MIT CEEPR

MIT Center for Energy and Environmental Policy Research

Working Paper Series

Optimal Commodity Taxation with a Non-renewable Resource

JULIEN X. DAUBANES AND PIERRE LASSERRE

MAY 2019

CEEPR WP 2019-008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Optimal Commodity Taxation with a Nonrenewable Resource*

by

Julien Daubanes
University of Geneva (GSEM), MIT (CEEPR), and CESifo

and

Pierre Lasserre
Université du Québec à Montréal (ESG), CIRANO and CIREQ

April 26, 2019

* Earlier versions of this paper have been presented at various seminars and conferences: Paris School of Economics; Montreal Workshop in Environmental and Resource Economics; Toulouse Business School; SURED; Bonn Max Planck Institute; Journées Louis-André Gérard-Varet; WCERE; Helsinki School of Economics; University of Basel; AERE; Tilburg University; University of Montpellier; University of Oxford; Université du Québec à Montréal; CPEG; CESifo Munich; Tinbergen Institute Conference; Montreal Workshop in Environmental and Resource Economics in Memory of Pierre Lasserre. Particular thanks to Saraly Andrade de Sá, Ujjayant Chakravorty, Helmuth Crémer, Philippe De Donder, Gérard Gaudet, Reyer Gerlagh, André Grimaud, Ben Groom, John Hartwick, Martin Hellwig, Beat Hintermann, Matti Liski, Ngo van Long, François Maniquet, Rick van der Ploeg, Larry Samuelson, and Eytan Sheshinski. Financial support from the Social Science and Humanities Research Council of Canada, the Fonds Québécois de recherche pour les sciences et la culture, the CESifo, the CIREQ, and the Agence Nationale de la Recherche (ANR-09-BLAN-0350-01) is gratefully acknowledged.
Pierre left us in the spring of 2017. He is missed so much.
Please address all correspondence to Julien Daubanes. E-mail address: *julien.daubanes@unige.ch*.
Phone: +41 22 379 0938.

Abstract

We obtain a formula for how nonrenewable resources should be taxed when governments need to collect commodity tax revenues: This tax rule is an augmented, dynamic version of the standard Ramsey inverse elasticity rule, which requires a novel interpretation of the optimal commodity tax distortions. We show the following results. First, flows of nonrenewable resources should be taxed at higher rates than otherwise identical conventional commodities. Second, our rule is compatible with the variety of observed resource tax systems, ranging from systems in which firms finance reserve production and are paid back by future after-tax extraction rents to the extreme case of nationalized industries. Third, optimal nonrenewable resource taxation distorts developed reserves, which are reduced, and their depletion, which is slowed down. These distortions go in the same direction as those prescribed for resolution of the climate externality. Our formula can be directly used to indicate how carbon taxation should be increased in the presence of public-revenue needs, as illustrated in a numerical example.

JEL classification: Q31; Q38; H21
Keywords: Optimal commodity taxation; Inverse elasticity rule; Nonrenewable resources; Supply elasticity; Carbon taxation

1 Introduction

The theory of optimal commodity taxation (OCT) addresses the following question: How should a government concerned with total welfare distribute the burden of commodity taxation across sectors in such a way as to collect a given amount of tax income? The literature originated with Ramsey's famous "A Contribution to the Theory of Taxation" (1927). It was further developed by Pigou (1947), Baumol and Bradford (1970), Diamond and Mirrlees (1971), and others. The influence, relevance, and modernity of Ramsey's approach were recently celebrated in Stiglitz's "In Praise of Frank Ramsey's Contribution to the Theory of Taxation" (2015). The most famous OCT result is the static inverse elasticity rule, which says that under simplifying conditions, the tax rate applied on each good should be proportional to the reciprocal of the price elasticity of its demand.[1]

In this paper we reexamine the problem of OCT in the presence of a natural nonrenewable resource. We obtain a formula for how nonrenewable resources should be taxed when governments need to collect commodity tax revenues. This new Ramsey tax formula is an augmented, dynamic version of the standard rule, and requires a novel analysis of Ramsey tax distortions to nonrenewable resource extraction and reserve development. First, our tax formula accounts for the variety of existing nonrenewable resource tax systems. Second, it can be directly used to indicate how carbon taxation should be augmented to take into account governments' revenue needs.

Surprisingly, the OCT problem was never extended to economies with natural resources despite the appearance of Hotelling's "The Economics of Exhaustible Resources" (1931) shortly after Ramsey's 1927 paper. Our extension of OCT to nonrenewable resources is necessary not only because it has been overlooked theoretically, but also for its relevance in the light of the current structure of resource tax systems.

On the one hand, Ramsey's original analysis does not seem sufficient to explain the special tax treatment received by the flow of most energy nonrenewable resource commodities.[2] For example, high levels of taxes on the use of energy resources are often rationalized

[1]In its textbook formulation, supply is typically neglected, either because it is considered infinitely elastic in a long-run perspective or because profits are assumed to be entirely taxed away at the outset. However, Ramsey's original paper does consider a static non-infinitely elastic supply, and presents a taxation rule that includes the supply price elasticity.

[2]The flow of hydrocarbon energy resources is taxed at both the production and consumption stages. At the production stage, resource-specific royalties are very common—see the descriptions provided by Nakhle

by the fact that energy demand is relatively price inelastic—for oil, see Berndt and Wood (1975), Pindyck (1979), and Hamilton (2009a). However, it is the peculiarity of their supply that makes nonrenewable resources special: The supply of a nonrenewable resource consists in extracting production from limited reserves over time. This peculiarity of nonrenewable resources has several important theoretical implications for the OCT problem. First, reserve limitations generate economic rents that Ramsey commodity taxes are able to bite (Stiglitz, 2015). Second, the non-renewability of a natural resource makes the OCT distortion intertemporal. Third, the OCT problem puts the government in a situation comparable to that of a profit-maximizing monopoly (Boiteux, 1956; Baumol and Bradford, 1970); in this context, Stiglitz (1976) showed that, depending on demand elasticity, it may be impossible to set a mark-up—such as a Ramsey tax—to generate additional revenues from a nonrenewable resource.[3]

On the other hand, apart from the peculiarity of resource supply, Ramsey's original framework fits particularly well with the characteristics of actual nonrenewable resource tax systems. Despite economists' recommendations—see, for example, Boadway and Keen (2010)[4]—the use of direct rent taxation proved limited in nonrenewable resource sectors, leaving large rents untaxed.[5] In this context, Ramsey commodity taxes are particularly

(2010) for oil and gas, and by Hogan (2008) for coal. At the consumption stage, excise taxes are specifically imposed on hydrocarbon products—see, for example, Newbery (2005) on taxes applied to final energy consumption on top of ordinary value-added taxes, and on taxes applied to the industrial intermediate consumption of energy inputs. See also the instructive review by the Energy Charter Secretariat (2008) of energy taxes at both stages.

[3] As Pindyck (1987) put it, "Potential monopoly power in extractive resource markets is reduced by the depletability of reserves."

[4] For more examples, see the references in Daubanes and Andrade de Sá (2014).

[5] Recent World Bank data suggest that, for instance, economic profits—including rents—from oil extraction worldwide exceeded US$ 609 billion in 2015. Data are available at `http://data.worldbank.org/indicator/NY.GDP.PETR.RT.ZS?end=2015&start=1970&view=chart` and at `http://data.worldbank.org/indicator/NY.GDP.MKTP.CD?end=2015&start=1960`. Most developed countries have partially adopted tax schemes that target nonrenewable resource extraction profit, although at relatively low tax rates, leading to an incomplete governments' take relative to the pre-tax net value of exploitation. For example, the overall UK government's take on new oil fields is about 50 % (Nackle, 2010). These schemes are mostly based on royalties that are reduced according to the cost of development investment in the early exploitation phase. However, the movement toward direct taxation is limited, because indirect taxes, such as production-based royalties, retain their attraction for all governments hosting extraction activities—see, for example, Hogan's (2008) description and analysis of tax arrangements, including the taxation of coal, for a selection of countries. The prevalence of indirect taxation may be explained by various reasons. According to Simmons (1977), quantity-based royalties can be used to address extraction externalities and market power, but seem to be mainly motivated by governments' revenue needs. From this perspective, one practical advantage of royalties over profit-based taxes is that the former provide an immediate and stable tax income. Other reasons for the prevalence of indirect taxes have to do with the administrative complexity of rent taxes and information asymmetry (Boadway and Keen, 2015).

useful, as they allow governments to indirectly tap such untaxed rents (Stiglitz, 2015); for instance, royalties and other indirect linear commodity taxes are dominant forms of resource taxation (Daniel, Keen, and McPherson, 2010).[6] Accordingly, the nonrenewable resource taxation literature has largely studied the distortions caused by, or the neutrality of, commodity taxes, although it has hitherto ignored governments' revenue needs—see, for influential examples, Dasgupta, Heal, and Stiglitz (1981), Long and Sinn (1985), Sinn (2008), and van der Ploeg and Withagen (2012), as well as Gaudet and Lasserre's (2015) recent synthesis. Moreover, most governments that impose special commodity taxes on nonrenewable resources are also struggling to meet their debt and budget constraints, especially in the aftermath of the financial crisis of 2008-2009.

Our analysis lies at the intersection of the above OCT and nonrenewable resource literatures, and directly contributes to both by examining how the taxation burden should be optimally spread over resource sectors when governments must raise commodity tax revenues. Our results are also complementary to the literature on carbon taxation, which has mostly disregarded public revenue needs; see the literature following Nordhaus (2008), including, among others, Chakravorty, Moreaux, and Tidball (2008), and Golosov, Hassler, Krusell, and Tsyvinski (2014). Some, however, have examined the ability of a carbon tax to transfer resource rents; see, among others, Liski and Tahvonen (2004), Dullieux, Ragot, and Schubert (2011), and Fischer and Salant (2017). One exception is Barrage (2017), who comes close to our analysis by introducing a nonrenewable resource sector in her study of OCT with carbon pollution, but she leaves Ramsey dynamic distortions to this sector unexplored. In addition, some of our results are reminiscent of the double dividend literature that addresses OCT in the presence of non-distortionary (corrective) taxes, although this literature has ignored natural resources; see, among others, Sandmo (1975), Bovenberg and de Mooij (1994), Fullerton (1997), and Cremer and Gahvari (2004).

Except for the production of resource reserves and their dynamic exploitation, our model adheres to the Ramsey OCT framework for theoretical and empirical reasons justified above. Lump-sum tax collection is impossible. Direct taxation is not a controllable option, whether it aims at income or at pure profits, such as resource rents. Indirect linear

[6]For example, even tax schemes that target profit in the most advanced tax systems are based on production-based royalties with linear tax-rate reductions that reflect the apparent development costs—see, for example, the Alberta Royalty Review (2007).

taxes or subsidies can be applied on the final consumption or on the production of any commodity or service; these taxes (or subsidies) may take the form of *ad valorem* taxes or of *unit* taxes, proportional to quantities. The government is not concerned with individual differences; we assume a representative consumer. Neither is the optimal supply of public goods addressed; we assume that the government faces exogenous financial needs in order to fulfill its role as a supplier of public goods, so the government's problem is to raise that amount of revenues in the least costly way, given the available tax instruments.

Moreover, we proceed in two steps. To start with, we adopt the assumptions of the traditional OCT literature and of the literature on Hotelling nonrenewable resources. In non-resource sectors, supply is infinitely elastic. In the resource sector, long-run supply is perfectly inelastic because reserves are fixed. In the second step, we present a more realistic model in which reserves are endogenously produced by exploration and development efforts.

Our main results are as follows. First, when reserves are fixed, the resource should be taxed at a higher rate than conventional commodities having the same demand elasticity. In our analysis, the Ramsey tax on the nonrenewable resource has a dynamic dimension: The taxation burden is spread not only across resource and non-resource sectors, but also over time. When public revenue needs are sufficiently high to warrant a Ramsey distortion to the resource sector, the optimal distortion takes the form of slower extraction, so that the path of reserves over time does not diminish as fast as it would if the tax were neutral or if there were no public revenue needs.

Second, we do away with the critical Hotelling assumption that reserves are fixed: Their production is determined by the net-of-tax rents derived during the extraction phase, and may be increased by subsidies toward the production of reserves, if any. In that more realistic case, we obtain the Ramsey tax on the nonrenewable resource, which is the main result of the paper. While the Ramsey resource tax varies according to the reserve subsidy, we show that the optimal amount of initial reserves and the optimal extraction path of these reserves do not depend on the extraction-tax reserve-subsidy combination. This implies that our results apply to a continuum of equivalent mixed tax systems, irrespective of the ability of governments to commit to leave after-tax exploitation rents to firms. This variety is observed empirically, including the polar case of a nationalized extraction sector. All such optimal combinations of extraction taxes with reserve development subsidies imply extraction taxes at least as high as the tax on other goods. Moreover, this tax causes a distortion to the nonrenewable resource sector that takes the form not only of slower extraction at a given level of remaining reserves, but also of lower induced reserves.

In a numerical application, we apply our new Ramsey resource tax formula to the case of oil, which provides orders of magnitude for the level of nonrenewable resources' OCT. The example illustrates how the need for public-revenue collection should affect both the development of oil reserves and their rate of exploitation.

Then, we show how our tax rule can be directly used to establish how much carbon taxation should be augmented in the presence of a public budget constraint, and how this affects the direction prescribed for resolution of the carbon externality.

The rest of the article is structured as follows. Section 2 presents our model in the case of a Hotelling resource. In Section 3, we consider that reserves are endogenous, obtain our main formula for the taxation of nonrenewable resources in the presence of public revenue needs, and examine its theoretical implications. Section 4 provides a numerical application for the case of oil. In Section 5, we present the theoretical and numerical implications of our formula for the case of a carbon resource taxed to both resolve the carbon externality and contribute to public tax revenue needs. Section 6 discusses, with technical details in the appendix, the taxation of resource substitutes and complements and the taxation of nonrenewable resources in open economies. Section 7 concludes.

2 OCT with a Nonrenewable Hotelling Resource

There are n produced commodities or services indexed by $i = 1, ..., n$, one nonrenewable resource indexed by s and extracted from a finite reserve stock S_0, and a numeraire that is not taxed.[7] We adopt the standard partial-equilibrium restrictions under which Baumol and Bradford (1970) obtain the inverse elasticity rule; that is: All goods or services $i = 1, ..., n$ and s are final-consumption goods.[8] Assuming a single resource simplifies the exposition without affecting the generality of the results.[9] At each date $t \geq 0$, quan-

[7] It is standard to interpret the untaxed numeraire as being leisure. The impossibility of lump-sum taxation means that neither labor endowment nor leisure consumption are taxable (e.g., Auerbach, 1985, p. 89).

[8] Final goods should be interpreted as non-leisure goods.

[9] See Appendix F for the case of multiple heterogenous resources.

tity flows are denoted by $x_t \equiv (x_{1t}, ..., x_{nt}, x_{st})$.[10] Storage is not possible, so goods and services must be consumed as they are produced. Producer prices $p_t \equiv (p_{1t}, ..., p_{nt}, p_{st})$ are expressed in terms of the numeraire. Goods and services are taxed[11] at unit levels $\theta_t \equiv (\theta_{1t}, ..., \theta_{nt}, \theta_{st})$ so that the representative consumer[12] faces prices $q_t = p_t + \theta_t$. In this autarkic economy, as in any situation in which production equals consumption, taxes may indifferently be interpreted as falling on consumers or producers, but must be such that they leave nonnegative profits to producers. In the case of the nonrenewable resource, this requires that at any date, the discounted profits accruing to producers over the remaining life of the mine must be nonnegative. Taxes that meet these conditions will be called feasible.

Since the resource is nonrenewable, it must be true that

$$\int_0^{+\infty} x_{st}\, dt \leq S_0, \tag{1}$$

where S_0 is the initial size of the depletable stock.

