$ and $\sigma \overline{P}_N(S)$, and is vertical at $(L+S)$ for lower prices. This shape is illustrated in Figure 1, where P_D is the daytime energy price, and Q_D is the corresponding quantity. The supply curve for energy in the daytime is a simple stair-step, as in Sections 2 and 3. There are three possible operating rules that competitive suppliers of storage can follow, following from the three possible relations between daytime marginal costs and expected nighttime prices.

Operating Rule 1: $\sigma \overline{P}_N(S) < \sigma \overline{P}_N(0) < c$. Under this configuration and the assumption of full nighttime discharge, competitive storage suppliers will charge as much as possible as long as the daytime market price is less than c, so only zero-marginal-cost output from renewables is stored.

In daytime periods under this configuration, the demand from storage, Z, gas output, marginal operating cost, and the market price of energy are determined by the relationship between native load, L, and generation and storage capacities. Maintaining the assumption $S < R$, this configuration implies a competitive operating rule with four daytime cases:

Case		Z	Gas Output	Marginal Cost	Energy Price
(a)	$L \leq R-S$	S	0	0	0
(b)	$R-S \leq L \leq R$	$R-L$	0	0	$\sigma \overline{P}_N(R-L)$
(c)	$R \leq L \leq R+G$	0	$L-R$	c	c
(d)	$R+G \leq L$	0	G	v	v

In case (a) there is sufficient renewable capacity both to satisfy native load and to charge storage to capacity without requiring gas generation. In case (b), charging is curtailed as necessary to avoid calling on gas generation and driving price above $\sigma \overline{P}_N(0)$. Marginal operating cost remains at zero, but the market price of energy is bid up by competitive storage to the expected value of additional nighttime sales, as illustrated in Figure 2.[16] In case (c) native load is high enough that gas is necessarily on the margin, and both the marginal operating cost and the energy price are c.

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[16] I am indebted to William Hogan and, independently, to Benjamin Hobbs and Haralampos Avraam for this key insight.

No charging occurs. Case (d) involves shortage, and the system marginal cost and price are the value of lost load, so there is, again, no charging.

Given the operating rule just described, suppose a competitive storage supplier considers storing a unit of energy at night at a cost of c for sale the next day. For this to be unprofitable, c must exceed the expected sales price the next day. Using the upper bound on the energy price in case (b), c, a sufficient condition for storage at night at a cost of c to be unprofitable is

(4.3)
$$c > v\frac{\sigma[1 - F(R+G)]}{1 - \sigma[F(R+G) - F(R-S)]}.$$

If (4.1) and (4.3) are satisfied, storage will be empty at the start of every daytime period under competition, as assumed. Since $v > c$, for (4.3) to be satisfied is the (endogenous) probability of a daytime shortage, $[1-F(R+G)]$, must be sufficiently low.

Operating Rule 2: $c < \sigma \overline{P}_N(S) < \sigma \overline{P}_N(0) < v$. There are, again, four cases under this configuration of costs and prices:

Case		Z	Gas Output	Marginal Cost	Energy Price
(a)	$L \leq R\text{--}S$	S	0	0	0
(b)	$R\text{--}S \leq L \leq R+G\text{--}S$	S	$L\text{--}(R\text{--}S)$	c	c
(c)	$R+G\text{--}S \leq L \leq R+G$	$R+G\text{--}L$	G	c	$\sigma \overline{P}_N(R+G-L)$
(d)	$R+G \leq L$	0	G	v	v

In cases (a) and (b), there is sufficient capacity both to satisfy native load and to charge storage fully without creating a shortage. The marginal generating cost is zero in case (a), and in case (b) it is c because gas is on the margin. In case (c), storage cannot be fully charged without creating a shortage and driving the market price to v. As in case (b) under Operating Rule 1, competitive storage suppliers bid the energy price above marginal cost, c, as illustrated in Figure 3. Finally, in case (d), there is an unavoidable shortage, marginal cost and price are the value of lost load, v, and storage demand is reduced to zero.

Using the upper bound on price in case (c), the reasoning that let to condition (4.3) yields a sufficient condition that rules out the profitability of purchasing energy at night at a price of c and reselling the next day under Operating Rule 2:

$$(4.4) \qquad c > v \frac{\sigma\left[1 - F(R+G-S)\right]}{1 - \sigma\left[F(R+G-S) - F(R-S)\right]}.$$

Operating Rule 3: $\sigma\overline{P}_N(S) < c < \sigma\overline{P}_N(0) < v$. The competitive operating rule implied by this final configuration is a bit more complicated than Operating Rules 1 and 2:

Case		Z	Gas Output	Marginal Cost	Energy Price
(a)	$L \leq R-S$	S	0	0	0
(b)	$R-S \leq L \leq R-\tilde{Z}$	$R-L$	0	0	$\sigma\overline{P}_N(R-L)$
(c)	$R-\tilde{Z} \leq L \leq R+G-\tilde{Z}$	$\tilde{Z}$	$L-(R-\tilde{Z})$	c	$\sigma\overline{P}_N(\tilde{Z})$
(d)	$R+G-\tilde{Z} \leq L \leq R+G$	$R+G-L$	G	c	$\sigma\overline{P}_N(R+G-L)$
(e)	$R+G \leq L$	0	G	v	v

Here $\tilde{Z}$ is implicitly defined by

$$(4.5a) \qquad c = \sigma\overline{P}_N(G, \tilde{Z}).$$

Increases in G reduce the expected nighttime price and thus reduce the amount of energy it is profitable to store for nighttime sale when its cost is c:

$$(4.5b) \qquad \frac{\partial \tilde{Z}(G,c)}{\partial G} = -\frac{1}{\sigma} < 0.$$

In case (a), demand is less than maximum renewable output, so storage is fully charged, and the energy price is zero. In case (b), as in case (b) under Rule 1, demand at a price of zero, $(L+S)$, exceeds maximum renewable output. Marginal operating cost remains at zero, but the market price of energy is bid up by competitive storage suppliers to the expected value of incremental nighttime sales. This resembles the situation illustrated in Figure 2, except that $\sigma\overline{P}_N(0)$ exceeds c here. In case (c), illustrated by Figure 4, gas is on the margin, and storage is limited to $\tilde{Z} < S$ because additional storage would have negative expected profit. In case (d), which resembles case (c) under Rule 2, as illustrated by Figure 3, marginal operating cost is c, but price is driven above c by competition among storage suppliers. Finally, in case (e), native load exceeds renewable plus gas capacity, there is a shortage, and storage is not charged. Note that storage capacity is fully utilized only in case (a).

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Using upper bounds on energy prices in cases (b) and (d) and the reasoning that led to conditions (4.3) and (4.4), a sufficient condition for purchasing energy at night at a cost of c and storing it for resale the next day to be unprofitable is

(4.6)
$$c > v \frac{\sigma\left[1 - F(R + G - \tilde{Z})\right]}{1 - \sigma\left[F(R + G - \tilde{Z}) - F(R - S)\right]}.$$

Because this condition employs two upper bounds on prices, it is likely considerably stronger than necessary.

5. Optima and Equilibria Under Operating Rule 1

This section analyzes minimization of expected total cost conditional on competitive storage suppliers following Operating Rule 1. This is algebraically simpler than the parallel analyses conditional on Operating Rules 2 and 3, which are sketched in Appendices B and C, respectively. Equation numbering in those Appendices tracks the numbering in this section to facilitate comparisons. The results in Appendices B and C are compared with those obtained here at the end of this section.

It follows from the discussion of Operating Rule 1 in Section 4 that the expected amount of energy in storage at the end of a daytime (and the start of a nighttime) is

(5.1a)
$$E(Z) = F(R - S)S + \int_{R-S}^{R} (R - X) f(X) dX.$$

Differentiation shows that increases in R increase the expected value of Z:

(5.1b)
$$\partial E(Z) / \partial R = F(R) - F(R - S) \geq 0,$$

with strict inequality if F is strictly increasing. In Figure 5, which follows from the description above of the four daytime cases under Operating Rule 1, the shaded area shows the impact of a small increase in R on Z. There is no impact on charging for loads below $(R - S)$ or above R, but a correspondingly small increase in storage demand for intermediate loads. Similarly, increasing storage capacity by a small unit increases the expected amount stored by less than one unit:

(5.1c).
$$\partial E(Z) / \partial S = F(R - S) < 1.$$

In Figure 6, the shaded area shows the effect of a small increase in S. There is no impact on charging for loads above $(R–S)$, but there is a small increase in the amount stored at lower loads, when storage is charged fully.

It also follows from the discussion of Operating Rule 1 in Section 4 that expected *daytime* operating cost is given by

$$\Lambda(R,G) \equiv c\int_R^{R+G}[L-R]f(L)dL + cG[1-F(R+G)]$$

(5.2a)

$$+ v\int_{R+G}^{\infty}[L-(R+G)]f(L)dL.$$

This is identical to the corresponding sum in (2.1). Expected daytime operating cost is independent of S under this operating rule because storage demand never causes gas to turn on or induces a shortage. Increases in either R or G reduce expected daytime operating cost:

(5.2b)
$$\partial\Lambda/\partial R = -c[F(R+G)-F(R)] - v[1-F(R+G)] < 0,$$

(5.2c)
$$\partial\Lambda/\partial G = -(v-c)[1-F(R+G)] < 0.$$

To get expected *nighttime* operating cost, first evaluate expected cost conditional on Z and then take the expectation over Z, which is in fact an expectation over L. If σZ kwh are sold from storage at night, expected nighttime operating cost is given by

$$\omega(G,Z) \equiv c\int_{\sigma Z}^{\sigma Z+G}[X-\sigma Z]h(X)dX + cG[1-H(G+\sigma Z)]$$

(5.3a)

$$+ v\int_{G+\sigma Z}^{\infty}[X-(G+\sigma Z)]h(X)dX.$$

When nighttime demand, X, is between σZ and $\sigma Z+G$, it can be met by gas generation and sales from storage. Higher levels of demand lead to shortages, gas generation runs at capacity, and the system marginal cost rises to the value of lost load, v.

Differentiating (5.3a) yields the impacts of increasing G and Z on expected cost:

(5.3b) $\partial\omega/\partial G = -(v-c)[1-H(G+\sigma Z)] < 0,$

(5.3c) $\partial\omega/\partial Z = -\sigma\{cH(G+\sigma Z) + v[1-H(G+\sigma Z)]\} = -\sigma\overline{P}_N(Z) < 0.$

The right-hand side of (5.3b) is minus the expected nighttime net earnings of a unit of gas capacity, conditional on G and Z. The right-hand-side of (5.3c) is minus σ times the expected nighttime marginal cost, again conditional on G and Z. Since nighttime demand is perfectly inelastic, it is also minus σ times the (conditional) expected nighttime market price of energy, which is the net revenue from a unit of charge in storage at the start of a nighttime.

Taking the expectation of ω over *Z,* using the characterization of Operating Rule 1 in Section 4, yields the unconditional expected nighttime operating cost:

(5.4a)
$$\Omega(R,G,S) \equiv F(R-S)\omega(G,S) + \int_{R-S}^{R} \omega(G,R-L)f(L)dL$$
$$+ \left[1 - F(R)\right]\omega(G,0).$$

Increases in renewables capacity reduce expected nighttime operating cost by increasing expected storage:

(5.4b)
$$\partial\Omega/\partial R = \int_{R-S}^{R} \frac{\partial\omega(G,R-L)}{\partial R} f(L)dL = -\sigma \int_{R-S}^{R} E_X[P_N(G,R-L)]f(L)dL$$
$$= -\left[F(R) - F(R-S)\right]\sigma E_L\left\{\overline{P}_N(R-L)\big| R-S \le L \le R\right\} < 0.$$

An increase in renewable capacity increases the expected end-of-day value of *Z,* per equation (5.1b), thus lowering expected nighttime cost by increasing the expected nighttime supply from storage. The second equality in (5.4b) follows from (5.3c). The third equality is obtained by multiplying and dividing the integral by the probability that the daytime native load is in the interval for which increases in *R* increase expected storage, per equation (5.1b). That probability is multiplied by the expected nighttime price conditional on the daytime load being in that interval.

Increases in gas capacity reduce expected nighttime operating costs by reducing the probability of a nighttime shortage:

(5.4c)
$$\partial\Omega/\partial G = F(R-S)\frac{\partial\omega(G,S)}{\partial G} + \int_{R-S}^{R} \frac{\partial\omega(G,R-L)}{\partial G}f(L)dL + \left[1 - F(R)\right]\frac{\partial\omega(G,0)}{\partial G}$$
$$= -(v-c)\left\{ \begin{array}{c} F(R-S)[1 - H(G+\sigma S)] + \int_{R-S}^{R}[1 - H(G+\sigma(R-L))]f(L)dL \\ + [1 - F(R)][1 - H(G)] \end{array} \right\}$$
$$= -(v-c)\Pr\left(P_N = v\right).$$

The term in curly brackets is the unconditional expected probability of a nighttime shortage, the probability that *X* exceeds the supply from storage plus the supply from gas generation operating at capacity. To see this, note that such a shortage can arise in three possible ways. The first term on the right of (5.4c) reflects the fact that with probability *F(R–S)*, storage is fully charged, and the conditional probability of a nighttime shortage is then [1–*H(G+σS)*]. At the other extreme,

the third term reflects the fact that with probability $[1–F(R)]$, daytime load exceeds R so that gas must be used, the marginal operating cost is c, no charging occurs, and $Z=0$. Conditional on no supply from storage, the probability of a nighttime shortage is just $[1–H(G)]$. Intermediate values of L, which imply $0 \le Z=R–L \le S$ from the discussion of Operating Rule 1, have density $f(L)$ and conditional night-time shortage probability $\{1–H[G+\sigma(R–L)]\}$.

Finally, increases in storage capacity reduce expected nighttime operating cost by increasing expected nighttime sales from storage:

$$(5.4d) \qquad \partial\Omega/\partial S = F(R-S)\frac{\partial\omega(G,S)}{\partial S} = -F(R-S)\sigma\overline{P}_N(S).$$

From (5.1c), the expected increase in Z from a small unit increase in S is $F(R–S)$, and that increase occurs when the daytime load is such that generation is being fully charged. (See Figure 6.) Since storage is only charged when the marginal cost of electricity is zero, there is no associated daytime cost increase. The reduction in expected nighttime operating cost is just expected marginal cost (equal to $\overline{P}_N(S)$) times the increase in expected energy sales from storage, $\sigma F(R–S)$.