In the rest of the paper, a "$\sim$" on top of a variable means that the variable is evaluated at the competitive market equilibrium. For given feasible taxes $\Theta \equiv \{\theta_t\}_{t\geq 0}$, competitive markets lead to the equilibrium allocation $\{\widetilde{x}_t\}_{t\geq 0}$ where $\widetilde{x}_t = (\widetilde{x}_{1t}, ..., \widetilde{x}_{nt}, \widetilde{x}_{st})$. Under the set of taxes Θ, this intertemporal allocation is second-best efficient.

Defining social welfare as the cumulative discounted sum of instantaneous utilities $\widetilde{W}_t$, the OCT problem consists in choosing a feasible set of taxes Θ in such a way as to maximize

[10]Production processes by which resources are transformed into final products are linear in the quantity of pre-transformed resource; hence, there is no need to distinguish the raw extracted resource from its derivative.

[11]Again, direct income or profit taxation is impossible. When income taxation is linear, there is no loss of generality in letting labor income be untaxed (e.g., Atkinson and Stiglitz, 1976). Even with nonlinear income taxation, the substantial literature that questions the relevance of Atkinson and Stiglitz's framework shows that commodity taxation remains an issue under sensible assumptions. Moreover, as long as direct profit and rent taxes are less than 100% and cannot be adjusted, assuming that profits and rents are not taxed directly at all amounts to rescaling the production cost function and does not involve any loss of generality. Finally, the absence of distinction between a resource-based final good or service and the quantity of resource inputs needed to produce it has been shown to be appropriate in situations in which the final good is difficult to tax (e.g., transportation; Stiglitz, 2015). In general, Stiglitz and Dasgupta (1971) further show that production efficiency is not required in the presence of untaxed profits or rents.

[12]Using a representative agent is a simplification that should not be interpreted to mean that a poll tax is feasible. With heterogeneous consumers and concerns about equity, the standard Ramsey tax formula for one commodity takes into account the social contribution of consumers' incomes (Diamond, 1975; see Belan et al., 2008, for a partial-equilibrium exposition closer to ours).

welfare while raising a given level of discounted revenue $R_0 \geq 0$:

$$\max_{\Theta} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt \tag{2}$$

$$\text{subject to} \int_0^{+\infty} \theta_t \widetilde{x}_t e^{-rt}\, dt \geq R_0. \tag{3}$$

The fact that all variables are evaluated at the competitive market equilibrium further indicates that the problem is implicitly constrained by the realization of this equilibrium. In particular, that means that the finiteness of resource reserves, as expressed in (1), does not need to be taken into account by the government, as long as it is a constraint of resource producers that is reflected in the market equilibrium, as explained below. It is assumed that the set of feasible taxes capable of collecting R_0 is not empty.

The tax revenue constraint (3) does not bind the government at any particular date, because financial markets allow expenditures to be disconnected from revenues. The government accumulates an asset a_t over time by saving tax revenues:

$$\dot{a}_t = r a_t + \theta_t \widetilde{x}_t, \tag{4}$$

where the initial amount of asset is normalized to zero and

$$\lim_{t \to +\infty} a_t e^{-rt} = R_0. \tag{5}$$

Thus the problem of maximizing (2) subject to (3) can be replaced with the maximization of (2) subject to (4) and (5) by choice of a feasible set of taxes.

As in Ramsey (1927, p. 55) and Baumol and Bradford (1970), we assume that the demand $D_i(q_{it})$ for each commodity or service i or s depends only on its own price, with $D_i'(.) < 0$.[13] Moreover, following Baumol and Bradford (1970) and many others, we assume in this section that supply is perfectly elastic, i.e., that marginal costs of production are constant in terms of the numeraire. Let $c_i \geq 0$ be the marginal cost of producing good or service $i = 1, ..., n$.[14]

[13]This means that the underlying consumer preferences are quasi-linear in the numeraire. In Section 6, with technical details in the appendix, we consider non-zero cross-price demand elasticities.

[14]See Appendix F for the case of rising marginal costs of production.

In the case of the nonrenewable resource, the supply is determined by Hotelling's rule under conditions of competitive extraction. Consistent with our assumption of constant marginal costs of production, we assume that the unit resource extraction cost is constant, equal to $c_s \geq 0$.

However, this does not imply that the producer price of the resource reduces to this marginal cost. Indeed, Hotelling's analysis shows nonrenewable resource supply to be determined in competitive equilibrium by the so-called "augmented marginal cost" condition:

$$\widetilde{p}_{st} = c_s + \widetilde{\eta}_t, \tag{6}$$

where $\widetilde{\eta}_t$ is the current-value unit Hotelling rent accruing to producers; it depends on the tax and the level of initial reserves, and must grow at the rate of discount over time. In competitive Hotelling equilibrium,

$$\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}. \tag{7}$$

At any date, the net consumer surplus, producer surplus, and resource rents in competitive equilibrium are, respectively,

$$\widetilde{CS}_t = \sum_{i=1,\dots,n,s} \int_0^{\widetilde{x}_{it}} D_i^{-1}(u)\, du - \sum_{i=1,\dots,n,s} (\widetilde{p}_{it} + \theta_{it})\widetilde{x}_{it}, \tag{8}$$

$$\widetilde{PS}_t = \sum_{i=1,\dots,n,s} \widetilde{p}_{it}\widetilde{x}_{it} - \sum_{i=1,\dots,n,s} c_i \widetilde{x}_{it} - \widetilde{\eta}_t \widetilde{x}_{st} \tag{9}$$

and

$$\widetilde{\phi}_t = \widetilde{\eta}_t \widetilde{x}_{st}. \tag{10}$$

Define $\widetilde{W}_t$ in problem (2) as the sum of net consumer surplus, net producer surplus, and resource rents accruing to resource owners.[15,16] The present-value Hamiltonian associated with the problem of maximizing cumulative discounted social welfare (2) under constraints

[15] Although changes in current taxes may affect current tax revenues, the budget constraint of the government applies only over the entire optimization period. The revenue requirements being treated as given over that period, they enter the general problem as a constant and thus no amount of redistributed taxes needs to enter the objective.

[16] This formulation has the advantage of making the value of the resource as a scarce input explicit; it would also apply if producers were not owners of the resource but were buying the resource from its owners at its in situ price $\widetilde{\eta}_t$.

(4) and (5) resulting from the budget requirement of the government is

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t \widetilde{x}_t), \tag{11}$$

where λ_t is the costate variable associated with a_t and θ_t is the vector of control variables. λ_t can be interpreted as the current unit cost of levying \$1 of present-value revenues through taxes. From the maximum principle, $\dot{\lambda}_t = -\frac{\partial \mathcal{H}}{\partial a_t}$, so that $\lambda_t = \lambda e^{-rt}$, where λ is the present-value unit cost of levying tax revenues. Indeed, tax revenues must be discounted according to the date at which they are collected. λ is equal to unity when there is no deadweight loss associated with taxation; it is higher than unity otherwise.

2.1 Optimal Taxation of Conventional Goods

Assuming that there exist feasible taxes that yield an interior solution to the problem, the first-order condition for the choice of tax θ_{it} on good $i = 1, ..., n$ is

$$[D_i^{-1}(\widetilde{x}_{it}) - \theta_{it} - c_i]\frac{d\widetilde{x}_{it}}{d\theta_{it}} - \widetilde{x}_{it} + \lambda(\widetilde{x}_{it} + \theta_{it}\frac{d\widetilde{x}_{it}}{d\theta_{it}}) = 0. \tag{12}$$

Since the competitive equilibrium allocation $\widetilde{x}_t$ satisfies $D_i^{-1}(\widetilde{x}_{it}) = c_i + \theta_{it}$, it is the case that $\frac{d\widetilde{x}_{it}}{d\theta_{it}} = \frac{1}{D_i^{-1\prime}(.)}$. The optimal tax is thus $\theta_{it}^* = \frac{1-\lambda}{\lambda}\widetilde{x}_{it}D_i^{-1\prime}(.)$ and the optimal tax rate is

$$\frac{\theta_{it}^*}{\widetilde{q}_{it}} = \frac{\lambda - 1}{\lambda}\frac{1}{-\widetilde{\varepsilon}_i}. \tag{13}$$

In this formula, the elasticity of demand $\varepsilon_i \equiv \frac{D_i^{-1}(.)}{x_{it}D_i^{-1\prime}(.)}$ is negative and is further assumed to be increasing in x_{it}; this standard monotonicity property guarantees that the optimal tax in (13) is unique. As $\lambda \geq 1$, the optimal tax rates on conventional goods $i = 1, ..., n$ are positive in general, lower than unity, and distortionary in the sense that they induce $\widetilde{x}_{it} = D_i(c_i + \theta_{it})$ to decrease; these tax rates vanish if $\lambda = 1$.

Formula (13) is Ramsey's formula for the optimal commodity tax rate. It provides an inverse elasticity rule for the case of perfectly elastic supplies. Since market conditions are unchanged from one date to the other, taxes and the induced tax rates are constant over time.

2.2 Optimal Taxation of the Nonrenewable Resource

The first-order condition for an interior solution to the choice of the resource tax is

$$[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = 0. \tag{14}$$

However, since resource supply is determined by condition (6), it follows that $D_s^{-1}(\widetilde{x}_{st}) - c_s - \theta_{st} = \widetilde{\eta}_t$, which is different from zero, unlike the corresponding expression in (12). Consequently, the Ramsey-type formula obtained for conventional goods does not apply.

If $\lambda = 1$, (14) reduces to $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = 0$. This means that the tax should not distort the Hotelling extraction path at all. Such a non-distortionary resource tax exists (Burness, 1976; Dasgupta et al., 1981); it must grow at the rate of interest to keep the path of consumer prices unchanged:[17] $\theta^*_{st} = \theta^*_{s0}e^{rt}$. Since θ^*_{st} grows at the rate of interest and the resulting $\widetilde{q}_{st}$ grows at a lower rate, the neutral tax rate is rising over time. The only exception is when the marginal cost of extraction is zero so that $\widetilde{q}_{st}$ grows at the rate of interest and the resulting optimal tax rate is constant.

As shown earlier, when $\lambda = 1$, commodity taxes on conventional goods are zero. Hence the totality of the tax burden falls on the nonrenewable resource. Since the tax on the resource is neutral in that case, a value of unity for λ is indeed compatible with taxing the natural resource exclusively. Consequently, provided the tax on the nonrenewable resource brings sufficient cumulative revenues, the government should tax the resource exclusively and should do so while taxing a proportion of the resource rent that remains constant over time.

The maximum revenue such a neutral resource tax can raise is the totality of gross cumulative scarcity rents that would accrue to producers in the absence of a resource tax. Since unit Hotelling rents are constant in present value, any reserve unit fetches the same rent, whatever the date at which it is extracted. The present value of total cumulative resource rents is thus $\widetilde{\eta}_0 S_0$ and its maximum possible value $\overline{\eta}_0 S_0$ corresponds to the absence

[17] Their proof goes as follows. Assume $\theta_{st} = \theta_{s0}e^{rt}$ for any θ_{s0} lower than the consumer price exclusive of the marginal cost in the absence of any resource tax. Then $\widetilde{q}_{st} = \widetilde{p}_{st} + \theta_{st} = c_s + \widetilde{\eta}_t + \theta_{st} = c_s + (\widetilde{\eta}_0 + \theta_{0t})e^{rt}$. Therefore, the price with the tax satisfies the Hotelling rule. The exhaustibility constraint must also be satisfied with equality: $\int_0^{+\infty} D_s(\widetilde{q}_{st})\, dt = S_0$. As a result, the extraction path under this tax is the same as in the absence of tax.

of taxation; the maximum tax revenue that can be raised by a neutral resource tax is thus

$$\overline{R}_0 = \overline{\eta}_0 S_0.$$

This maximum is implemented with a tax equal to the unit rent in the absence of taxation: $\theta^*_{st} = \overline{\eta}_0 e^{rt}$. Both $\widetilde{\eta}_0$ and $\overline{\eta}_0$ are determined in Appendix B. If the tax revenues needed by the government are lower than $\overline{R}_0$, the level of the neutral resource tax θ^*_{st} is set in such a way as to exactly raise the required revenue: $\theta^*_{st} = \theta^*_{s0} e^{rt}$ with

$$\theta^*_{s0} = \overline{\eta}_0 - \widetilde{\eta}_0 = \frac{R_0}{S_0}. \tag{15}$$

If $R_0 > \overline{R}_0$, revenue needs cannot be met by non-distortionary taxation of the resource sector and $\lambda > 1$; this case will be discussed further below. The following proposition summarizes our findings when government revenue needs are low, in the sense that $\lambda = 1$.

Proposition 1 *(Low government revenue needs) The maximum tax revenue that can be raised by neutral taxation of the nonrenewable resource sector is $\overline{R}_0 = \overline{\eta}_0 S_0$, where $\overline{\eta}_0$ is the unit present-value Hotelling rent under perfect competition and in the absence of taxation.*

1. *If $R_0 \leq \overline{R}_0$, then $\lambda = 1$, and government revenue needs are said to be low; if $R_0 > \overline{R}_0$, then $\lambda > 1$, and government revenue needs are said to be high;*

2. *When $R_0 \leq \overline{R}_0$, the optimal unit tax on the nonrenewable resource is positive and independent of demand elasticity, while the optimal unit tax on produced goods is zero. The resource tax raises exactly R_0 over the extraction period.*

As long as the government's revenue needs are low, Proposition 1 indicates that the archetypal distortionary tax of the OCT literature should not be applied to conventional commodities; taxation should be applied to the sole resource according to a rule that has nothing to do with Ramsey's rule, is independent of the elasticity of demand and does not induce any distortion.[18] Except for a few resource-rich economies—for example, Saudi

[18] The fact that neutral taxation of the Hotelling commodity is possible does not mean that neutral profits taxation, as in Stiglitz and Dasgupta (1971), or capital levy, as in Lucas (1990), or some form of resource rent tax, as in Boadway and Keen (2010), have been allowed into the model. It should be clear from the formulation that neutral resource taxation is reached by commodity taxation only.

Arabia's non-oil government revenues reached[19] 6.5% in 2010—this case of low revenue needs is less realistic than the second-best case studied hereafter.

If government revenue needs are high in the sense that $R_0 > \overline{R}_0$ and $\lambda > 1$, revenue needs cannot be met by neutral taxation; then we have shown that both the resource and the conventional goods should be taxed. Furthermore, the question arises whether the government can and should collect more resource revenues by departing from neutral taxation of the resource sector.[20] This possibility was not addressed by Dasgupta, Heal, and Stiglitz (1981), nor by followers in the nonrenewable resource taxation literature. Barrage (2017) came closest by introducing a nonrenewable resource sector in her study of OCT with carbon pollution (p. 50), but she left Ramsey dynamic distortions to this sector unexplored.

The neutral tax that maximizes tax revenues does not leave any resource rent to producers: $\widetilde{q}_{st} = c_s + \theta_{st}$. Assume, as will be seen to be true later on, that the government can maintain its complete appropriation of producers' resource rents while further increasing tax revenues: The condition $\widetilde{q}_{st} = c_s + \theta_{st}$ remains true, while θ_{st} is set so as to further extract some of the consumer surplus. This implies that when $\lambda > 1$, $\widetilde{p}_{st} = c_s$, $\widetilde{\eta}_t = 0$, and $\widetilde{x}_{st} = D_s(c_s+\theta_{st})$. With $\widetilde{\eta}_t = 0$, resource extraction is no longer determined by the Hotelling supply condition (6): Since $\widetilde{p}_{st} = c_s$, producers are indifferent to the quantity they supply so that quantity is determined on the demand side by the condition $\widetilde{q}_{st} = c_s + \theta_{st}$. Consequently, the choice of θ_{st} by the government determines extraction so that the finiteness of reserves, if it turns out to be binding, comes as a constraint faced by the government in its attempt to increase cumulative tax revenues rather than as a constraint faced by producers in maximizing cumulative profits. Thus the government's problem is now to maximize (2), not only subject to (4) and (5), but also subject to

$$\dot{S}_t = -\widetilde{x}_{st}, \tag{16}$$

where S_t denotes the size of the remaining depletable stock at date t.

[19]See `http://data.imf.org/?sk=4CC54C86-F659-4B16-ABF5-FAB77D52D2E6&sId=1390030109571`.

[20]Clearly, at each date, a nonlinear tax on the resource extraction rate reaching the level of the maximum constant neutral tax at the Pareto-optimal extraction rate would achieve such a goal. However, such a non-distortionary tax is ruled out in the conventional Ramsey-Pigou optimal taxation analysis. If it were feasible, the Ramsey-Pigou problem would be meaningless.

The Hamiltonian is modified to

$$\mathcal{H}\left(a_t,\theta_t,\lambda_t,\mu_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t \widetilde{x}_t) - \mu_t \widetilde{x}_{st}, \tag{17}$$

where $\widetilde{CS}_t$, $\widetilde{PS}_t$, and $\widetilde{\phi}_t$ are defined as before but with $\widetilde{\eta}_t = 0$, and μ_t is the co-state variable associated with the exhaustibility constraint (16). From the maximum principle, $\lambda_t = \lambda e^{-rt}$, as above, and $\mu_t = \mu \geq 0$. If the exhaustibility constraint is binding, that is to say if optimal taxation induces complete exhaustion of the reserves, $\mu > 0$; if optimal taxation leads to incomplete exhaustion, then $\mu = 0$.

The first-order condition for the choice of the tax on the resource becomes

$$[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}. \tag{18}$$

Since no resource rent is left to producers above the marginal cost of extraction, $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s = 0$, $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$, and the optimal tax on the resource is thus

$$\theta_{st}^* = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}, \tag{19}$$

where the elasticity of resource demand $\varepsilon_s \equiv \frac{D_s^{-1}(.)}{x_{st}D_s^{-1\prime}(.)}$ is negative and is further assumed to be increasing for the same reason as for conventional commodities.

Provided the resource is scarce ($\mu > 0$) from the government's social welfare point of view, (19) implies that the resource is taxed at a higher rate than would be the case according to (13) for a conventional commodity having the same demand elasticity. Furthermore, while the first term on the right-hand side of (19) is neutral as it rises at the rate of discount, the presence of the second term implies that the tax is not constant in present value, so that it is distortionary in general.

Can the tax revenue collection motive cause the government to assign no scarcity value to a resource that would otherwise be extracted until exhaustion? The answer is negative. Suppose that $\mu = 0$ in (19). This implies that the tax rate is constant over time, so that the extraction rate is also constant and strictly positive, which in turn implies that the exhaustibility constraint must be violated in finite time.

The following proposition summarizes the results on the optimal taxation of the resource when neutral taxation is not sufficient to collect the revenue needs.

Proposition 2 *(High government revenue needs) If $R_0 > \overline{R}_0$, then commodity taxation is distortionary ($\lambda > 1$) and both the resource sector and conventional sectors are subject to taxation. In that case:*

1. *Taxes on conventional commodities are given by Ramsey's rule (13) and the tax on the nonrenewable resource is given by (19), where λ is determined by the condition that total tax revenues levied from the non-resource and resource sectors equal R_0;*
2. *The nonrenewable resource is taxed at a higher rate than a conventional commodity having the same demand elasticity;*
3. *The after-tax resource rent to producers is nil: $\widetilde{\eta}_t = \widetilde{\eta}_0 = 0$;*
4. *The OCT distortion to the nonrenewable resource extraction is determined by the time path of taxes (19);*
5. *OCT of the nonrenewable resource causes extraction to slow down, but not for reserves to be left unexploited.*

Although the resource tax rate is higher than the rate on conventional commodities of identical elasticities, its distortionary effect is not higher; it is chosen so as to minimize the total welfare effect of all commodity tax distortions. The distortion slows down extraction, thus working in the same direction as prescribed by, for example, Withagen (1994) to deal with cumulative pollution.

Propositions 1 and 2 also have implications on the evolution of the total flow of tax revenues over time. When the government's revenue needs are low, the total flow of tax revenues decreases in present value as the resource unit tax is constant in present value, while extraction diminishes. Tax revenues from conventional sectors being nil, total tax revenues decrease in present value; they vanish entirely if the resource is exhausted in finite time, or they always remain positive but tend to zero if the resource is only exhausted asymptotically.[21] When government revenue needs are high, the flow of tax revenues from conventional sectors is constant in current value. If the resource is exhausted in finite time,

[21] More details on the case of asymptotic exhaustion are presented in Appendix D.

the total tax revenue flow is thus lower at and after the date of exhaustion than before exhaustion. In either case, the government's assets accumulated at resource exhaustion must be sufficient to ensure that expenditures taking place after exhaustion can be financed.

When the government cannot avoid the introduction of distortions, as when revenue needs are high, its problem acquires a revenue-maximizing dimension. This confers OCT a resemblance with monopoly pricing, as the term $\frac{1}{-\tilde{\varepsilon}_s}$ in (13) is nothing but a monopoly mark-up (for details see Appendix E). The resource monopoly literature has shown that the exercise of market power by a Hotelling resource monopoly is constrained by exhaustibility. The sharpest example is Stiglitz (1976), who showed that a resource monopoly facing a constant-elasticity demand and zero extraction costs must adopt the same behavior as a competitive firm; such a monopoly cannot increase its profits above the value of the mine under competition by distorting the extraction path. This limitation also applies to the OCT problem. With zero extraction cost and isoelastic demand, the tax defined by (19) is neutral and rises at the discount rate. In that case, OCT requires that no distortion be imposed on the nonrenewable resource extraction. We make use of it in Section 3, where initial reserves are treated as endogenous.

From Propositions 1 and 2, the resource should be taxed in priority whatever its demand elasticity and whatever the demand elasticity of regular commodities. This irrelevance of demand elasticities contrasts sharply with the standard rationalization of OCT, but not with Ramsey's original message. The message is "tax inelastic sectors," whether the source of inelasticity is demand or supply. Once it is realized that long-run reserve supply fixity results in reduced short-run resource supply elasticity, it becomes clear that the emphasis should shift from demand to supply in the case of a Hotelling resource.

In Appendix F, we extend the analysis to the case of increasing marginal costs of production and increasing marginal costs of extraction, so that the supply elasticity of conventional goods is no longer infinite. While the inverse elasticity rule then acquires a supply elasticity component, the finiteness of ultimate reserves implies that nonrenewable resources should be taxed in priority and at higher rates than otherwise identical conventional commodities. The inelasticity of long-run resource supply dominates other considerations. We also examine the role of resource heterogeneity. Again, the results are altered but not in any fundamental way.

In Appendix G (see the discussion in Section 6) we do away with the assumption that demands are independent of each other—a standard assumption under which the inverse elasticity rule is usually derived. This allows us to examine the specific tax treatment that resource substitutes and complements should receive from an OCT perspective.

In Section 3, the Hotelling assumption that reserves are exogenously given is relaxed. Doing away with this assumption introduces the long-run supply elasticity of the resource and also allows us to highlight the distinction between a nonrenewable resource and conventional capital.

3 Endogenous Reserves

In order to focus on the role of the long-run supply of reserves, we assume in this section, as in Section 2, that marginal extraction costs are constant, equal to $c_s \geq 0$. This means that the supply of the natural resource is only limited by the availability of reserves. Consistently, we assume that marginal costs of production are constant, equal to $c_i \geq 0$.

The stock of reserves exploited by a mine does not become available without some prior exploration and development investment. Although exploration for new reserves and exploitation of current reserves often take place simultaneously at the industry level—see, e.g., Pindyck (1978) and Quyen (1988)—most exploration is aimed at the discovery of new deposits, and deposit-specific exploration becomes limited once the mine is in exploitation. It is thus a meaningful simplification to adopt the microeconomic view that exploration and exploitation take place in a sequence, as in Gaudet and Lasserre (1988) and Fischer and Laxminarayan (2005). This way of modeling the supply of reserves is particularly adapted to the OCT problem under study[22] because it provides a simple and natural way to distinguish short-run supply elasticity from long-run supply elasticity. It also raises the issue of the government's ability to tax and subsidize, as well as its ability to commit.[23]

Most commonly observed extractive resource tax systems feature royalties and levies

[22]Firms seldom exploit only one site, so that exploration is an ongoing process at the firm level (as opposed to the site level). However, linear Ramsey taxes do not need to be industry or firm specific; they can be site specific in theory. In practice they sometimes are. For example, Alberta's taxation of conventional oil and natural gas commits to royalty rate reductions that depend on a well's discovery date (Alberta Royalty Review, 2007); those reductions amount to exploration subsidies that aim at recognizing exploration costs; they are made dependent on discovery date to reflect the changing costs of exploration across deposits over time. To the extent that their sum depends on cumulative extraction, they are based on the amount of discovered reserves.

[23]On issues of commitment and regime changes in resource taxation, see Daniel et al. (2010).

based on extraction revenues or quantities, which are often combined with tax incentives for exploration and development. During the extraction phase, i.e., once reserves are established, these systems let some Hotelling rents accrue to producers, perhaps to compensate firms for the prior production of reserves.[24] On the other hand, state-owned extraction sectors are common. A nationalized industry means that no extraction rents are left to private producers.

Thus two situations are common empirically: In the first instance extraction is taxed in such a way that strictly positive rents are left to firms; in the second instance no extraction rents are left to firms. Results from the previous section point to the importance of that distinction. Indeed, when S_0 is given, as in Section 2, if the government has high revenue needs in the sense of Proposition 2, it should use the nonrenewable resource commodity tax to take the totality of extraction rents away from producers. If it did so when S_0 were endogenous, it would tax quasi-rents together with scarcity rents, thus removing incentives for producers to generate reserves in the first place. If the government wants to create a tax environment that allows net extraction profits to compensate firms for the cost of reserve production, it must be able to commit, prior to extraction, to a system of ex post extraction taxation that leaves enough rents to producers. Alternatively, if the government taxes away extraction rents, including quasi-rents sunk into them, it must compensate firms by subsidies prior to extraction. In practice, these subsidies often take the form of commitments to reductions in the tax rate applied to future extraction;[25] thus they are formally equivalent to subsidies that are linear in the quantity developed and put into exploitation. We will show that there exists a continuum of mixed systems, combining subsidies (negative commodity taxes) on reserve production with positive taxes on extraction, that leave some rents in the hands of firms while meeting the government's revenue needs.[26]

For simplicity, assume that ex ante reserve producers (explorers) are the same firms as ex post extractors. Assume that the stock of reserves to be exploited is determined

[24]Clearly, Ramsey's tax setup rules out the direct taxation of rents, but not their indirect taxation by commodity taxes (Stiglitz, 2015).

[25]See Footnote 22 for the example of Alberta's relatively advanced system.

[26]These mixed systems are feasible if the government is able to commit to leave firms the prescribed after-tax extraction rent; otherwise, an optimal system relying on reserve supply subsidies while not leaving the firms any extraction surplus can also achieve the same objective.

prior to extraction by a supply process that reacts to the sum of the subsidies obtained by the firms for reserve production and the cumulative net present-value rents accruing to resource producers during the exploitation stage; also for simplicity, assume that reserve production is instantaneous.

Express total cumulative present-value rents from extraction as $\eta_0 S_0$. Suppose further that a negative linear tax $-\rho$ may be applied to the production of reserves, for a total subsidy of ρS_0. Then the initial stock of reserves may be written as a function of $\eta_0 + \rho$.[27] This function $\mathcal{S}(\eta_0 + \rho)$ can be interpreted as the long-run after-tax supply of reserves as follows. Suppose that reserves can be obtained, via exploration or purchase, at a cost $E(S_0)$. As not only known reserves, but also exploration prospects, are finite, the long-run supply of reserves is subject to decreasing returns, so that $E'(S_0) > 0$ for any $S_0 > 0$, and $E''(.) > 0$. Then the profit from the production of a stock S_0 of initial reserves is $(\widetilde{\eta}_0 + \rho) S_0 - E(S_0)$. Given ρ and $\widetilde{\eta}_0$, its maximization requires $\widetilde{\eta}_0 + \rho = E'(S_0)$. We define $\mathcal{S}(\widetilde{\eta}_0 + \rho) \equiv E'^{-1}(\widetilde{\eta}_0 + \rho)$, making the following assumption.

Assumption 1 *(Long-run supply) The supply of initial reserves $\mathcal{S}(.)$ is continuously differentiable and such that $\mathcal{S}(0) = 0$, $\mathcal{S}(\eta_0 + \rho) > 0$ for any strictly positive value of $\eta_0 + \rho$, and $\mathcal{S}'(\eta_0 + \rho) > 0$.*

The property $\mathcal{S}(\eta_0 + \rho) > 0$ for any strictly positive value of $\eta_0 + \rho$ is introduced because it is sufficient to rule out the uninteresting situation in which the demand for the resource does not warrant the production of any reserves.

3.1 Optimal Resource Taxation with a Strictly Positive Producer Rent

Even when the government can subsidize exploration, i.e., when $\rho > 0$, leaving some positive after-tax extraction rent to producers may be desirable. Two reasons make it interesting to analyze situations in which the government leaves positive extraction rents to producers. First, they are empirically relevant. Second, they will be shown to constitute a general case that includes no-commitment as a limiting case. In this subsection, we assume that ρ is given and is not high enough to remove the need for the government to leave producers positive after-tax extraction rents. Later on, we will analyze the choice of

[27] Clearly, at the equilibrium, for a given subsidy ρ, $\widetilde{\eta}_0$ will depend on the stock of reserves in the same way as in Section 2.

ρ and study whether it is desirable for the government to leave positive extraction rents to producers at all.

Ex post, once reserves have been established, producers face a standard Hotelling extraction problem and the government chooses taxes. Furthermore, we assume that the government is committed to leaving the producers a Hotelling rent $\widetilde{\eta}_t > 0$, with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$, as defined in (6) and (7), for a total rent commitment of $\widetilde{\eta}_0 S_0$. Clearly, given ρ, the level of initial reserves will be determined ex ante by that commitment; it will be denoted $\widetilde{S}_0$, with

$$\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho), \tag{20}$$

and discussed further below.

Thus the government chooses optimal taxes on extraction given $\widetilde{\eta}_0$, or, equivalently, given any positive $\widetilde{S}_0$. The problem is thus identical to the problem with exogenous reserves analyzed in Section 2, except that the government is now subject to its ex ante rent commitment. The Hamiltonian is thus (17), with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$ rather than $\widetilde{\eta}_t = 0$:

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t, \mu_t\right) = (\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t)e^{-rt} + \lambda_t(ra_t + \theta_t \widetilde{x}_t) - \mu_t \widetilde{x}_{st}, \tag{21}$$

where $\widetilde{CS}_t$, $\widetilde{PS}_t$ and $\widetilde{\phi}_t$ are respectively defined by (8), (9), and (10), with $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$. The control variables are the taxes θ_t.

Suppose, as an assumption to be contradicted, that revenue needs are low ($\lambda = 1$); then, according to Proposition 1, conventional goods are not taxed and a tax is imposed on the resource during the extraction phase to satisfy revenue needs. This reduces the rent accruing to extracting firms and, by (20), reduces the initial amount of reserves relative to the no-tax situation. Consequently, any attempt to satisfy revenue needs by taxing the resource extraction sector results in a distortion, so that in contradiction with the initial assumption, λ is strictly higher than unity whatever the revenue needs. It follows that the tax on conventional goods is given by (13) with $\lambda > 1$.