Using (5.2a) and (5.4a), expected total system cost is simply

$$(5.5a) \qquad E(TC) = rR + gG + sS + \Lambda(R,G,S) + \Omega(R,G,S).$$

The first-order conditions for minimizing this quantity are given by:

$$(5.5b) \qquad \begin{aligned} \partial E(TC)/\partial R &= r - \{c[F(R+G)-F(R)] + v[1-F(R+G)]\} \\ &\quad - \sigma E_L\{\overline{P}_N(R-L)|R-S \le L \le R\}[F(R)-F(R-S)] = 0, \end{aligned}$$

$$(5.5c) \qquad \partial E(TC)/\partial G = g - (v-c)\{[1-F(R+G)] + \Pr(P_N = v)\} = 0,$$

$$(5.5d) \qquad \partial E(TC)/\partial S = s - \sigma\overline{P}_N(S)F(R-S) = 0.$$

Renewable suppliers earn revenue whenever the daytime market price of energy is positive. With probability $[F(R+G)-F(R)]$, that price is c, and with probability $[1-F(R+G)]$, that price is v. The discussion of Operating Rule 1 in Section 4 establishes that when L is between $R–S$ and R, the market price is $\sigma\overline{P}_N(R-L)$. The third term in condition (5.5b) is just the probability that L is between $R–S$ and R, times the expected market price conditional on L being in that

range. Condition (5.5b) thus says that the capital cost of a unit of renewable capacity must equal the expected per-unit operating revenue. This is a zero-expected-profit condition that must hold in long-run competitive equilibria in the energy market.

Gas generators earn revenue in excess of marginal operating cost in both daytime and nighttime only when conditions of shortage prevail. The second term in (5.5c) is just per-unit net revenues conditional on shortage, times the (total) probability of shortage conditions. Condition (5.5c) thus requires that capital cost equal expected operating revenue, another zero-expected-profit condition that must hold in long-run competitive energy market equilibrium.

Condition (5.5d) is also a zero-expected-profit condition, even though the development just above might suggest otherwise. It equates the per-unit capital cost of storage with the incremental expected nighttime revenue from an incremental unit of storage capacity. However, we noted in the discussion of condition (5.5b) that the daytime market price of energy, paid by storage suppliers and received by renewable generators, is positive whenever native load is between $(R–S)$ and R. Thus with probability $[F(R)–F(R–S)]$ storage suppliers bear a cost that is not reflected in condition (5.5d). But the market price in this case is exactly equal to the corresponding expected nighttime revenue, so the daytime expected cost term that seems to be missing from (5.5d) is exactly cancelled by a missing nighttime expected revenue term. Condition (5.5d) is thus a zero-expected-profit condition with these two offsetting terms removed.

It does not seem possible to prove that the Hessian corresponding to (5.5) is positive definite for all parameter values and native load distributions. This suggests the possible existence of competitive equilibria of this simple model that do not correspond to minima of expected cost. If such equilibria do exist, the fact that the own second partials with respect to R, G and S are positive, consistent with minimization, at least suggests that they are uncommon and, more importantly, that conditional on the values of two of these variables, the zero-expected-profit equilibrium value of the third minimizes expected system cost:

(5.6a)
$$\partial^2 E(TC)\big/\partial R^2 = f(R)[c - \sigma \overline{P}_N(0)] + (v-c)f(R+G) + \sigma[f(R-S)\overline{P}_N(S)]$$
$$+ \sigma^2 \int_{R-S}^{R} \left\{ (v-c)h[G + \sigma(R-L)] + ch[\sigma(R-L)] \right\} f(L)dL > 0,$$

$$(5.6b) \quad \partial^2 E(TC)/\partial G^2 = (v-c)\left\{ \begin{array}{c} f(R+G)+F(R-S)h(G+\sigma S) \\ +\int_{R-S}^{R} h[G+\sigma(R-L)]f(L)dL+[1-F(R)]h(G) \end{array} \right\} > 0.$$

$$(5.6c) \quad \begin{aligned} \partial^2 E(TC)/\partial S^2 &= \sigma f(R-S)\overline{P}_N(S) \\ &+ \sigma^2 F(R-S)\left\{ ch(\sigma S)+(v-c)h(G+\sigma S) \right\} > 0. \end{aligned}$$

The first term on the right in (5.6a) is positive by the assumption on expected prices from which Operating Rule 1 was derived.

The results in Appendices B and C that assume Operating Rules 2 and 3 for competitive storage behavior, respectively, exactly parallel those above. In all three cases, the first-order necessary conditions for minimization of expected total cost imply that competitive suppliers of renewable generation, gas generation, and storage earn zero expected profit. Thus all minima of expected system cost can be supported as zero-expected-profit competitive equilibria, just as under the simple operating rule of Section 3. Moreover, all own second partial derivatives are positive, so that conditional on the values of any two of the three stock variables, R, G, and S, the long-run competitive equilibrium value of the third minimizes expected total cost.

However, under Operating Rules 2 and 3, as here and as under the simple operating rule of Section 3, it does not seem possible to show that the relevant Hessian is always positive definite. That suggests, though it certainly does not establish, that for some parameter values and native load distributions there may exist zero-expected-profit competitive equilibria that do not correspond to minima of expected total cost.

6. Concluding Remarks

Economists learn early and often that in the absence of market failures, competitive equilibria are efficient. Thus there is a strong presumption that FERC Order 841, which allows merchant storage suppliers to participate in bulk energy markets, would not upset the efficiency of equilibria in those markets. But, as I noted above, one cannot defend that presumption by pointing to general efficiency results that rely on complete markets, since the basic question raised by FERC Order 841 is whether energy markets *alone* are sufficient to induce efficient investment.

To explore the validity of this presumption more directly, this essay adds competitively-supplied storage and two types of periods to the classic Boiteux-Turvey model of an electric power system with multiple generation technologies and stochastic demand. This converts a simple static model into a dynamic stochastic model that is inherently complex. As a result of that complexity, the analysis here has been limited to a special case, albeit the most interesting special case, and it must be considered preliminary.

It is nonetheless reassuring that under rational expectations, long-run competitive equilibria in which all arbitrage flows in the expected direction satisfy the necessary conditions for minimization of expected total cost. Moreover, the second-order conditions for conditional optimality of each stock variable are also always satisfied. However, the corresponding second-order conditions for optimality of all stock variable at competitive equilibria cannot be shown always to be satisfied, raising the surprising possibility that inefficient competitive equilibria exist. More work is clearly required to understand this possibility.

It would be inappropriate to make policy recommendations on the basis of the preliminary analysis here of a highly abstract model, particularly since I have followed most of the academic literature and have assumed throughout that energy prices are not capped below the value of lost load, even though they are capped in most real markets. The possibility of inefficient equilibria observed here might be taken to imply that it would be unwise for power system operators and regulators to rely exclusively on energy markets to determine generation and storage capacities. But such reliance is waning in any case in the face of problems posed by intermittent renewable generation at scale.[17] In particular, while FERC Order 841 envisions merchant storage suppliers participating in bulk power markets, it seems likely that at least in the near term, storage deployment in the U.S. will be driven at least as much by state mandates and subsidies of various sorts as by incentives provided by competitive markets.[18] To the extent that energy markets drive investments in storage, the analysis here suggests that, conditional on generation capacities, the competitive equilibrium supply of storage is likely to be efficient.