Now consider the taxation of the resource sector, with $\lambda > 1$. In Appendix I, we show that the optimal extraction tax differs from its value when reserves are exogenous, in that it now depends on the rent that the government is committed to as follows:

$$\theta^*_{st} = \frac{1}{\lambda}(\mu - \widetilde{\eta}_0)e^{rt} + \frac{\lambda - 1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}. \tag{22}$$

The second term on the right-hand side of the expression is the familiar inverse elasticity rule; it appears in the same form as in Formula (19) that describes the resource tax when reserves are exogenous. As in that case, the tax rate on the resource thus exceeds the tax rate on a conventional good of identical demand elasticity if and only if the first term is nonnegative. Such is clearly the case with exogenous reserves when the first term on the right-hand side is $\frac{1}{\lambda}\mu e^{rt}$, but perhaps not so with endogenous reserves, as the sign of the first term on the right-hand side of (22) depends on the sign of $(\mu - \widetilde{\eta}_0)$. Intuition suggests that the government would not commit ex ante to leaving a unit after-tax rent of $\widetilde{\eta}_0$ to firms if this were not at least equal to its ex post implicit valuation μ of a reserve unit. One can validate this intuition by analyzing the choice of $\widetilde{\eta}_0$, which we now turn to.

Let us characterize the ex ante choice of the rent $\widetilde{\eta}_0$ left to firms after payment of the extraction taxes, for a given level of ρ.[28] The choice of $\widetilde{\eta}_0$ is dual to the choice of reserves $\widetilde{S}_0$ since (20) must hold. The marginal cost of establishing reserves at a level S_0 is $E'(S_0) = \mathcal{S}^{-1}(S_0)$, implying a total cost of reserves $\int_0^{S_0} \mathcal{S}^{-1}(S)\, dS$. This cost should be deducted from the ex ante objective of the government, which is given by (2) when reserves are exogenous. The objective should also include as benefit the total subsidy payment to producers ρS_0.

The ex ante problem of the government is thus

$$\max_{\widetilde{\eta}_0,\ \Theta} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt + \rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS \tag{23}$$

subject to (20) and subject to the tax revenue constraint, adapted to take account of the additional liability associated with the reserve subsidy:

$$\int_0^{+\infty} \theta_t \widetilde{x}_t e^{-rt}\, dt \geq R_0 + \rho \widetilde{S}_0 \equiv R. \tag{24}$$

In Appendix I, we address the ex ante problem (23)-(20)-(24) while taking account of

[28] Clearly, the subsidy must be low enough to necessitate the presence of after-tax rents at the extraction stage. This will be addressed further below.

the ex post situation—once reserves have been determined—first analyzed, and obtain the following relationship:

$$\mu = \lambda\rho + \widetilde{\eta}_0. \tag{25}$$

Indeed, as hinted earlier, the marginal unit value of reserves for the government in its taxation exercise exceeds the private marginal cost $\rho + \widetilde{\eta}_0$ of developing those reserves by a factor that reflects the cost of raising funds ($\lambda > 1$) to finance the subsidy payment.

With $\mu - \widetilde{\eta}_0 \geq 0$, it follows from (22) and (13) that the tax rate on the nonrenewable resource is higher than the tax rate on a conventional good with the same demand elasticity. Precisely, the unit tax θ^*_{st} on the resource exceeds the common inverse-elasticity term by ρe^{rt}. This component of the unit tax grows at the discount rate so that, alone, it would leave the extraction profile unchanged. In contrast, the component that is common to the resource tax and the tax on the conventional good[29] causes a distortion to the extraction profile; its value is $\frac{\lambda-1}{\lambda}\frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}$, exactly that of a conventional Ramsey tax. This is stated in Proposition 3.

Proposition 3 *(Optimal extraction taxes; endogenous reserves) When the supply of reserves is elastic and is subsidized at the unit rate $\rho \geq 0$, and the supply of conventional goods or services is infinitely elastic,*

1. *The resource tax rate is given by (26); it is made up of a component that is neutral at given initial reserves complemented by a distortionary Ramsey inverse-elasticity component;*

2. *The nonrenewable resource is taxed at a strictly higher rate than a conventional good or service having the same demand elasticity if $\rho > 0$ and is taxed at the same rate if $\rho = 0$.*

Substituting (25) into (22) implies

$$\frac{\theta^*_{st}}{\widetilde{q}_{st}} = \frac{\rho e^{rt}}{\widetilde{q}_{st}} + \frac{\lambda - 1}{\lambda}\frac{1}{-\widetilde{\varepsilon}_s}, \tag{26}$$

[29] As previously mentioned, an exception arises when the demand has constant elasticity and the extraction cost is zero (Stiglitz, 1976), in which case the tax has no effect on extraction, given reserves. More on this below.

where $\widetilde{q}_{st} = c_s + \widetilde{\eta}_0 e^{rt} + \theta^*_{st}$. This expression identifies the role of the reserve subsidy on the tax rate at any extraction date explicitly. Its analysis, presented in Appendix K, reveals that as the subsidy ρ changes, the government should adjust its tax θ^*_{st} in such a way as to induce an unchanged subsidy-inclusive rent $\widetilde{\eta}_0 + \rho$, unchanged developed reserves $\widetilde{S}_0$, unchanged extraction prices $\widetilde{q}_{st}$, and quantities.[30] In other words, the optimal after-tax rent depends negatively on the ex ante subsidy: $\widetilde{\eta}_0 = \widetilde{\eta}_0(\rho)$. Similarly, the resource tax is an increasing function $\theta^*_{st} = \theta^*_{st}(\rho)$. These functions satisfy both the equality $\widetilde{\eta}_0(\rho) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(\rho)$ and the fact that $\widetilde{\eta}_0(\rho) + \rho$ is independent of ρ.

In fact, this is true within an admissible range for ρ. Indeed, the subsidy must not exceed the threshold level above which it would not be necessary for the government to leave firms a rent during the extraction phase. The threshold can be determined as follows. The unit after-tax extraction rent induced by the optimal policy is $\widetilde{\eta}_0(\rho) + \rho = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(\rho) + \rho = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0)$. Therefore, the condition ensuring that the after-tax rent $\widetilde{\eta}_0$ remains strictly positive is

$$\rho < \bar{\rho} \equiv \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0), \tag{27}$$

where $\widetilde{S}_0$ must satisfy (20), or $\mathcal{S}^{-1}(\widetilde{S}_0) = \overline{\eta}_0(\widetilde{S}_0) - \theta^*_{s0}(0) = \bar{\rho}$.

Proposition 4 *(Tax-subsidy mix) For $0 \leq \rho \leq \bar{\rho}$, the optimal initial reserve level and the optimal extraction profile are independent of the combination of tax and subsidy by which they are induced.*

An immediate corollary is that subsidies are not necessary to achieve the optimum if the government can commit to extraction taxes that leave sufficient rents to extractors; vice versa commitment is not necessary if the government is willing to subsidize sufficiently, at $\rho = \bar{\rho}$. This subsidy level corresponds to the special case of Section 2 taken with initial reserves at $\widetilde{S}_0$. By Proposition 2, the tax is then given by (19), where $\mu = \lambda\bar{\rho}$ according to (25). Thus the observed variety in nonrenewable resource taxation systems is compatible with optimal Ramsey taxation. To the extent that commitment is not costly, the government is financially indifferent between the proportion of ex ante subsidies and

[30] Again, clearly, the subsidy must be low enough to necessitate the presence of after-tax rents at the extraction stage. More on this below.

ex post rents left to extracting firms in order to finance exploration and development expenditures.

3.2 Distortion to Reserve Production under Optimal Nonrenewable Resource Taxation

The Ramsey tax distortion to a nonrenewable resource should not only be evaluated in terms of distortion to current resource extraction. It must simultaneously be appraised in terms of the distortion that it causes to initial reserves.

Indeed, the OCT analysis of a nonrenewable resource sharply contrasts with the usual interpretation of Ramsey taxation: Were the stock of reserves given, as assumed in Section 2, an optimal resource tax may leave its path of extraction undisturbed even when government revenue needs call for distortions in other sectors of the economy. This is true, more precisely, under Stiglitz' (1976) special case of an isoelastic demand and zero extraction cost.[31] In general, however, resource taxation affects the development of reserves to be exploited.

When reserves are endogenous, given the subsidy, initial reserves are determined by the optimal level of the unit after-tax rent $\widetilde{\eta}_0$ via (20). Since $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ and $\widetilde{q}_{st} = c_s + \widetilde{\eta}_t + \theta^*_{st}$, $\widetilde{\eta}_0$ also affects the optimal tax rate given by (26). However, it is very difficult in general to isolate its effect because there is a continuum of relationships such as (26)—one at each date—and it is their combined influence over the whole extraction period that determines initial reserves. An exception is the special case of Stiglitz (1976): With an isoelastic demand and zero extraction cost, the optimal tax does not cause any distortion to the extraction profile of a given stock of reserves.

Since the Ramsey distortion on the extraction profile has been examined in Section 2, under the assumption of fixed reserves, now consider the distortion to initial reserves. Stiglitz' (1976) case, just discussed, provides the ideal laboratory for this analysis.

When the tax is neutral at given initial reserves, it grows at the rate of discount, so that the optimal tax can be characterized at any date by its initial level and alternative

[31] As underlined by Stiglitz (1976) in his analysis of monopoly pricing in the Hotelling model, confronted with the dilemma of raising the price at some date while increasing supply at some other date, a zero-cost monopoly facing an isoelastic demand ends up choosing the same price as would prevail under competitive equilibrium given the same amount of initial reserves. Under the same cost and demand conditions the Ramsey tax will not distort the resource extraction profile for the same reason.

tax profiles can be compared by comparing initial levels. A higher initial tax level implies a lower after-tax rent to firms, which implies lower initial reserves by (20). In the spirit of Ramsey taxation, one would then expect the optimal initial tax to be inversely related to supply elasticity. This is precisely the message of the following expression established in Appendix L for the optimal long-run resource tax rate:

$$\frac{\theta_{s0}^*}{\widetilde{q}_{s0}} = \frac{\rho}{\widetilde{q}_{s0}} + \frac{\lambda - 1}{\lambda} \left[\frac{1 - \frac{\theta_{s0}^*}{\widetilde{q}_{s0}}}{\widetilde{\zeta}} - \frac{1}{\widetilde{\xi}} \right], \tag{28}$$

where $\widetilde{\zeta} \equiv \frac{\widetilde{\eta}_0}{\widetilde{S}_0 \mathcal{S}^{-1\prime}(.)}$ is the *long-term* elasticity of reserve supply measured using (20) at the resource scarcity rent induced by the tax at the beginning of extraction; and where $\widetilde{\xi} \equiv \left(\frac{d\widetilde{\mathcal{D}}}{dq_{s0}} \right) \frac{\widetilde{q}_{s0}}{\widetilde{\mathcal{D}}}$ is the elasticity of the *cumulative* demand for the resource $\widetilde{\mathcal{D}} \equiv \int_0^{+\infty} D_s(\widetilde{q}_{st})\, dt$ with respect to the initial price q_{s0}, measured over the path of equilibrium prices $\{\widetilde{q}_{st}\}_{t \geq 0}$ induced by the optimal tax.

Keeping in mind that by Proposition 4, the optimal resource tax adjusts to changes in the reserve subsidy in such a way that optimal initial reserves are the same for any admissible value of ρ, let us again assume that $\rho = 0$. Then (28) looks similar to the well-known expression for the optimal rate of tax that applies to conventional goods whose supply is not perfectly elastic.[32] Its interpretation is also standard: Tax more when elasticity is lower, whether the source of elasticity is on the supply or demand side. Hence, to the extent that the supply of conventional commodities is more elastic than the supply of reserves $\left(\widetilde{\epsilon}_i > \widetilde{\zeta}\right)$, (28) implies that the resource is taxed at a higher rate than commodities of identical demand elasticity. There is an important difference, however, between the nonrenewable resource and conventional goods or services, having to do with the notions of elasticities involved.

Indeed, in (28), supply elasticity $\widetilde{\zeta}$ measures the long-run adjustment of the stock of initial reserves relative to the percentage change in the unit producer rent. This *stock* elasticity depends on how sensitive exploration is to the rent. In usual formulas of the inverse elasticity rule applying to commodities whose supply is not perfectly elastic, the concept of supply elasticity is standard; it measures the instantaneous percentage change

[32] This expression involves the sum of the reciprocals of demand and supply elasticities. See, for instance, Expression (11) in Ramsey (1927); for a formula derived under our notations, see (F.3) in Appendix F.

in (the *flow* of) production relative to the percentage change in the unit producer price.[33]

Similarly, while the elasticity of demand is the standard flow notion in (13), its counterpart in (28) is defined as the elasticity of cumulative resource demand—over the whole extraction period—with respect to the initial resource price. In the current special case, the long-run elasticity of cumulative demand is the same as the standard flow demand elasticity: $\widetilde{\xi} = \widetilde{\varepsilon}_s$.

Results are gathered in the following proposition.

Proposition 5 *(Time profile and initial reserves) When the supply of reserves is elastic and subsidized at the unit rate $\rho \geq 0$,*

1. *The Ramsey tax profile described by (26) implies distortions in both the time profile of extraction and the level of initial reserves;*

2. *When the demand for the nonrenewable resource is isoelastic and the extraction cost is zero, the optimal extraction tax is neutral with respect to the time profile of extraction and only affects the level of initial reserves. In that case, the optimal tax rate is given by (28), a rule resembling that for conventional goods and services whose supply is not perfectly elastic.*

When reserves are endogenous, distortions are unavoidable, whatever the revenue needs of the government; less reserves are to be developed. Again, if Pigovian policies were in place that penalized the carbon-emitting use of nonrenewable resources, the Ramsey tax would go in the same direction as, and would reinforce, the Pigovian tax.

The analogy underlined in Section 2 between Ramsey taxation and monopoly pricing when reserves are exogenous is thus even more pronounced when reserves are endogenous. Whether one considers the tax on the production flow of conventional goods or the extraction flow of a resource as in (13) and (26), or the long-run formula (28), the optimal tax rate approaches a monopoly mark-up as the factor $\frac{\lambda-1}{\lambda}$ approaches unity, i.e., when the government's revenue needs are at their highest.

[33]If the supply elasticity of a conventional good is finite, it must be the case that some input, e.g., the stock of capital, does not fully adjust to price and tax changes, which implies decreasing returns to scale. For a nonrenewable resource, the increasing scarcity of exploration prospects makes decreasing returns unavoidable in the long run.

4 A Numerical Example: OCT of Oil

Whether reserves are taken as given or are endogenous, the supply of a nonrenewable resource is never infinite. As we just demonstrated, when reserves are endogenous, OCT affects both the level of discovered reserves and the flow of extraction they constrain. We have illustrated how distortion on the latter vanishes when demand is isoelastic and the unit extraction cost is zero. In general, however, the tax described by Formula (26) causes both distortions.

The numerical example we provide now for the case of fossil oil not only illustrates the yield of the tax, but the distortions it implies on extraction and reserve development. The application follows the steps explained in Section 3 to resolve the OCT problem. Formula (26) gives the optimal resource tax as a function of the gross resource price

$$\widetilde{q}_{st} = c_s + \widetilde{\eta}_0 e^{rt} + \theta^*_{st}, \tag{29}$$

where the unit present-value rent $\widetilde{\eta}_0$ solves the ex ante problem (23)-(24). By Proposition 4, distortions in extraction over time and in the level of reserves discovered are independent of the repartition of the taxes or subsidies between the exploration phase and the extraction period. Thus, we can use Formula (26) without loss of generality while assuming that $\rho = 0$. Under that assumption, tax revenues and subsidies are confined to the extraction phase, so that (26) accounts for the full extent of Ramsey taxation applied to fossil oil over both the exploration phase and extraction.[34]

The solution is obtained in two steps: First, solve the system that consists of equations (26) and (29) for the trajectories of θ^*_{st} and $\widetilde{q}_{st}$ while treating $\widetilde{\eta}_0$ as parametric. Then, use the level of $\widetilde{S}_0$ from (20) and the trajectory of $\widetilde{x}_t$ implied by $\widetilde{q}_{st}$ via the demand function to establish the value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt$ maximized with respect to Θ given $\widetilde{\eta}_0$ in the ex ante problem (23). Finish the maximization by choosing $\widetilde{\eta}_0$.

Such an application implies a further simplification: treating the unit cost λ of levying \$1 of tax revenues as exogenous rather than endogenous, as in the full analysis of Section 3. This allows our application to focus on the resource sector; indeed, λ summarizes the extent to which the resource sector must contribute to public finance needs, taking into

[34]The analysis has been confined to a closed economy. However, the highlighted principles remain valid in an open economy, as shown in Appendix N and as explained in Section 6.

account the contribution of the other sectors.

We specify the model as follows. Values are in US dollars ($) of 2015. The short-run extraction cost c_s is assumed to be $35 per barrel of oil; this is the total cost—including development cost—of producing the first barrel developed (e.g., van der Ploeg and Rezai, 2017). Resource costs rise in the long run because the marginal cost of developing additional reserves increases: We assume a reserve supply function $\mathcal{S}(.)$ of constant elasticity $\zeta = 0.5$. Similarly, we assume an isoelastic demand function $D_s(.)$ with $\varepsilon_s = -0.5$.[35] Both elasticities are chosen to be relatively high in absolute values so that the Ramsey resource tax tends to be underestimated; compare with the long-run estimates for the demand and reserve elasticities suggested by Krichene (2005) and Hamilton (2009b). We use a middle of the road value of 3% for the discount rate (Nordhaus, 2014). As for the social cost of $1 of public funds, values of $1.1 to $1.2 are considered sensibly low for both developed and developing countries (e.g., Dahlby, 2008, and Auriol and Warlters, 2012): We consider $\lambda = 1$, which represents the baseline case in which no distortionary taxes are needed, as well as $\lambda = 1.1$ and $\lambda = 1.2$.

Given these parameter values, computations are carried out yearly over a period of 100 years, and the calibration is performed by choosing the shift terms K_D and K_S of the demand and reserve supply functions. Therefore, the demand for fossil oil is $\widetilde{x}_{st} = K_D \widetilde{q}_{st}^{-0.5}$ and the supply of reserves is $S_0 = K_S \eta_0^{0.5}$. By choosing $K_D = 260$ and $K_S = 500$, we obtain that the world extraction rate and price are, respectively, slightly below 35 billion barrels (BB) and slightly below $57 a barrel in 2015 when $\lambda = 1$, i.e., in the absence of any Ramsey taxation. The corresponding cumulative extraction of $2,327$ BB over 100 years exceeds the current proven world oil reserves of 1,662 BB by about 40%.[36]

In Table 1, Column $\lambda = 1$ represents business as usual, as no Ramsey tax is then necessary. The 2015 computed extraction level and producer price (gross of other taxes) approximately match observed values in 2015, while the total reserves to be exploited are sensibly higher than official proven reserves. When the cost of $1 of government funds is

[35] Under these specifications, the finiteness of the time horizon—inherent in the simulation exercise—allows us to eliminate some arbitrarily small strictly positive extraction levels from the integration of the consumer surplus. This avoids the objective in (23) taking an infinite value.

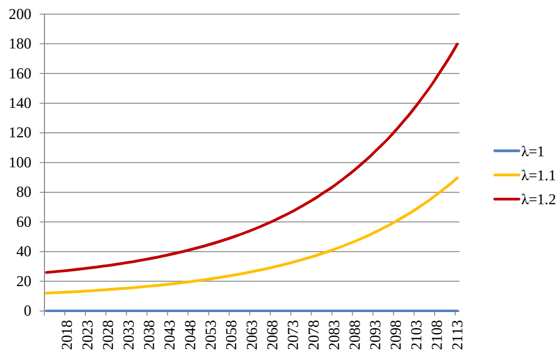
[36] This might be considered conservative, given that fear of running out of reserves has been proven wrong in the past repeatedly. However, mean estimations of undiscovered reserves by the US Geological Survey (Schenk, 2012) were 565 BB in 2012, i.e., 38% of proven reserves that year.

$\lambda = 1.1$, the Ramsey tax is set at \$12 in 2015 so that the producer price is driven to \$54 instead of \$57, causing extraction to be reduced from 35 BB to 32 BB that year. The corresponding tax yield is \$385 billion. If the US were to collect a share of that yield corresponding to its 20% share of world oil consumption, this would correspond to roughly 17% of its \$440 billion 2015 federal deficit. At the world level, cumulative discounted tax revenues over the horizon of 100 years would amount to about \$16,000 billion.

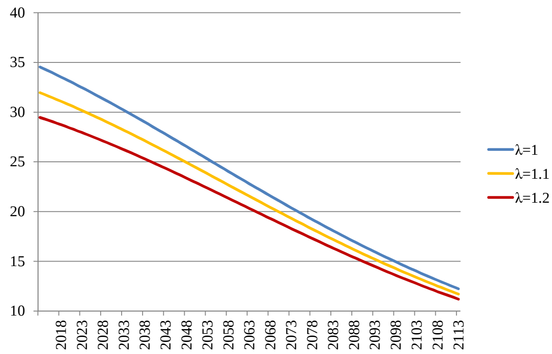
Table 1: 2015 OCT of oil

Cost of \$1 of tax revenues	$\lambda = 1$	$\lambda = 1.1$	$\lambda = 1.2$
2015 Unit optimal tax (\$)	0	12	26
2015 Extraction rate (BB)	35	32	29
2015 Producer price (\$)	57	54	52
2015 Tax yield (\$ billion)	0	385	765
Total exploited reserves (BB)	2,327	2,189	2,055
Total cumulative discounted tax yield (\$ billion)	0	16,002	31,352

Table 1 gives a static reading of the magnitude and impact of the Ramsey oil tax on 2015 variables for alternative levels of λ. It also indicates effects on total exploited reserves and cumulative tax yield. Dynamic aspects are presented in the two graphs of Figure 1, respectively giving trajectories of the Ramsey oil tax and the corresponding extraction paths.



(a) OCT unit tax of oil



(b) OCT-induced oil extraction

Figure 1: Dynamics of oil OCT

In Figure 1, the extraction trajectories implied by the OCT of the fossil oil resource (right-hand graph) become both flatter and lower as λ increases. Since they do not cross, this also means that exploited reserves become lower with λ, as can be verified in Table 1: Compared with their level of 2,327 BB in the absence of Ramsey taxation, initial reserves diminish to 2,189 BB; for a more financially constrained government ($\lambda = 1.2$), developed and exploited reserves further diminish to 2,055 BB. This illustrates an important result established in Section 3: The need to collect tax revenues requires slower extraction of lower reserves.

Since the climate impact of using carbon resources increases both with the total amount of reserves to be exploited and the speed at which those reserves are consumed, the Ramsey taxation objective of collecting public funds also serves the objective of fighting climate change. As mentioned in the introduction, the Ramsey tax may be imposed on top of other taxes. In the next section, we show how the presence of a carbon tax, as in Nordhaus (2014), modifies the Ramsey tax and the industry.

5 OCT-augmented Carbon Taxation and Numerical Implications for Oil

This section draws the theoretical and numerical implications of our results for the taxation of carbon-emitting nonrenewable resources. Nonrenewable resources are sources of carbon emissions and should be subject to Pigovian taxation to correct the climate externality they generate. Whatever the actual Pigovian tax on a resource—whether it is appropriate or not—our results can directly be used to indicate the OCT tax that should augment it in the presence of tax revenue needs.

Introduce such a carbon tax in the analysis of Section 3. Assume, for simplicity, that it is set at each date t at an exogenous level τ_{st} per unit of resource consumed. In this case, it is straightforward to see that Formula (26) is unchanged, although it now applies to the higher gross resource price

$$\widetilde{q}_{st} = c_s + \tau_{st} + \widetilde{\eta}_0 e^{rt} + \theta^*_{st}, \tag{30}$$

which now includes the carbon tax, unlike (29) in Section 3 and in the application of Section 4. The relevant resource price $\widetilde{q}_{st}$ being adjusted in this way, the obtained level of the OCT tax θ^*_{st} establishes by how much the carbon tax τ_{st} should be augmented, without any reinterpretation of the model.

This simple application is consistent with the famous "additionality property" highlighted by Sandmo (1975) in his analysis of OCT with explicit externality-generating commodities. In Sandmo's paper, the OCT and externality taxation problems are resolved simultaneously, unlike the above simplifying assumption that the Pigovian tax component is imposed at the outset, while the OCT component endogenously adjusts to it. In general, Sandmo (1975) shows—and Kopczuk (2003) confirms—that in the presence of public-revenue requirements, Pigovian taxes should simply be augmented by a component that corresponds to the Ramsey tax. This result has an important and convenient implication: Establishing by how much carbon taxation should be augmented in the presence of revenue needs does not require treating the externality issue explicitly.

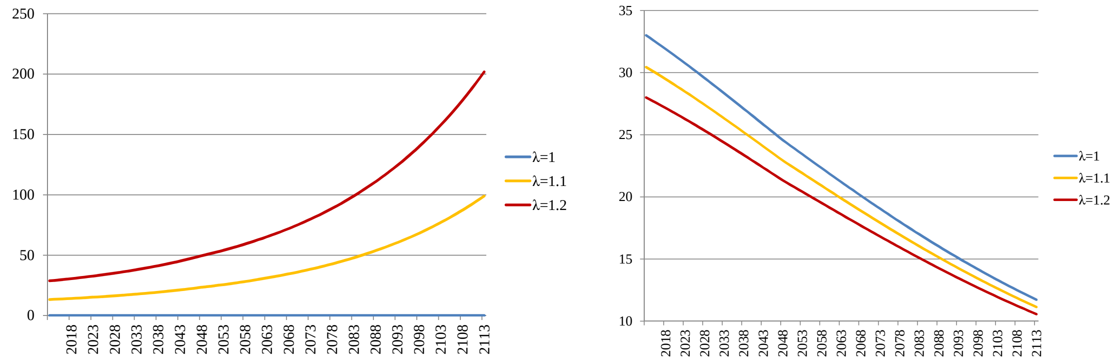
Therefore, one can conclude from our previous results that the Ramsey resource tax causes a distortion to the extraction of carbon resources that goes further than the Pigovian tax in the direction prescribed for the resolution of carbon externalities: The reserves are lower, and so is the speed at which they are exploited.

In the numerical example in Section 4, assume that the Ramsey tax is imposed on top of a carbon tax. The carbon tax is taken from Nordhaus (2014), starting in 2015 at about \$22 per ton of CO2 equivalent or about \$8 per barrel, rising in real terms at an annual rate of 3.1% until 2050, and rising at 2.1% thereafter. Table 2 and Figure 2 show how the results carry over in that case. For $\lambda = 1.1$, the Ramsey tax is set at \$13 and the induced extraction rate is 30 BB, lower than if the Ramsey tax were alone. As a result, its yield is lower than if there were no carbon tax. The yield of the carbon tax is also lower than in the absence of a Ramsey tax, and the more so the higher λ. Nevertheless, the joint yield of the two taxes is higher than if either of them were alone. Discoveries are also lower than if either of the two taxes were present in isolation. Clearly, both contribute to the objectives of increasing revenue and protecting the climate.

To sum up, public financial hardness does not need to obscure or delay environmental decisions; on the contrary, it calls for policies that go even further than correcting externalities.

Table 2: 2015 OCT of oil on top of a carbon tax

Cost of \$1 of tax revenues	$\lambda = 1$	$\lambda = 1.1$	$\lambda = 1.2$
2015 Unit carbon tax (\$)	8	8	8
2015 Unit optimal tax (\$)	0	13	29
2015 Extraction rate (BB)	33	30	28
2015 Producer price (\$)	54	52	50
2015 Tax yield from carbon tax (\$ billion)	264	243	223
2015 Tax yield from Ramsey tax (\$ billion)	0	404	805
Total exploited reserves (BB)	2,185	2,043	1,905



(a) OCT unit tax of oil

(b) OCT-induced oil extraction

Figure 2: Dynamics of oil OCT on top of a carbon tax

6 Extensions: Nonrenewable Resource Substitutes and Complements, and the Capture of Foreign Resource Rents

Our analysis purposely focuses on the most fundamental aspects of our theory: the indirect taxation of a nonrenewable resource which is endogenously developed and exploited over time. Two main aspects are omitted in our model and deserve further discussion: first, the presence and, therefore, taxation of nonrenewable resource substitutes and complements; second, the taxation of nonrenewable resources in an open economy. Each of them are examined in extensions presented in the appendix, which are summarized in this section.

6.1 Nonrenewable Resource Substitutes and Complements

In Appendix G, we extend the analysis of Section 2 by introducing non-zero cross-price demand elasticities across commodities. The nonrenewable resource should still be taxed at a higher rate than otherwise-identical commodities in that context. However, substitutes for the resource should receive a special tax treatment as such: Precisely, resource substitutes should be taxed at lower rates than commodities that are substitutes for non-resource commodities; vice versa complements to the resource should be taxed at higher rates.

The rationale behind the special tax treatment of substitutes or complements is clear. On the one hand, a higher mark-up on any good positively affects the demand for substitutes, thus their tax base. On the other hand, this effect is less pronounced when the impacted substitute or complement is a nonrenewable resource. Indeed, the tax on a resource substitute shifts demand towards a sector with a relatively inelastic supply (the nonrenewable resource), unlike the tax on other conventional goods.

This specific treatment differs from the treatment of substitutes for, or complements of, externality-generating commodities in the absence of government financial restrictions. In Sandmo (1975), the "marginal social damage . . . does not enter the formulas for the other [non externality-creating] commodities, regardless of the pattern of complementarity and substitutability" (p. 92). In contrast, financially constrained governments should tax substitutes to fossil nonrenewable resources more lightly, and complements more heavily, than commodities unrelated to nonrenewable resources.

The analysis further indicates that the optimal tax on the substitutes for, or complements to, a nonrenewable resource depends on time, unlike the optimal tax on conventional commodities without links to a resource. The link with the resource confers a dynamic dimension to the tax on its substitutes and complements. More precisely, the tax on a nonrenewable resource substitute should fall as the resource scarcity increases; vice versa the tax on a resource complement should rise.

6.2 Ramsey Taxes and the Capture of Foreign Resource Rents

More often than not, Ramsey's commodity taxes are applied domestically to an open economy. This has two immediate implications for OCT. First, taxes applied on demand and supply are not equivalent; consequently distinct optimal taxes must be chosen for the domestic supply and demand of each traded good or service, rather than one tax applying indifferently to demand or supply. Second, prices are formed on international markets, so that the effects of commodity taxes on prices are diluted.[37] The limited elasticity of world resource supply makes these effects all the more relevant.

It is well known that the combination of domestic demand taxes and domestic supply subsidies plays the role of a tariff (Friedlander and Vandendorpe, 1968; Dornbusch, 1971). When these instruments can impact international prices, commodity taxes are then capable of pursuing both a tax revenue objective and a rent capture objective as optimal tariffs do.[38] The rent capture objective has been addressed in a rich literature on optimal tariffs—see, among others in the resource taxation literature, Bergstrom's (1982) paper on the capture of foreign nonrenewable resource rents.

In Appendix N, we extend Section 3's analysis of OCT with endogenous resource reserves to the case of an open economy. We formally derive the open-economy counterparts of the tax formulas (26) and (28), and we reexamine optimal distortions on resource extraction and reserve production.