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[17] See Joskow (2019) for a detailed discussion.
[18] See note 5, above.

Appendix A: Stochastic Renewables Without Storage

Following Llobet and Padilla (2018), assume that total renewable output is equal to $R\theta$, where $\theta \in [0,1]$ has smooth distribution function K and density function k,[19] and that θ is distributed independently of L and X. Expected total system cost is then

$$E(TC) = rR + gG +$$

$$(A.1) \qquad \int_0^1 \left\{ c \int_{R\theta}^{R\theta+G} (L - R\theta) f(L) dL + cG[1 - F(R\theta + G)] + v \int_{R\theta+G}^{\infty} [L - (R\theta + G)] f(L) dL \right\} k(\theta) d\theta$$

$$+ c \int_0^G X h(X) dX + cG[1 - H(G)] + v \int_G^{\infty} (X - G) h(X) dx.$$

This is identical to (2.1), except that renewable output, $R\theta$, is stochastic, and expected cost is the expectation over θ of expected cost conditional on θ.

The two first-order conditions for minimization of expected cost are

$$(A.2a) \qquad \partial E(TC)/\partial R = r - \int_0^1 \theta \left\{ c[F(R\theta + G) - F(R\theta)] + v[1 - F(R\theta + G)] \right\} k(\theta) d\theta = 0,$$

$$(A.2b) \qquad \partial E(TC)/\partial G = g - (v - c) \left\{ \int_0^1 [1 - F(R\theta + G)] k(\theta) d\theta + [1 - H(G)] \right\} = 0.$$

In (A.2a), the expression in curly brackets is the expected daytime price, conditional on θ. It is thus expected revenue per unit of renewable *output*. To get expected revenue per unit of *capacity*, it is necessary to multiply by θ, output per unit of capacity, and take the expectation over θ. Equation (A.2a) is thus a break-even condition that corresponds exactly to equation (2.2a). Equation (A.2b) is a similar break-even condition that corresponds exactly to equation (2.2b): the unconditional probability of a daytime shortage is the expectation of the conditional probability given θ. Thus the break-even conditions for a competitive equilibrium are again identical to the first-order conditions for minimization of expected social cost.

With the first row and column containing derivatives of (A.2a) and the second row and column containing derivatives of (A.2b), the Hessian for this minimization problem can be written as

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[19] A sufficient condition for the distribution of θ to be independent of R is that all renewable generators have identical outputs. I conjecture but have not proven that the basic results of this Appendix hold for less restrictive stochastic specifications.

(A.3)
$$\begin{bmatrix} T(\theta^2)+cE_\theta[\theta^2 f(R\theta)] & T(\theta) \\ T(\theta) & T(1)+(v-c)h(G) \end{bmatrix},$$
$$where\ T(x)\equiv(v-c)\int_0^1 xf(R\theta+G)k(\theta)d\theta.$$

Both diagonal terms are positive as in (2.3), and the structure of this matrix resembles that of (2.3). Multiplying out the determinant yields the sum of three positive terms, corresponding to those in (2.4), plus a fourth term:

(A.4)
$$T(\theta^2)[(v-c)h(G)]+E_\theta[cf(R\theta)]T(1)+E_\theta[cf(R\theta)][(v-c)h(G)]$$
$$+[T(\theta^2)T(1)-T(\theta)^2]$$

A sufficient condition for the determinant of (A.3) to be positive is that the fourth term be non-negative.[20] If F is uniform, this quantity is proportional to the variance of θ and is thus positive. While there may be distribution functions F and K for which this quantity is negative and large enough to offset the three positive terms in the determinant, it seems reasonable to conclude that as a general matter, the zero-expected-profit first-order conditions (A.2) signal a minimum of expected total cost.

Appendix B: Optima and Equilibria Under Operating Rule 2

It follows from the discussion of Operating Rule 2 in Section 4 that expected charge at the end of a daytime period is given by

(B.1a) $\qquad E(Z)=F(R+G-S)S+\int_{R+G-S}^{R+G}(R+G-L)f(L)dX.$

Increases in either R or G increase the expected value of Z:

(B.1b) $\qquad \partial E(Z)/\partial R=\partial E(Z)/\partial G=F(R+G)-F(R+G-S)\ge 0,$

(B.1c) $\qquad \partial E(Z)/\partial S=F(R+G-S)<1.$

These derivatives are illustrated by Figures 5 and 6 in the text, with $R+G$ replacing R in both figures.

Expected *daytime* operating cost follows from the description of Operating Rule 2:

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[20] The corresponding quantity in the determinant of (2.3) replaces the integrals with the values of the corresponding integrands when $\theta=1$ and is accordingly zero.

(B.2a)
$$\Lambda(R,G,S) \equiv c\int_{R-S}^{R+G-S}\left[L-(R-S)\right]f(L)dL + cG\left[1-F(R+G-S)\right]$$
$$+v\int_{R+G}^{\infty}\left[L-(R+G)\right]f(L)dL.$$

Daytime operating cost now depends on S because demand from storage may cause gas to turn on. The derivatives of the daytime cost function are the following:

(B.2b)
$$\partial\Lambda/\partial R = -c\left[F(R+G-S)-F(R-S)\right]-v\left[1-F(R+G-S)\right],$$

(B.2c)
$$\partial\Lambda/\partial G = -(v-c)\left[1-F(R+G)\right]+c\left[F(R+G)-F(R+G-S)\right],$$

(B.2d)
$$\partial\Lambda/\partial S = c\left[F(R+G-S)-F(R-S)\right].$$

If σZ kwh are sold from storage, expected nighttime operating cost is given by equation (5.3a) in the text:

(B.3)
$$\omega(G,Z) \equiv c\int_{\sigma Z}^{\sigma Z+G}\left[X-\sigma Z\right]h(X)dX + cG\left[1-H(G+\sigma Z)\right]$$
$$+v\int_{G+\sigma Z}^{\infty}\left[X-(G+\sigma Z)\right]h(X)dX.$$

Equations (5.3b) and (5.3c) are thus also valid under Operating Rule 2.

Taking the expectation over Z from the description of Operating Rule 2 in Section 4 yields unconditional expected nighttime operating cost:

(B.4a)
$$\Omega(R,G,S) \equiv F(R+G-S)\omega(G,S) + \int_{R+G-S}^{R+G}\omega(G,R+G-L)f(L)dL$$
$$+\left[1-F(R+G)\right]\omega(G,0).$$

The derivatives of this function parallel equations (5.4b) – (5.4d) in the text:

(B.4b)
$$\partial\Omega/\partial R = \int_{R+G-S}^{R+G}\frac{\partial\omega(G,R+G-L)}{\partial Z}f(L)dL = -\sigma\int_{R+G-S}^{R+G}E_X[P_N(G,R+G-L)]f(L)dL$$
$$= -\left[F(R+G)-F(R+G-S)\right]\sigma E_L\left\{\bar{P}_N(R+G-L)\big|R+G-S\leq L\leq R+G\right\},$$

(B.4c)

$$\partial\Omega/\partial G = F(R+G-S)\frac{\partial\omega(G,S)}{\partial G} + \int_{R+G-S}^{R+G}\frac{\partial\omega(G,R+G-L)}{\partial G}f(L)dL$$

$$+[1-F(R+G)]\frac{\partial\omega(G,0)}{\partial G} + \int_{R+G-S}^{R+G}\frac{\partial\omega(G,R+G-L)}{\partial Z}f(L)dL$$

$$= -(v-c)\left\{\begin{array}{l}F(R+G-S)[1-H(G+\sigma S)] + \int_{R+G-S}^{R+G}[1-H(G+\sigma(R+G-L))]f(L)dL \\ +[1-F(R+G)][1-H(G)]\end{array}\right\}$$

$$-[F(R+G)-F(R+G-S)]\,\sigma E_L[\overline{P}_N(G,R+G-L)|R+G-S\le L\le R+G]$$

$$= -(v-c)\Pr(P_N=v)$$

$$-[F(R+G)-F(R+G-S)]\,\sigma E_L[\overline{P}_N(G,R+G-L)|R+G-S\le L\le R+G],$$

(B.4d) $\partial\Omega/\partial S = F(R+G-S)\dfrac{\partial\omega(G,S)}{\partial S} = -F(R+G-S)\sigma\overline{P}_N(S).$

Aside from the general substitution of (R+G) for R, the main difference between equations (B.4) and equations (5.4) in the text is the $\overline{P}_N$ term in (B.4c) that does not appear in (5.4c). That term stems from case (c), in which competitive suppliers bid the price of energy above c, thus enabling gas generators to more than cover their marginal cost. (See Figure 3.)

Expected total cost is given by equation (5.5a) in the text, modified to reflect the fact that S affects daytime operating costs under Operating Rule 2:

(B.5a) $\qquad E(TC) = rR + gG + sS + \Lambda(R,G,S) + \Omega(G,S).$

The first-order conditions for minimizing this quantity are given by

(B.5b)
$$\partial E(TC)/\partial R = r - c[F(R+G-S)-F(R-S)] - v[1-F(R+G)]$$
$$-[F(R+G)-F(R+G-S)]\sigma E_L\left[P_N(G,R+G-L)|R+G-S\le L\le R+G\right]=0,$$

(B.5c)
$$\partial E(TC)/\partial G = g - (v-c)\left\{[1-F(R+G)]+\Pr(P_N=v)\right\}$$
$$-[F(R+G)-F(R+G-S)]\left\{\left[\sigma E_L\{\overline{P}_N(R+G-L)|R+G-S\le L\le R+G\}\right]-c\right\}=0,$$

(B.5d) $\partial E(TC)/\partial S = s + c\left[F(R+G-S)-F(R-S)\right] - F(R+G-S)\sigma\overline{P}_N(S)]=0$

These are, again, zero-expected-profit conditions. Condition (B.5b) compares capital cost to revenue earned by renewables when the market price is c, when it is v, and in case (c) when competition among storage suppliers raises the market price above marginal generation cost.

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Condition (B.5c) reflects the fact that in case (c) competition among storage suppliers raises the market price above c, so that gas suppliers earn positive operating profits. Condition (B.5d) says that capital cost plus expected daytime charging cost in case (b) must equal expected nighttime revenue. Charging cost in case (a) is zero. Expected charging cost in case (c), which does not appear in (B.5.d), is exactly offset by expected nighttime revenues, which also do not appear.

Thus all minima of expected cost are zero-expected-profit competitive equilibria. As in Section 5, it seems impossible to show that the complicated Hessian associated with (B.5) is positive definite without additional restrictions on the parameters and native load distribution functions. This suggests, as in Section 5, that there could exist competitive equilibria that do not correspond to minima of expected total cost. The second-order own-partial derivatives of (B.5b)-(B.5d) are the following:

(B.6a)
$$\partial^2 E(TC)/\partial R^2 = f(R+G-S)[\sigma \overline{P}_N(S)-c] + f(R+G)[v-\sigma \overline{P}_N(0)]$$
$$+ cf(R-S) + \sigma^2(v-c)\int_{R+G-S}^{R+G} h[G+\sigma(R+G-L)]f(L)dL > 0,$$

(B.6b) $$\partial^2 E(TC)/\partial G^2 = (v-c)\left\{ \begin{array}{l} F(R+G-S)h(G+\sigma S)+[1-F(R+G)]h(G) \\ +(1+\sigma)^2\int_{R+G-S}^{R+G} h[G+\sigma(R+G-L)]f(L)dL \end{array} \right\} > 0.$$

(B.6c)
$$\partial^2 E(TC)/\partial S^2 = f(R+G-S)[\sigma \overline{P}_N(S)-c] + cf(R-S)$$
$$+ F(R+G-S)\sigma[(v-c)h(G+\sigma S)] > 0.$$

The first and second terms on the right of (B.6a) and the first term on the right of (B.6c) are positive by the assumption about $\sigma \overline{P}_N$ from which Operating Rule 2 was derived.

C. Optima and Equilibria under Operating Rule 3

Operating Rule 3 implies the following equation for expected end-of-daytime charge:

(C.1a)
$$E(Z) = F(R-S)S + \int_{R-S}^{R-\tilde{Z}} (R-L)f(L)dL + \tilde{Z}[F(R+G-\tilde{Z}) - F(R-\tilde{Z})]$$
$$+ \int_{R+G-\tilde{Z}}^{R+G} (R+G-L)f(L)dL.$$

Differentiation yields

(C.1b) $\partial E(Z)/\partial R = [F(R-\tilde{Z})-F(R-S)]+[F(R+G)-F(R+G-\tilde{Z})] \geq 0,$

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(C.1c) $\partial E(Z)/\partial G = \dfrac{\partial \tilde{Z}}{\partial G}[F(R+G-\tilde{Z}) - F(R-\tilde{Z})] + [F(R+G) - F(R+G-\tilde{Z})],$

(C.1d) $\partial E(Z)/\partial S = F(R-S) > 0.$

Increasing R shifts the $Z(L)$ curve to the right, as Figure C.1 illustrates, and Z is increased when L is in two distinct intervals. Increasing G decreases $\tilde{Z}$, shifting the curve down for L between $(R-\tilde{Z})$ and $(R+G-\tilde{Z})$, as shown by the cross-hatched area in Figure C.2, and shifting it to the right beyond $(R+G-\tilde{Z})$, as the shaded area in that figure illustrates. Thus, in contrast to Operating Rule 1 (in which $\partial E(Z)/\partial G = 0$) and Operating Rule 2 (in which $\partial E(Z)/\partial G > 0$), here the impact of increases in G on Z cannot be signed in general. Increasing S just shifts the curve up to the left of $(R-S)$, as in Figure 6.