The main results are the following. The distinction between low and high revenue needs emerges in an open economy with endogenous reserves as it does in a closed economy with exogenous reserves (Section 2). Revenue needs are low or high according to whether government needs are covered or not by the amount raised when resource taxes are set so as to maximize welfare in absence of tax-revenue constraint, that is with the sole purpose of capturing foreign nonrenewable resource rents.

When revenue needs are low in the sense defined above, our open-economy OCT rule for nonrenewable resources extends Bergstrom's (1982) analysis to the case of endogenous reserves but does not modify his message: Importing countries should tax resource consumption and exporting countries should subsidize resource consumption. As he showed for fixed reserves, the two-country Nash equilibrium is then such that importing countries capture some foreign resource rents while exporting countries limit the cut to their rents by subsidizing.

[37] The literature on resource oligopolies and oligopsonies is relevant to the problem of OCT in an open economy. According to Karp and Newbery (1991) "the evidence for potential market power on the side of importers is arguably as strong as for oil exporters" (p. 305); the more so when suppliers and/or buyers act in concert as suggested by Bergstrom (1982). On market power on the demand side, see also Liski and Montero (2011).

[38] The OCT problem and the optimal tariff problem differ only by the constraint for the government to collect a minimum revenue. The latter characterizes an unconstrained optimum while optimal commodity taxes are distortionary: As Boadway et al. (1973) put it "domestic commodity taxes *introduce* a distortion while optimal tariffs *eliminate* a distortion" (p. 397, their italics).

When revenue needs are high, the revenue collection constraint becomes active, and Bergstrom's rent capture tax component is complemented by the two components described in the inverse elasticity formula (28) for the closed economy with endogenous reserves.[39] For an importing country, the result that the nonrenewable resources should be taxed at a higher rate than commodities of identical demand elasticity comes reinforced because the three terms of the open-economy inverse elasticity formula push in the same direction. In the case of exporters, Bergstrom's component is negative and counteracts the traditional Ramsey components: Exporting countries may even subsidize resource consumption despite a pressing tax revenue constraint.

7 Conclusion

The standard Ramsey-Pigou framework used in this paper considers indirect, linear taxes or subsidies on any commodity or service. This includes linear subsidies to the production of natural resource reserves (exploration) as well as linear taxes on extraction and on consumption of the natural resource. In that framework, the objective of the government is to maximize the welfare of producers and consumers while securing a given level of revenues for the production of public goods.

Nonrenewable resources are a form of capital, while discoveries and extraction are forms of positive and negative investments. When reserves are produced endogenously, the situation is close to that analyzed in influential papers on capital taxation by Judd (1985) and Chamley (1986). In the long run, the development of capital relies on investment and investment becomes less profitable the more capital is taxed. In this context, Chamley and Judd find that the revenue from capital should not be taxed at all if the horizon of the government is long enough. The famous Chamley-Judd result obeys the standard OCT logic: The social cost of capital taxation over the long run is so high that it is impossible to evenly spread distortions across sectors while imposing a positive capital tax.

Our OCT analysis of nonrenewable resources, however, yields a very different result when capital is a natural resource rather than a conventional capital. Indeed, we find that a natural resource should be taxed even if the government has a long-run horizon, despite

[39] The definitions of the demand and supply elasticities must be adjusted to reflect the fact that there is a home and a foreign sector.

the fact that the supply of reserves is responsive to the tax. In fact, it should always be taxed at a higher rate than otherwise identical commodities.

When the supply of initial reserves is elastic and determined by the combination of after-tax rents to extraction and ex ante subsidies to reserve production, we establish an optimal dynamic Ramsey tax formula for nonrenewable resources. In the absence of any subsidies to the production of reserves and provided the government can commit to leaving after-tax rents to firms, the optimal tax rule for resource extraction resembles the inverse elasticity rule applying to conventional goods. This resemblance hides a crucial difference: Due to the dynamic nature of the extraction problem, a similar rule must hold at all dates during the extraction period, so that the distortion to extraction cannot be simply measured according to the tax applying at any particular date.

In general, our Ramsey tax formula for nonrenewable resources affects both reserves developed in the long run, which are reduced, and their depletion over time, which is slowed down. These distortions go in the same direction as those prescribed for resolution of the climate externality. Therefore, the Ramsey taxation objective of collecting public funds also serves the objective of fighting climate change. To sum up, public financial constraints do not need to obscure or delay environmental decisions; on the contrary, they call for policies that go even further than correcting externalities.

Another noticeable result is that, although the optimal extraction tax varies according to the reserve subsidy, the optimal amount of initial reserves and the optimal extraction path of these reserves do not depend on the extraction-tax reserve-subsidy combination. This means that the ability of governments to commit to a future tax system is irrelevant. For example, a government that were unable to commit to leaving positive after-tax rents to firms during the extraction period, could finance reserve production by subsidies exclusively and achieve the same objective as a government that were able to commit. This implies that Ramsey taxation is compatible with all observed institutional forms ranging from a nationalized industry, in which the entire reserve production effort is subsidized while the total surplus from extraction is taxed away, to a system in which firms finance reserve production and are paid back by future extraction rents.

As a matter of fact, most governments are financially constrained, often severely. The figures given in the numerical example of Section 4 suggest that this reality has significant implications for both the level of taxation of nonrenewable resources and government finances. In particular, our results can directly be used to indicate the OCT tax that should augment Pigovian taxation of externality-generating carbon resources in the presence of tax revenue needs. The numerical example of Section 5 illustrates the implications of our analysis for the case of oil. Besides significant effects on public revenues, the application shows how an OCT-augmented carbon tax goes further in the direction of opposing the carbon externality.

Besides public revenue needs and externalities, the instruments considered in this paper have in principle the ability to correct potential market power distortions. Although monopoly power is a common aspect of nonrenewable resource markets, our analysis, for the sake of brevity, has focused on the benchmark case of perfect competition, avoiding the issue of dynamic strategic interactions between resource suppliers and taxing authorities. Clearly, OCT of natural resources under more realistic market structures that take account of suppliers' influence on markets is an exciting and promising field for future research.[40]

[40] Assuming a conventional nonrenewable resource monopoly (e.g., Stiglitz, 1976), Bergstrom, Cross, and Porter's (1981) analysis suggests that optimal taxation should induce a faster exploitation of fixed reserves—see also Gaudet (2007). With endogenous reserves, Gaudet and Lasserre (1988) showed that optimal taxation should further encourage markets to develop more reserves. Under these conditions, one might suppose that the extension of resources' OCT to a monopoly be relatively intuitive. However, little is known about optimal taxation in the face of a limit-pricing cartel—see, for example, Andrade de Sá and Daubanes (2016).

References

Alberta Royalty Review (2007), "Final Report to the Finance Minister," Government of Alberta, Edmonton
http://www.albertaroyaltyreview.ca/panel/final_report.pdf.

Andrade de Sá, S., and J.X. Daubanes (2016), "Limit Pricing and the (In)Effectiveness of the Carbon Tax," *Journal of Public Economics*, 139: 28-39.

Atkinson, A.B., and J.E. Stiglitz (1976), "The Design of Tax Structure: Direct versus Indirect Taxation," *Journal of Public Economics*, 6: 55-75.

Auerbach, A.J. (1985), "The Theory of Excess Burden and Optimal Taxation," in: Auerbach, A.J., and M. Feldstein (Eds.), *Handbook of Public Economics*, 1: 61-127, Elsevier.

Auriol, E., and M. Warlters (2012), "The Marginal Cost of Public Funds and Tax Reform in Africa," *Journal of Development Economics*, 97: 58-72.

Barrage, L. (2017), "Optimal Dynamic Carbon Taxes in a Climate-Economy Model with Distortionary Fiscal Policy," *Review of Economic Studies*, forthcoming.

Baumol, W.J., and D.F. Bradford (1970), "Optimal Departures From Marginal Cost Pricing," *American Economic Review*, 60: 265-283.

Belan, P., S. Gauthier and G. Laroque (2008), "Optimal Grouping of Commodities for Indirect Taxation," *Journal of Public Economics*, 92: 1738-1750.

Bergstrom, T.C. (1982), "On Capturing Oil Rents with a National Excise Tax," *American Economic Review*, 72: 194-201.

Bergstrom, T.C., J.G. Cross and R.C. Porter (1981), "Efficiency-Inducing Taxation for a Monopolistically Supplied Depletable Resource," *Journal of Public Economics*, 15: 23-32.

Berndt, E.R., and D.O. Wood (1975), "Technology, Prices, and the Derived Demand for Energy," *Review of Economics and Statistics*, 57: 259-268.

Boadway, R.W., and M. Keen (2010), "Theoretical Perspectives on Resource Tax Design," in: Daniel, P., M. Keen and C.P. McPherson (Eds.), *The Taxation of Petroleum and Minerals: Principles, Problems and Practice*, 13-74, Routledge.

Boadway, R.W., and M. Keen (2015), "Rent Taxes and Royalties in Designing Fiscal Regimes for Nonrenewable Resources," in: Halvorsen, R, and D.F. Layton (Eds.), *Handbook on the Economics of Natural Resources*, 97-139, Edward Elgar.

Boadway, R.W., S. Maital and M. Prachowny (1973), "Optimal Tariffs, Optimal Taxes and Public Goods," *Journal of Public Economics*, 2: 391-403.

Boiteux, M. (1956), "Sur la Gestion des Monopoles Publics Astreints à l'Equilibre Budgétaire," *Econometrica*, 24: 22-40.

Bovenberg, A.L., and R.A. de Mooij (1994), "Environmental Levies and Distortionary Taxation," *American Economic Review*, 84: 1085-1089.

Burness, H.S. (1976), “On the Taxation of Nonreplenishable Natural Resources,” *Journal of Environmental Economics and Management*, 3: 289-311.

Chakravorty, U., M. Moreaux and M. Tidball (2008), “Ordering the Extraction of Polluting Nonrenewable Resources,” *American Economic Review*, 98: 1128-1144.

Chamley, C. (1986), “Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives,” *Econometrica*, 54: 607-622.

Cremer, H., and F. Gahvari (2004), “Second-Best Taxation of Emissions and Polluting Goods,” *Journal of Public Economics*, 80: 169-197.

Dahlby, B. (2008), *The Marginal Cost of Public Funds: Theory and Applications*, The MIT Press.

Daniel, P., M. Keen and C.P. McPherson (2010), *The Taxation of Petroleum and Minerals: Principles, Problems and Practice*, Routledge.

Dasgupta, P.S., G.M. Heal and J.E. Stiglitz (1981), “The Taxation of Exhaustible Resources,” NBER Working Papers, 436.

Daubanes, J., and S. Andrade de Sá (2014), “Taxing the Rent of Non-Renewable Resource Sectors: A Theoretical Note,” OECD Working Papers, 1149.

Diamond, P.A. (1975), “A Many-Person Ramsey Tax Rule,” *Journal of Public Economics*, 4: 335-342.

Diamond, P.A., and J.A. Mirrlees (1971), “Optimal Taxation and Public Production I: Production Efficiency,” *American Economic Review*, 61: 8-27.

Dornbusch, R. (1971), “Optimal Commodity and Trade Taxes,” *Journal of Political Economy*, 79: 1360-1368.

Dullieux, R., L. Ragot and K. Schubert (2011), “Carbon Tax and OPEC’s Rents Under a Ceiling Constraint,” *Scandinavian Journal of Economics*, 113: 798-824.

Energy Charter Secretariat (2008), “Taxation along the Oil and Gas Supply Chain” `https://energycharter.org/fileadmin/DocumentsMedia/Thematic/Taxation_Study_2008_en.pdf`.

Fischer, C., and R. Laxminarayan (2005), “Sequential Development and Exploitation of an Exhaustible Resource: Do Monopoly Rights Promote Conservation?,” *Journal of Environmental Economics and Management*, 49: 500-515.

Fischer, C., and S.W. Salant (2017), “Balancing the Carbon Budget for Oil: The Distributive Effects of Alternative Policies,” *European Economic Review*, 99: 191-215.

Friedlander, A.F., and A.L. Vandendorpe (1968), “Excise Taxes and the Gains from Trade,” *Journal of Political Economy*, 76: 1058-1068.

Fullerton, D. (1997), “Environmental Levies and Distortionary Taxation: Comment,” *American Economic Review*, 87: 245-251.

Gaudet, G., and P. Lasserre (1988), "On Comparing Monopoly and Competition in Exhaustible Resource Exploitation," *Journal of Environmental Economics and Management*, 15: 412-418.

Gaudet, G., and P. Lasserre (2015), "The Taxation of Nonrenewable Natural Resources," in: Halvorsen, R, and D.F. Layton (Eds.), *Handbook on the Economics of Natural Resources*, 66-96, Edward Elgar.

Gaudet, G. (2007), "Natural Resource Economics under the Rule of Hotelling," *Canadian Journal of Economics*, 40: 1033-1059.

Golosov, M., J. Hassler, P. Krusell and A. Tsyvinski (2014), "Optimal Taxes on Fossil Fuel in General Equilibrium," *Econometrica*, 82: 41-88.

Hamilton, J.D. (2009a), "Causes and Consequences of the Oil Shock of 2007-08," *Brookings Papers on Economic Activity*, 40: 215-261.

Hamilton, J.D. (2009b), "Understanding Crude Oil Prices," *Energy Journal*, 30: 179-206.

Hogan, L. (2008), "International Minerals Taxation: Experiences and Issues," ABARE Conference Papers 08/11.

Hotelling, H. (1931), "The Economics of Exhaustible Resources," *Journal of Political Economy*, 39: 137-175.

Judd, K.L. (1985), "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics*, 28: 59-83.

Karp, L., and D.M. Newbery (1991), "OPEC and the U.S. Oil Import Tariff," *Economic Journal*, 101: 303-313.

Kopczuk, W. (2003), "A Note on Optimal Taxation in the Presence of Externalities," *Economics Letters*, 80: 81-86.

Krichene, N. (2005), "A Simultaneous Equations Model for World Crude Oil and Natural Gas Markets," IMF Working Papers 05/32.

Lewis, T.R., S.A. Matthews and H.S. Burness (1979), "Monopoly and the Rate of Extraction of Exhaustible Resources: Note," *American Economic Review*, 69: 227-230.

Liski, M., and J.-P. Montero (2011), "On the Exhaustible-Resource Monopsony," mimeo, Aalto University.

Liski, M., and O. Tahvonen (2004), "Can Carbon Tax eat OPEC's Rents?," *Journal of Environmental Economics and Management*, 47: 1-12.

Long, N.V., and H.-W. Sinn (1985), "Surprise Price Shifts, Tax Changes and the Supply Behaviour of Resource Extracting Firms," *Australian Economic Papers*, 24: 278-289.

Lucas, R.E. (1990), "Supply-Side Economics: An Analytical Review," *Oxford Economic Papers*, 42: 293-316.

Nakhle, C. (2010), "Petroleum Fiscal Regimes: Evolution and challenges," in: Daniel, P., M. Keen and C.P. McPherson (Eds.), *The Taxation of Petroleum and Minerals: Principles, Problems and Practice*, 89-121, Routledge.

Newbery, D.M. (2005), "Why Tax Energy? Towards a More Rational Policy," *Energy Journal*, 26: 1-39.

Nordhaus, W.D. (2014), "Estimates of the Social Cost of Carbon: Concepts and Results from the DICE-2013R Model and Alternative Approaches," *Journal of the Association of Environmental and Resource Economists*, 1: 273-312.

Nordhaus, D.M. (2008), *A Question of Balance: Weighing the Options on Global Warming Policies*, Yale University Press.

Pigou, A.C. (1947), *A Study in Public Finance*, Macmillan.

Pindyck, R.S. (1978), "The Optimal Exploration and Production of Nonrenewable Resources," *Journal of Political Economy*, 86: 841-861.

Pindyck, R.S. (1979), *The Structure of World Energy Demand*, MIT Press.