Expected daytime operating cost under Operating Rule 3 is given by

(C.2a)
$$\Lambda(R,G) = c\int_{R-\tilde{Z}}^{R+G-\tilde{Z}}[L-(R-\tilde{Z})]f(L)dL + cG[1 - F(R+G-\tilde{Z})]$$
$$+ v\int_{R+G}^{\infty}[L-(R+G)]f(L)dL.$$

Note that S does not affect daytime operating cost under this operating rule, since changes in S only affect charging in case (a), when marginal operating cost is zero. Differentiation yields

(C.2b) $\qquad \partial \Lambda/\partial R = -c[F(R+G-\tilde{Z}) - F(R-\tilde{Z})] - v[1 - F(R+G)],$

(C.2c) $\partial \Lambda/\partial G = c[1 - F(R+G-\tilde{Z})] + c\dfrac{\partial \tilde{Z}}{\partial G}[F(R+G-\tilde{Z}) - F(R-\tilde{Z})] - v[1 - F(R+G)].$

If σZ kwh are sold from storage, conditional expected nighttime operating cost, $\omega(G,Z)$, is given by equation (5.3a) in the text, as under Operating Rules 1 and 2:

(C.3)
$$\omega(G,Z) \equiv c\int_{\sigma Z}^{\sigma Z + G}[X - \sigma Z]h(X)dX + cG\left[1 - H(G+\sigma Z)\right]$$
$$+ v\int_{G+\sigma Z}^{\infty}\left[X - (G+\sigma Z)\right]h(X)dX.$$

Equations (5.3b) and (5.3c) are accordingly also valid under this operating rule.

Taking the expectation of ω over Z, using the characterization of Operating Rule 3 in Section 4, yields the unconditional expectation of nighttime operating costs:

$$\Omega(R,G,S) = F(R-S)\omega(G,S) + \int_{R-S}^{R-\tilde{Z}} \omega(G,R-L)f(L)dL$$

(C.4a)
$$+ [F(R+G-\tilde{Z}) - F(R-\tilde{Z})]\omega(G,\tilde{Z})$$

$$+ \int_{R+G-\tilde{Z}}^{R+G} \omega(G,R+G-L)f(L)dL + [1 - F(R+G)]\omega(G,0).$$

Differentiation yields

$$\partial\Omega/\partial R = \int_{R-S}^{R-\tilde{Z}} \frac{\partial\omega(G,R-L)}{\partial Z}f(L)dL + \int_{R+G-\tilde{Z}}^{R+G} \frac{\partial\omega(G,R+G-L)}{\partial Z}f(L)dL$$

(C.4b)
$$= -\left[F(R-\tilde{Z}) - F(R-S)\right]\sigma E_L\left\{\overline{P}_N(R-L)\big|R-S \leq L \leq R-\tilde{Z}\right\}$$

$$-\left[F(R+G) - F(R+G-\tilde{Z})\right]\sigma E_L\left\{\overline{P}_N(R+G-L)\big|R+G-\tilde{Z} \leq L \leq R+G\right\},$$

$$\partial\Omega/\partial G = -(v-c)\left\{\begin{array}{c} F(R-S)[1-H(G+\sigma S)] + \int_{R-S}^{R-\tilde{Z}}[1-H(G+\sigma(R-L))]f(L)dL \\ + [F(R+G-\tilde{Z}) - F(R-\tilde{Z})][1-H(G+\sigma\tilde{Z})] \\ + \int_{R+G-\tilde{Z}}^{R+G}[1-H(G+\sigma(R+G-L))]f(L)dL \\ + [1-F(R+G)][1-H(G)] \end{array}\right\}$$

(C.4c)
$$- c\frac{\partial\tilde{Z}}{\partial G}[F(R+G-\tilde{Z}) - F(R-\tilde{Z})] - \sigma\int_{R+G-\tilde{Z}}^{R+G}\overline{P}_N(R+G-L)f(L)dL$$

$$= -(v-c)\Pr(P_N = v) - c\frac{\partial\tilde{Z}}{\partial G}[F(R+G-\tilde{Z}) - F(R-\tilde{Z})]$$

$$-[F(R+G) - F(F+G-\tilde{Z})]\sigma E_L[\overline{P}_N(G,R+G-L)\big|R+G-\tilde{Z} \leq L \leq R+G],$$

(C.4d) $\partial\Omega/\partial S = -F(R-S)\sigma\overline{P}_N(G,S).$

Expected total cost is again given by a slight modification of equation (5.5a) in the text:

(C.5a) $\qquad E(TC) = rR + gG + sS + \Lambda(R,G) + \Omega(R,G,S).$

Differentiation yields the first-order necessary conditions for a minimum of expected total cost

$$\partial E(TC)/\partial R = r - \{c[F(R+G-\tilde{Z}) - F(R-\tilde{Z})] - v[1-F(R+G)]\}$$

(C.5b)
$$-\left[F(R-\tilde{Z}) - F(R-S)\right]\sigma E_L\left\{\overline{P}_N(R-L)\big|R-S \leq L \leq R-\tilde{Z}\right\}$$

$$-\left[F(R+G) - F(R+G-\tilde{Z})\right]\sigma E_L\left\{\overline{P}_N(R+G-L)\big|R+G-\tilde{Z} \leq L \leq R+G\right\} = 0,$$

$$(\text{C.5c}) \quad \partial E(TC)/\partial G = g - (v-c)\{\Pr(P_N = v) + [1 - F(R+G)]\}$$
$$- [F(R+G) - F(R+G-\tilde{Z})]E_L\{[\sigma\bar{P}_N(G, R+G-L) - c]\big| R+G-\tilde{Z} \le L \le R+G\} = 0,$$

$$(\text{C.5d}) \quad \partial E(TC)/\partial S = s - F(R-S)\sigma\bar{P}_N(G, S) = 0.$$

These conditions once again imply zero expected profits for each technology. Condition (C.5b) compares unit capital cost for renewable generation with the sum of expected per-unit revenues in cases (b)-(e). Similarly, condition (C.5c) compares unit capital cost for gas generation with the sum of expected net revenues above variable cost in cases (d) and (e) and in nighttime shortage conditions. (Comparing (C.2c) and (C.4c) reveals that the change in $\tilde{Z}$ induced by a marginal increase in G has equal and opposite effects on expected gas generation costs in daytime and nighttime periods.) Finally, (C.5d) compares unit capital cost with the marginal expected revenue from the increased charging in case (a) that a unit increase in storage capacity would induce. As in the other operating rules, payments by storage suppliers above marginal generation costs, in cases (b) and (d) here, show up as revenues for renewable and gas generators but not as costs for storage suppliers, since those payments exactly equal the expected nighttime revenue from sales of the incremental stored energy.

As under Operating Rules 1 and 2, it does not seem possible to show that the Hessian corresponding to (C.5a) is positive definite without further restrictions. The diagonal elements of that matrix are the following:

$$(\text{C.6a}) \quad \begin{aligned} \partial^2 E(TC)/\partial R^2 &= \sigma\tilde{P}_N(S)f(R-S) + [v - \sigma\bar{P}_N(0)]f(R+G)] \\ &+ \sigma^2(v-c)\int_{R-S}^{R-\tilde{Z}} h[G+\sigma(R-L)]f(L)dL \\ &+ \sigma^2(v-c)\int_{R+G-\tilde{Z}}^{R+G} h[G+\sigma(R+G-L)f(L)dL > 0, \end{aligned}$$

$$(\text{C.6b}) \quad \partial^2 E(TC)/\partial G^2 = (v-c)\left\{ \begin{array}{l} F(R-S)h(G+\sigma S) + \int_{R-S}^{R-\tilde{Z}} h[G+\sigma(R-L)]f(L)dL \\ + h(G+\sigma\tilde{Z})[F(R+G-\tilde{Z}) - F(R-\tilde{Z})] \\ + (1+\sigma)^2\int_{R+G-\tilde{Z}}^{R+G} h[G+\sigma(R+G-L)]f(L)dL \\ + h(G)[1 - F(R+G)] \end{array} \right\}$$
$$+ [v - \sigma\bar{P}_N(0)]f(R+G) > 0,$$

$$(\text{C.6c}) \quad \partial^2 E(TC)/\partial S^2 = f(R-S)\sigma\bar{P}_N(S) + F(R-S)(v-c)\sigma^2 h(G+\sigma S) > 0.$$