Pindyck, R.S. (1987), "On Monopoly Power in Extractive Resource Markets," *Journal of Environmental Economics and Management*, 14: 128-142.

van der Ploeg, F., and A. Rezai (2017), "Abandoning Fossil Fuel: How Fast and How Much?," *Manchester School*, forthcoming.

van der Ploeg, F., and C.A.A.M. Withagen (2012), "Is There Really a Green Paradox?," *Journal of Environmental Economics and Management*, 64: 342-363.

Quyen, N.V. (1988), "The Optimal Depletion and Exploration of a Nonrenewable Resource," *Econometrica*, 56: 1467-1471.

Ramsey, F.P. (1927), "A Contribution to the Theory of Taxation," *Economic Journal*, 37: 47-61.

Sandmo, A. (1975), "Optimal Taxation in the Presence of Externalities," *Swedish Journal of Economics*, 77: 86-98.

Schenk, C.J. (2012), "An Estimate of Undiscovered Conventional Oil and Gas Resources of the World," U.S. Geological Survey Fact Sheets, 2012-3042.

Simmons, P. (1977), "Optimal Taxation and Natural Resources," *Recherches Economiques de Louvain / Louvain Economic Review*, 43: 141-163.

Sinn, H.-W. (2008), "Public Policies Against Global Warming: A Supply Side Approach," *International Tax and Public Finance*, 15: 360-394.

Stiglitz, J.E. (1976), "Monopoly and the Rate of Extraction of Exhaustible Resources," *American Economic Review*, 66: 655-661.

Stiglitz, J.E. (2015), "In Praise of Frank Ramsey's Contribution to the Theory of Taxation," *Economic Journal*, 125: 235-268.

Stiglitz, J.E., and P.S. Dasgupta (1971), "Differential Taxation, Public Goods, and Economic Efficiency," *Review of Economic Studies*, 38: 151-174.

Tirole, J. (1988), *The Theory of Industrial Organization*, MIT Press.

Withagen, C.A.A.M. (1994), "Pollution and Exhaustibility of Fossil Fuels," *Resource and Energy Economics*, 16: 235-242.

Optimal Commodity Taxation with a Nonrenewable Resource:

Online Appendix (Literature, Proofs and Extensions)

by

Julien Daubanes
University of Geneva (GSEM), MIT (CEEPR), and CESifo

and

Pierre Lasserre
Université du Québec à Montréal (ESG), CIRANO and CIREQ

April 26, 2019

A More on the Related Literature

A.1 Direct Rent Taxation, and its Limitations, Especially in Nonrenewable Resource Sectors

Ramsey's original approach rules out the direct taxation of profits, rents, or incomes, which leaves it open to the criticism that it ignores the possibility of neutral taxation. Nevertheless, as Sandmo pointed out in 1976 ". . . it seems definitely sensible to admit the unrealism of the assumption that the public sector can raise all its revenue from neutral . . . taxes, and once we admit this we face the second-best problem of making the best of a necessarily distortionary tax system. This is the problem with which the optimal tax literature is mainly concerned." This view was shared by Stiglitz and Dasgupta (1971), who pointed out that "no government imposes 100% taxes on profits and the income of fixed factors, in spite of the desirability of such non-distortionary taxes."[41] Their remark applies more definitely nowadays: Most countries are struggling to meet debt and budget constraints in institutional environments in which profits or rents may be only partially taxed but in which no increase in the amount raised by profit taxes is considered feasible. According to a recent Ernst and Young's (2015) report, one major feature of the world tax landscape is that "Indirect taxes continue to grow while direct taxes stagnate." In this paper, whatever the amount of profits or rents that is taxed away, we assume that governments do not have enough control on direct profit or rent taxes to use them as instruments.

This limitation to the ability to use direct taxes is very apparent in nonrenewable resource sectors. On the demand side, resource taxes are almost exclusively linear commodity taxes. On the supply side, direct nonlinear taxes, such as the resource rent tax, have been advocated as non distortionary—see, e.g., Boadway and Flatters (1993), and Boadway and Keen (2010). However, they are contested on theoretical grounds—e.g., Gaudet and Lasserre (1986), and Garnaut (2010)—and meet strong opposition in practice, for reasons ranging from institutions (property rights) to feasibility (information and agency issues,

[41] According to Stiglitz and Dasgupta (1971), "two possible explanations for this limitation suggest themselves. (1) It is difficult if not impossible . . . to separate out pure profits from, say, income to capital, and few if any governments—or national income accountants—have even attempted the task. (2) In at least some western economies, where the rights of private property are considered to be very important, a 100% profits tax would be considered equivalent to nationalization of the fixed factors."

e.g., Boadway and Keen, 2014). In any case, royalties and other linear commodity taxes are dominant forms of resource taxation (see Daniel et al., 2010) and are the apparently most significant tax instruments available to governments nowadays.[42]

A.2 OCT Analysis in Presence of Untaxed Profits

While Diamond and Mirrlees (1971) showed that Ramsey's results held in a fully specified general equilibrium framework, OCT was dealt a serious theoretical blow by Atkinson and Stiglitz (1976) who showed that, under some conditions, direct income taxation suppresses any need for commodity taxes. The Atkinson-Stiglitz result does not hold unless profits are fully taxed or are absent because of constant returns to scale.[43] While constant returns to scale may be considered a useful simplification for economies limited to conventional goods, doing away with that assumption and with the assumption of 100% profit taxation is an empirical and theoretical necessity in the presence of nonrenewable resources:[44,45] Decreasing returns and the presence of untaxed rents are fundamental characteristics of nonrenewable resource sectors. In this context, as Stiglitz (2015) reminded us, commodity taxes have the ability to indirectly tap profits and rents when they would otherwise be left untaxed. Therefore, while we treat conventional sectors in line with much of the literature by assuming constant returns to scale, hence no profits, we make the rents arising in the resource sector explicit. These rents arise from the total or partial inelasticity of reserves. As is well known, when reserves are given, commodity taxes have the ability to tax resource rents in a neutral way. Therefore, the optimization of nonrenewable resource commodity

[42]For a good practical example of a relatively advanced system, see Alberta Royalty Review (2007, pp. 54-60).

[43]As a matter of fact, Corlett and Hague showed as early as 1953 that uniform commodity taxation may be optimal under specific conditions. As the Atkinson-Stiglitz theorem, this case of uniform OCT relies on the assumption that returns to scale are constant. Under otherwise similar conditions, decreasing returns to scale and the incomplete taxation of the resulting profits would justify differential commodity taxes along the lines explained by Ramsey (1927). Indeed, as recalled by Stiglitz (2015, p. 237), Corlett and Hague's analysis is a "a special application of Ramsey's analysis." For a derivation of Ramsey's and Corlett and Hague's results in the same framework of analysis, see Ley's (1992) note.

[44]This includes, a fortiori, models in which reserves are allowed to be endogenous as in this paper.

[45]Another parallel issue is taxes that interfere with productive efficiency such as taxes on intermediate inputs. In the case of resource-based final goods or services, this often occurs through linear technologies, that is in a given proportion of the final consumption. In such a situation, a linear tax on a resource input does not compromise efficiency and does not need to be distinguished from a linear tax on the resource-based final good or service; that distinction was avoided in the original partial equilibrium formulation of the OCT problem, an option further validated by Baumol and Bradford (1970). Stiglitz and Dasgupta (1971) further showed that production efficiency is not in general required in presence of untaxed profits or rents.

taxes in the model of this paper combines the proceeds of neutral rent taxation with the proceeds of possibly distortionary resource taxation into government tax revenues.

Ramsey's framework seems perfectly adapted to examine the fact that nonrenewable resources usually receive a special commodity tax treatment. That is, to the extent that Ramsey taxes can be set at different levels according to the commodity involved.[46] In our paper, a crucial assumption is that resources can be (linearly) taxed independently of other commodities. This assumption is satisfied by nonrenewable resource tax regimes; whether they are applied to the demand or supply sides, they are largely independent of the commodity taxes applied to conventional goods or clusters of conventional goods.

A.3 Taxation of Rents and Capitals

The renewed interest in the profit-capturing dimension of commodity taxes is pervasive in the double dividend literature and in the carbon taxation literature. Barrage (2017) shows that, absent 100% profits taxes, the optimal tax on carbon resources acquires a Ramsey component. The recent literature on capital taxation (e.g., Piketty, 2015) sees commodity taxation as an alternative to the direct taxation of wealth. As Auerbach and Hassett (2015) put it, consumption taxation has "the ability . . . to hit existing sources of wealth." In our paper, a similar logic applies; the dynamic formulation highlights the role of resource taxes as taxes on resource capital and rents.

As a matter of fact, Ramsey's framework explicitly rules out the direct taxation of capital income, whether in the form considered by Chamley (1986), or in a form mimicking profit taxation as with Lucas' (1990) capital levies, or via some form of resource rent taxes, as described by Boadway and Keen (2010). However, a nonrenewable resource is a form of capital and applying a commodity tax to resource extraction over time is not unlike taxing the income of that capital. We find that taxing resource extraction is optimal, in apparent contradiction with Chamley (1886), who shows that no tax should be applied on the income of capital in the long run. These results differ despite their similar OCT logic. The elasticity of supply again plays the central role, although in a way completely distinct from that in Piketty and Saez (2013).

[46]Ramsey himself had worried about possible restrictions to the set of linear taxes that can be imposed on various commodities; the extension of Ramsey's original work with limited groups of commodities was undertaken by Belan et al. (2008).

A.4 Taxation of Carbon Resources and their Substitutes

Given recent governmental commitments to penalize CO2 emissions generated by the use of fossil resources, a currently important application of our research concerns carbon taxation. Our formulas can directly be used to calculate by how much carbon taxation should be augmented when the regulator is budget constrained, as well as to qualify the distortion required by public revenue needs.

This is in line with Sandmo's (1975) analysis of commodity taxation in presence of externality-generating goods, which highlights the famous "additionality property:" In the presence of public-revenue requirements, Pigovian taxes should simply be augmented by a component that corresponds to the Ramsey tax. Recent applications of this property include, for example, Sandmo (2011), and d'Autume, Schubert and Withagen (2016). The robustness of Sandmo's result has been confirmed by Kopczuk (2003). It implies that establishing by how much carbon taxation should be augmented in presence of revenue needs does not require treating the externality issue explicitly.

We show that public policies facing financial constraints should go further in the direction prescribed for the resolution of carbon externalities (Withagen, 1994) than in the absence of these constraints: Less reserves should be developed and they should be depleted less rapidly. As far as the treatment of non-carbon substitutes to fossil resources is concerned, our results importantly differ from Sandmo (1975). He found that public-revenue needs did not warrant a special treatment of substitutes to externality-generating goods, which should be taxed solely according to Ramsey's rule. In contrast, we find that they should receive a favorable tax treatment, not because they are substitutes to carbon-containing goods, but because they are substitutes to nonrenewable resources.

A.5 Capture of Foreign Rents by an Open Economy

A nonrenewable resource importer cannot apply any form of direct resource rent taxation to foreign suppliers; in that sense, Ramsey's assumption that direct taxation is not possible applies to the foreign suppliers of an importing country implacably. However, that country can apply commodity taxes to home consumption as substitute for the taxation of foreign resource rents. Since the capture of foreign rents involves the exercise of market power, the OCT problem for a resource importer connects with the famous result of Bergstrom (1982) on rent capture. Here again, in an extension presented in Appendix N and discussed in Section 6, we find that the OCT tax rate in a resource importing country is higher than the rate on conventional commodities having the same demand elasticity.

B The Hotelling Rent and the Neutral Tax

A Hotelling resource is a homogenous nonrenewable natural asset, such as an oil deposit. As an asset, it should provide the same return as any traded asset if it is to be held. Since a unit of oil underground does not provide any return other than the value realized upon extraction, its return consists of capital gains over time. If oil reseerves underground were traded, absent any uncertainty, non arbitrage would thus require its current price to rise at the risk-free rate of interest. The value of such a non-traded asset is known as Hotelling rent and the non-arbitrage rule that it should satisfy is known as Hotelling's rule (Hotelling, 1931; Dasgupta and Heal, 1979, pp. 153-156; Gaudet, 2007).

This appendix defines the Hotelling rent with tax $\widetilde{\eta}_0$ and the Hotelling rent without tax $\overline{\eta}_0$ in competitive equilibrium. In competitive equilibrium with linear taxation, Hotelling's current-value unit rent to producers equals producer price minus marginal cost. At time zero, with constant unit extraction cost, this is $\widetilde{\eta}_0 = \widetilde{q}_{s0} - \theta_{s0} - c_s$. By Hotelling's rule, the rent is constant in present value so that, at any date, its present value is $\widetilde{\eta}_0$; it can be computed as follows.

If there exists a finite choke price $\overline{q} = D_s^{-1}(0)$ for the resource, the resource will be depleted in finite time, at a date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$, where $\widetilde{T}$ is defined by the condition that reserves are exactly exhausted over the period $\left[0, \widetilde{T}\right]$: $\int_0^{\widetilde{T}} D_s(\widetilde{q}_{st})dt = S_0$, with $\widetilde{q}_{st} - \theta_{st} - c_s = (\overline{q} - \theta_{s\widetilde{T}} - c_s)e^{-r\left(\widetilde{T}-t\right)}$. At time zero, the rent is thus $\widetilde{\eta}_0\left(S_0\right) = \widetilde{q}_{s0} - \theta_{s0} - c_s = (\overline{q} - \theta_{s\widetilde{T}} - c_s)e^{-r\widetilde{T}}$. If there is no finite choke price for the resource and the resource is not exhausted in finite time, then similar conditions must hold in the limit and define the present-value rent $\widetilde{\eta}_0\left(S_0\right)$ implicitly: $\lim\limits_{T\to+\infty} \int_0^T D_s(\widetilde{\eta}_t + \theta_{st} + c_s)dt = S_0$, where $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$. It can be shown that $\widetilde{\eta}_0$ is a positive and decreasing function of S_0.

The maximum value that can be raised from the mine by non-distortionary taxation is its discounted cumulative rent under competitive extraction and in the absence of taxation. That is $\overline{\eta}_0\left(S_0\right) = \widetilde{\eta}_0\left(S_0\right)$, where $\widetilde{\eta}_0$ is computed as above for the values of $\widetilde{q}_{st}$ implied by $\theta_{st} = 0$, $\forall\ t$. The present value of the mine in the absence of tax is thus $\overline{\eta}_0\left(S_0\right) S_0$.

If taxes are neutral, $\theta_{st} = \theta_{s0}e^{rt}$ and part of the unit scarcity rent is captured. The present value of the net-of-tax unit rent earned by the owner of the mine is thus $\widetilde{\eta}_0(S_0) = \overline{\eta}_0(S_0) - \theta_{s0}$ and the after-tax present value of the mine is $\widetilde{\eta}_0(S_0)S_0$.

C Proof of Proposition 1

1. We have shown in the main text that $\lambda = 1$ implies $\theta_i^* = 0$, $i = 1, ..., n$, and $\theta_{st}^* = \theta_{s0}^* e^{rt}$, so that the totality of tax revenues is raised from the resource sector. Moreover, we have argued that, if $\lambda = 1$, it must be the case that $R_0 \leq \overline{\eta}_0 S_0$. The contrapositive of that statement is that if $R_0 > \overline{\eta}_0 S_0$, then $\lambda > 1$. In that case, we have shown in the main text that $\theta_i^* > 0$, $i = 1, ..., n$, and that θ_{st}^* must be set in such a way as to raise more than $\overline{\eta}_0 S_0$ from the resource sector.