Equations (4.5) were used in the derivation of (C.6a) and (C.6b). Equation (C.6c) demonstrates that as under Operating Rules 1 and 2, in long-run competitive equilibrium, the value of *G* minimizes expected total cost conditional on the values of *R* and *G.*

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Xu, Bolun, Magnus Korpås, Audun Botterud, and Francis O'Sullivan (2019). "A Lagrangian Policy for Optimal Energy Storage Control," *arXiv*:1901.09507v2.[math.OC], 9 February.
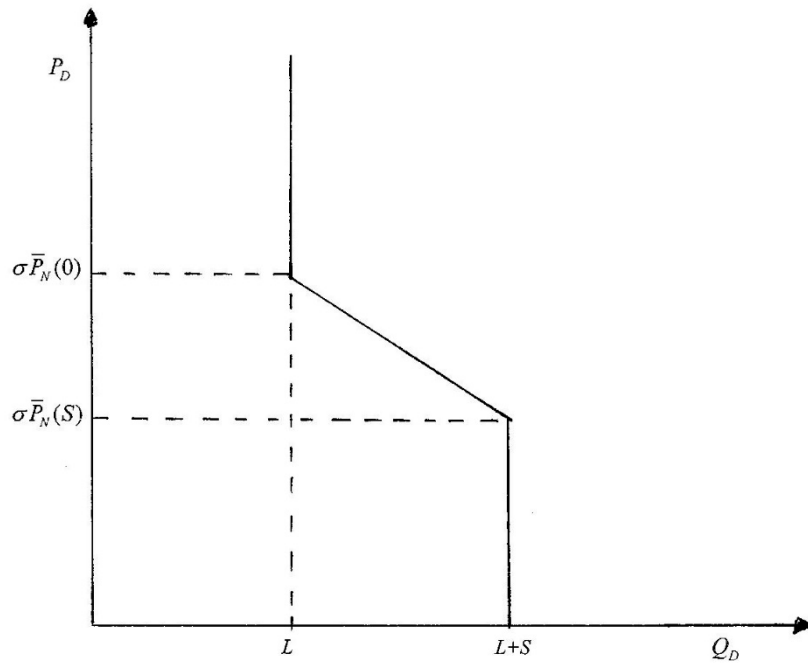
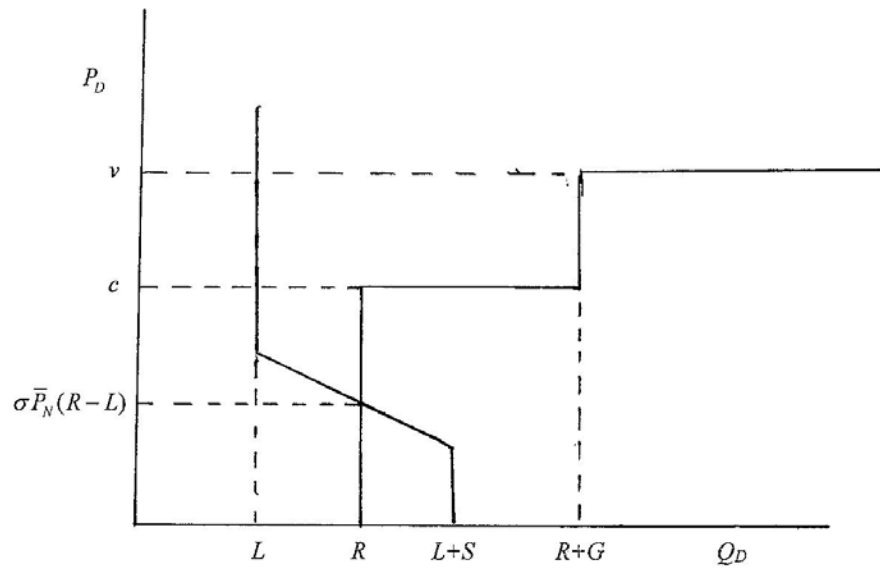Figure 1. The Daytime Competitive Demand for Energy
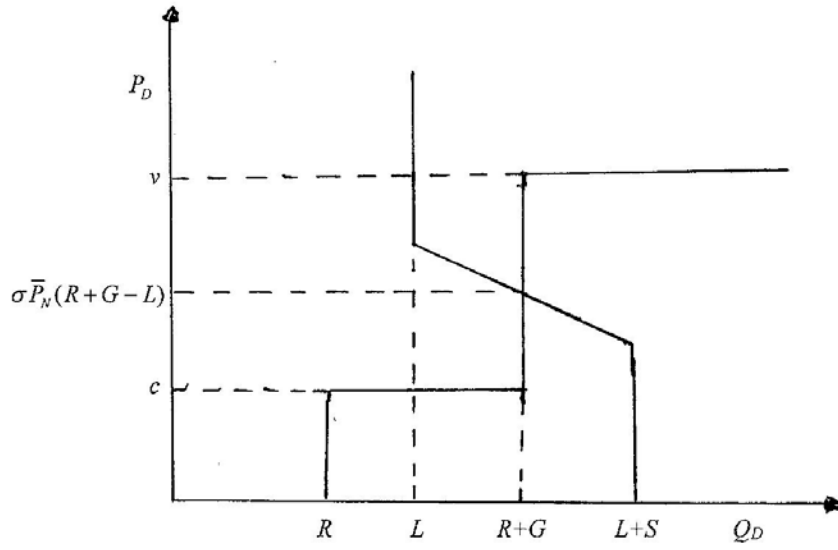

Figure 2. Operating Rule 1, Case (b)

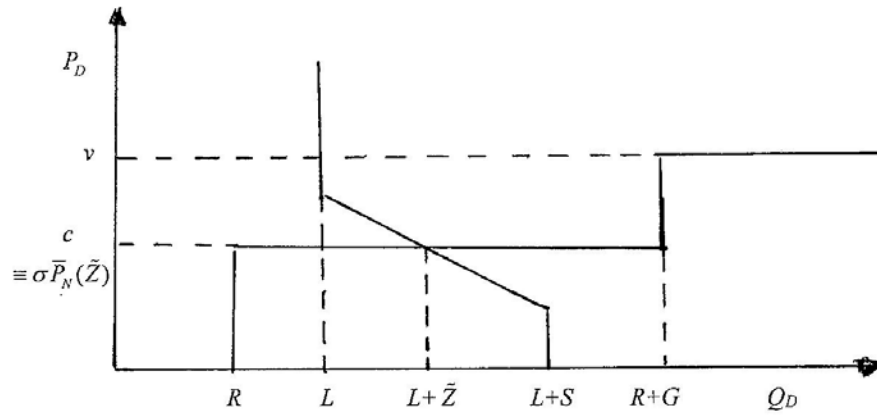Figure 3. Operating Rule 2, Case (c)
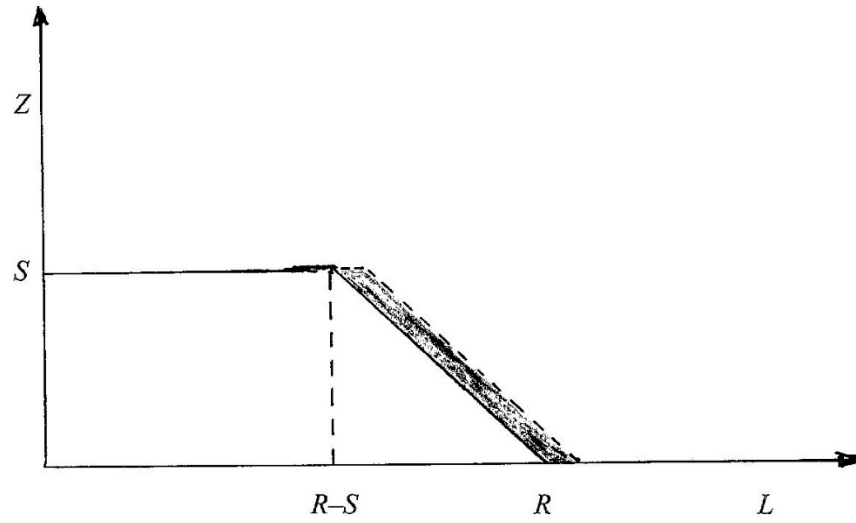

Figure 4. Operating Rule 3, Case (c)

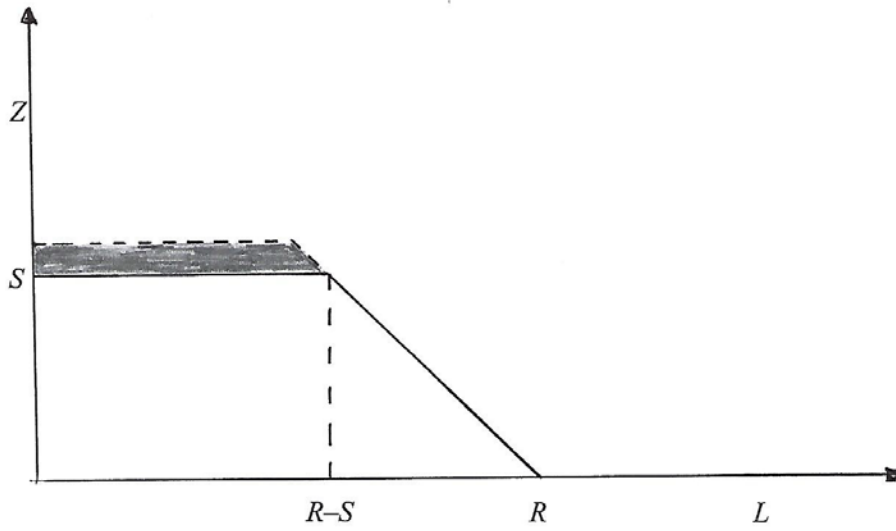Figure 5. A Small Increase in R under Operating Rule 1

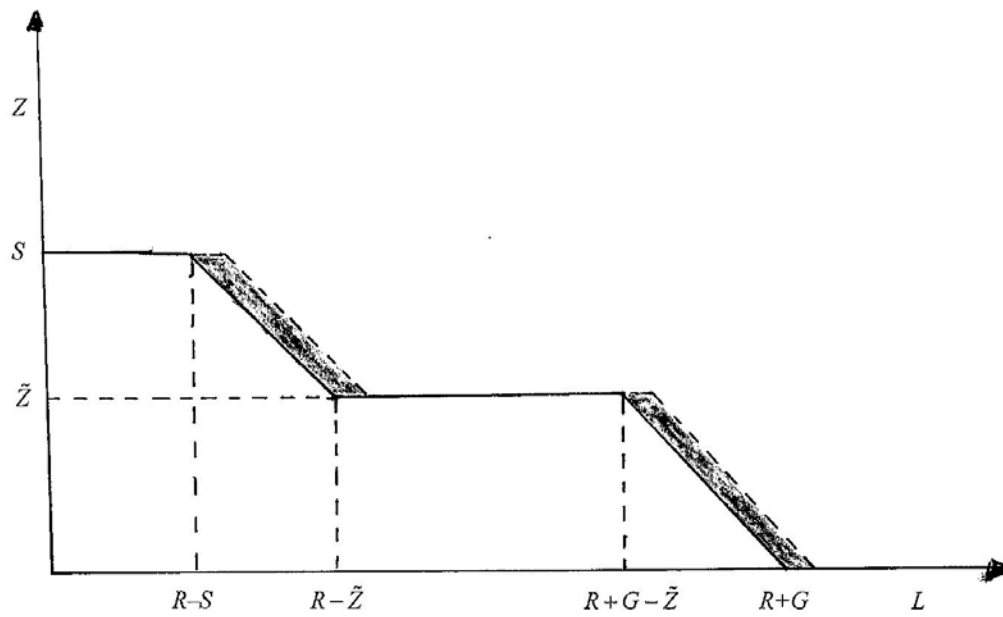Figure 6. A Small Increase in S under Operating Rule 1

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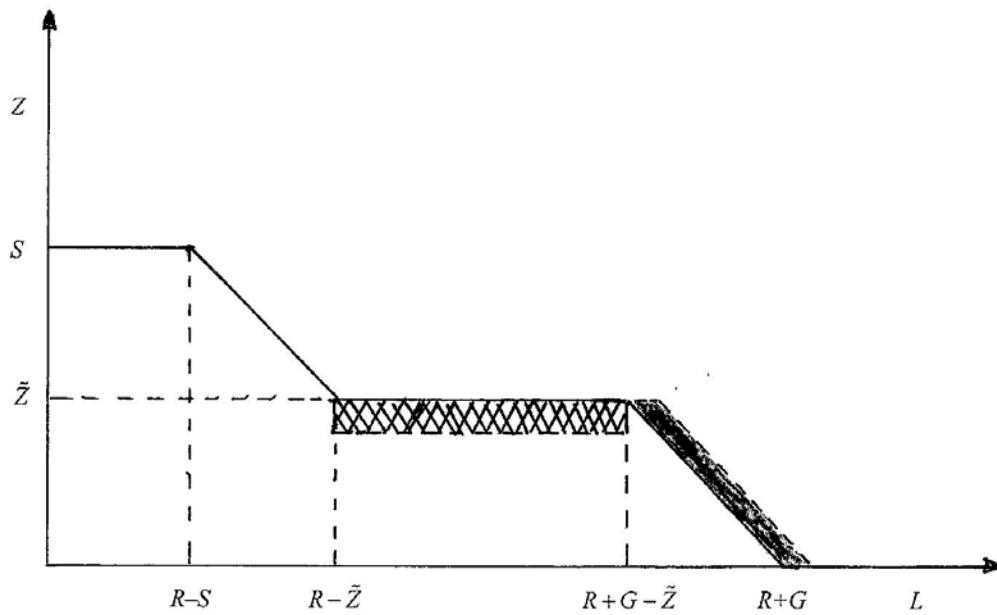Figure C.1. A Small Increase in R under Operating Rule 3

Figure C.2. A Small Increase in G under Operating Rule 3

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