There remains to show that $R_0 \leq \overline{\eta}_0 S_0$ implies $\lambda = 1$. Assume $R_0 \leq \overline{\eta}_0 S_0$ and $\lambda > 1$. Then, taxes on conventional goods θ_i^*, $i = 1, ..., n$, raise a strictly positive revenue and cause distortions. Since it is possible to generate $\overline{\eta}_0 S_0 \geq R_0$ without imposing any distortions by taxing the natural resource, this cannot be optimal. Hence, $R_0 \leq \overline{\eta}_0 S_0$ implies $\lambda = 1$.
2. Shown in the main text, once it is observed that the tax formula given by (15) is independent of demand elasticities. ■

D Proof of Proposition 2

1. As shown in the main text, when $\lambda > 1$, the optimal tax rate on conventional good $i = 1, ..., n$ is θ_{it}^* as given in (13) and depends on λ. The optimal tax on the resource is given by (19), where $\mu > 0$ is determined to satisfy (1) with equality. Together, taxes on conventional goods and the tax on the resource must exactly raise $R_0 > \overline{\eta}_0 S_0$, which requires that $\sum\limits_{i=1,...,n,s} \int_0^{+\infty} \theta_{it}^* \widetilde{x}_{it} e^{-rt}\, dt = R_0$. Substituting for θ_{it}^* implicitly defines λ.
$2-4$. Shown in the main text.
5. The result that OCT of the nonrenewable resource never induces reserves to be left unexploited is shown in the main text. There remains to show that the optimal resource tax does not induce a more rapid extraction than in the non-distortionary case.

In any non-distortionary equilibrium—a fortiori in absence of resource tax—the resource price at any date of the exploitation period is

$$\widetilde{q}_{st} = c_s + \overline{\eta}_0 e^{rt}, \tag{D.1}$$

where $\overline{\eta}_0$ is defined in Appendix B. If there exists a finite choke price $\overline{q} = D_s^{-1}(0)$ for the resource, the resource is depleted in finite time, at a date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$, where $\widetilde{T}$ is defined by the condition that reserves are exactly exhausted over the period $\left[0, \widetilde{T}\right]$:

$$\int_0^{\widetilde{T}} D_s(\widetilde{q}_{st})dt = S_0. \tag{D.2}$$

If there is no finite choke price for the resource and the resource is not exhausted in finite time, then similar conditions must hold in the limit and define $\overline{\eta}_0$ implicitly: $\lim\limits_{T\to+\infty} \int_0^T D_s(\widetilde{q}_{st})dt = S_0$.

In the second-best equilibrium, in which high revenue needs imply that the resource should be taxed at the rate θ^*_{st} given by (19) with $\lambda > 1$, the resource price at any date of the exploitation period is $\widetilde{q}_{st} = c_s + \theta^*_{st}$; indeed, the producer rent $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ must be zero in this case, as explained in the main text. Making use of the presence of $\widetilde{q}_{st}$ on the right-hand side of (19), the resource price may be written as follows:

$$\widetilde{q}_{st} = \frac{c_s + \frac{1}{\lambda}\mu e^{rt}}{1 + \frac{\lambda-1}{\lambda}\frac{1}{\widetilde{\varepsilon}_s}}. \tag{D.3}$$

In this expression, $\lambda > 1$, and $\mu > 0$ is determined in a way similar as $\overline{\eta}_0$ in the non-distortionary case. It can easily be verified that the denominator on the right-hand side of (D.3) is strictly positive. Indeed, rearranging (19) immediately yields

$$\frac{\lambda - 1}{\lambda}\frac{1}{\widetilde{\varepsilon}_s} = \frac{\theta^*_{st} - \frac{1}{\lambda}\mu e^{rt}}{\theta^*_{st} + c_s} < 1.$$

When $c_s = 0$ and the ε_s is constant, as in Stiglitz's particular case, both the second-best equilibrium price (D.3) and the non-distortionary equilibrium price (D.1) reduce to a single term that rises at the rate of discount. Their levels are also identical as they are both determined in such a way that (D.2) holds. Indeed, we have established that the optimal resource tax induces reserves to be entirely depleted. In this case, the second-best equilibrium with $\lambda > 1$ implies the same extraction profile as prevails when no distortions are needed at all.

In all other cases, extraction cost c_s is strictly positive or the resource demand elasticity $\varepsilon_s < 0$ decreases with q_{st} along the demand (increases in absolute value), and the second-best extraction profile strictly differs from the non-distortionary case. In (D.3), the numerator takes the same form as (D.1); it consists of the same constant c_s and of a term rising at the rate of discount. However, as time t goes and price $\widetilde{q}_{st}$ increases, the denominator in (D.3) increases. Clearly, price $\widetilde{q}_{st}$ increases less rapidly in the second-best equilibrium. Since (D.2) must hold, the resource price must be higher at early dates and lower at more distant dates relative to the non-distortionary case.

When there exists a finite choke price $\overline{q}$ for the resource, the resource is depleted at a finite date $\widetilde{T} > 0$ such that $\widetilde{q}_{s\widetilde{T}} = \overline{q}$. Since the resource price rises less rapidly in the second-best case, it is immediate that the exhaustion date $\widetilde{T}$ is postponed relative to the non-distortionary case. ■

E OCT and Monopoly Pricing

If the need of tax revenues were extreme, that is to say if λ tended towards infinity, the optimal tax rate implied by (19) would be[47] $\frac{\theta^*_{st}}{\widetilde{q}_{st}} = \frac{1}{-\widetilde{\varepsilon}_s}$, corresponding to static monopoly pricing; indeed, $\frac{\theta_{st}}{\widetilde{q}_{st}} = \frac{\widetilde{q}_{st}-c_s}{\widetilde{q}_{st}}$ is the static Lerner index for the resource industry. Under such an extreme condition, the optimal resource tax rate would be determined by the same inverse elasticity rule as the tax rate applying to other commodities according to (13).

When revenue needs equal total rents ($\lambda = 1$), the second term in the right-hand side of (19) vanishes so that the optimal extraction tax is neutral.

Since $\frac{1}{\lambda}$ and $\frac{\lambda-1}{\lambda}$ sum to unity, the optimal tax on the resource industry given by (19) is a weighted sum of two elements. The first element μe^{rt} can be interpreted as the neutral component of the tax since it rises at the rate of discount, as does a neutral Hotelling tax. The second element was just seen to correspond to monopoly pricing.

[47] Although μ varies as λ changes, this scarcity rent cannot become infinite as $\lambda \to \infty$ so that the first term on the right-hand side of (19) indeed vanishes as required for this statement to be true.

F Extension to Rising Marginal Costs and Resource Heterogeneity

One may wonder whether the results of Section 2 are not due to the parsimony of the model, in particular the assumption that the supply of all conventional commodities is perfectly elastic and the assumption that marginal extraction costs are not only constant but independent of the source of resource supply. It will be shown that the basic message—tax the resource more than similar conventional commodities—is not much affected by relaxing these assumptions, although several new insights are derived from the analysis.

The assumption of infinite supply elasticity made by so many contributors to the OCT literature may be justified on the ground that they adopt a long-run perspective, where all commodities can be produced at constant marginal costs because all inputs are variable. The natural counterpart of constant marginal production cost for conventional commodities is constant marginal extraction cost. In this appendix, this will be replaced by rising marginal production and extraction costs.

There is another important aspect. The conditions of extraction of a nonrenewable resource may be quite variable over time, as resources are not necessarily homogeneous; a possibility that is ruled out by the simple Hotellian formulation adopted so far. Even with a rising marginal extraction cost, the extraction technology does not provide for resource heterogeneity. Two approaches have been used in the literature to deal with this issue. The Ricardian approach considers a single stock of reserves but assumes that the extraction cost increases with cumulative extraction (see, e.g., Levhari and Liviatan, 1977; Pindyck, 1978); this approach has been criticized because it implicitly assumes that the economically most accessible reserves are used first, which is not always optimal.[48] The second approach consists in modeling the resource as originating from different deposits, each with its own cost function and its own stock of reserves. It underlies the manner in which advanced systems, such as the Alberta oil and gas taxation regime, approach resource taxation[49]—see

[48] As Slade (1988) put it "The idea that the least-cost deposits will be extracted first is so firmly embedded in our minds that it is an often-made but rarely tested assumption underlying the construction of theoretical exhaustible-resource models." (p. 189). See her references.

[49] Conrad and Hool (1981) pointed at the relevance of deposits' differences for resource taxation: In the ". . . mining problem, . . . differences in the composition of the ore bodies cause differences in response to a given economic change. In part because of this, mineral tax policy in some countries has been negotiated on a mine-by-mine basis. Geological features must therefore be an essential part of any model that is to be used for policy or empirical analysis" (p. 18).

Slade (1988) for a theoretical formulation, empirical considerations, and references.

We start with the introduction of rising marginal costs; then, we further add multiple deposits. Thus, assume that conventional good i is supplied according to the function $S_i(p_{it})$, with $S_i'(.) > 0$, for $i = 1, ..., n$; $S_i^{-1}(x_{it})$ is the increasing marginal cost of producing a quantity x_{it}. Regarding the nonrenewable resource, assume an increasing marginal cost of extraction. For notational simplicity, this marginal cost is denoted by $S_s^{-1}(x_{st})$. However, this does not denote the inverse supply function. In competitive equilibrium, the supply of resource is determined by the "augmented marginal cost" condition:

$$\widetilde{p}_{st} = S_s^{-1}(\widetilde{x}_{st}) + \widetilde{\eta}_t, \tag{F.1}$$

where the current-value Hotelling's rent $\widetilde{\eta}_t$ grows at the rate of discount.

The OCT problem of maximizing (2) subject to (4) and (5), and the associated Hamiltonian are only modified to the extent that the producer surplus becomes

$$\widetilde{PS}_t = \sum_{i=1,...,n,s} \widetilde{p}_{it}\widetilde{x}_{it} - \sum_{i=1,...,n} \int_0^{\widetilde{x}_{it}} S_i^{-1}(u)\, du - \int_0^{\widetilde{x}_{st}} \left(S_s^{-1}(u) + \widetilde{\eta}_t\right) du. \tag{F.2}$$

Given this change, the structure of the analysis is quite similar to that with constant marginal costs. Assuming that there exist feasible taxes that yield an interior solution to the problem, the first-order condition for the choice of the tax θ_{it} on conventional good i is $[D_i^{-1}(\widetilde{x}_{it}) - \theta_{it} - S_i^{-1}(\widetilde{x}_{it})]\frac{d\widetilde{x}_{it}}{d\theta_{it}} - \widetilde{x}_{it} + \lambda(\widetilde{x}_{it} + \theta_{it}\frac{d\widetilde{x}_{it}}{d\theta_{it}}) = 0$. Since the competitive equilibrium allocation $\widetilde{x}_t$ satisfies $D_i^{-1}(\widetilde{x}_{it}) = S_i^{-1}(\widetilde{x}_{it}) + \theta_{it}$, it follows that $\frac{d\widetilde{x}_{it}}{d\theta_{it}} = \frac{1}{D_i^{-1\prime}(.) - S_i^{-1\prime}(.)}$. The optimal tax is thus such that $\theta_{it}^* = \frac{1-\lambda}{\lambda}\widetilde{x}_{it}\left(D_i^{-1\prime}(.) - S_i^{-1\prime}(.)\right)$. Consequently the optimal tax rate on conventional commodity i is

$$\frac{\theta_{it}^*}{\widetilde{q}_{it}} = \frac{\lambda - 1}{\lambda}\left(\frac{1 - \frac{\theta_{it}^*}{\widetilde{q}_{it}}}{\widetilde{\epsilon}_i} - \frac{1}{\widetilde{\varepsilon}_i}\right), \tag{F.3}$$

where $\epsilon_i \equiv \frac{S_i^{-1}(.)}{x_{it}S_i^{-1\prime}(.)}$ is the elasticity of supply, positive by assumption. As before, λ is strictly greater than unity when taxes are distortionary and equals unity if there is a non-

For example in Alberta, royalties depend on the type of resource (conventional oil, gas, oil sands) and the date at which the deposit was discovered, because exploration targets different deposits as extraction technology evolves, as oil prices increase, and as exploration prospects become exploited (Alberta Royalty Review, 2007).

distortionary way to collect enough revenues. Formula (F.3) provides an inverse elasticity rule for the case of non-perfectly-elastic supplies, as in Ramsey (1927, p. 56).

The first-order condition for an interior tax on the resource is now $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st})]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = 0$. Since resource supply is determined by condition (F.1), it follows that $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st}) = \widetilde{\eta}_t$, which is different from zero. If tax revenue needs are low, the other commodities are not taxed at all and the resource is the sole provider of tax revenues; the resource should be taxed in priority even when supply elasticities in the other sectors are not assumed to be infinite.

If the revenues needed cannot be raised neutrally so that λ exceeds unity, all sectors are taxed in such a way that the distortions are spread across sectors; the tax on the resource sector is distortionary as in Section 2. What is new, however, is that the distortion aims at capturing part of the consumer surplus and part of the producer surplus while no producer surplus was available when marginal extraction were assumed to be constant. In that case, as in Section 2, the government's problem is subject to the exhaustibility constraint (16); taxation completely expropriates producers' resource rents, so that $\widetilde{\eta}_t = 0$ and $\widetilde{q}_{st} = S_s^{-1}(\widetilde{x}_{st}) + \theta_{st}$; the first-order condition for the resource tax becomes $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - S_s^{-1}(\widetilde{x}_{st})]\frac{d\widetilde{x}_{st}}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}$, where μ is the present-value co-state variable associated with the exhaustibility constraint. The competitive equilibrium allocation satisfies $D_{si}^{-1}(\widetilde{x}_{st}) = S_s^{-1}(\widetilde{x}_{st}) + \theta_{st}$; transforming the first-order condition as for conventional goods yields the optimal tax on the resource

$$\theta_{st}^* = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda}\left(\frac{\widetilde{p}_{st}}{\widetilde{\epsilon}_s} - \frac{\widetilde{q}_{st}}{\widetilde{\varepsilon}_s}\right), \tag{F.4}$$

where $\epsilon_s \equiv \frac{S_s^{-1}(.)}{x_{st}S_s^{-1\prime}(.)}$, the reciprocal of the elasticity of marginal extraction costs, can also be interpreted as the elasticity of short-run resource supply. Consequently, the resource should be taxed at a higher rate than conventional commodities having identical elasticities.

Now consider that the resource may be extracted from m deposits using an extraction technology characterized by rising marginal costs, as above, but possibly different for each deposit. Each deposit $l = 1, ..., m$ makes a contribution z_{lt} to total production so that consumption of the homogeneous final commodity is $x_{st} = \sum_{l=1,...,m} z_{lt}$. While the consumer price q_{st} is unique, producer prices and scarcity rents typically differ because extraction costs and reserves may differ from one deposit to the next: $p_{lt} = S_l^{-1}(z_{lt}) + \eta_{lt}$, $l = 1, ..., m$. However, since each deposit is homogeneous, the corresponding rent satisfies Hotelling's rule and must grow at the rate of interest, so that its supply is determined in competitive equilibrium by $\widetilde{p}_{lt} = S_l^{-1}(\widetilde{z}_{lt}) + \widetilde{\eta}_{lt}$, where the Hotelling rent $\widetilde{\eta}_{lt}$ corresponds to the exhaustibility constraint applying to deposit l: $\int_0^{+\infty} z_{lt}\, dt \leq S_{l0}$. We assume that the government has the ability to tax each deposit individually[50] so that $q_{st} = p_{lt} + \theta_{lt}$, $l = 1, ..., m$. Precisely, the tax θ_{st} that could indifferently fall on demand or supply in the previous cases, is replaced with a vector of taxes that fall on the supply of individual deposits; resource demand is not taxed. For any feasible tax trajectory and Hotelling rent, the output from each deposit adjusts in such a way that marginal extraction cost plus rent equals producer price, as required in equilibrium.

The government budget constraint is only modified by the increase in the size of the tax vector, which becomes $\theta_t \equiv (\theta_{1t}, ..., \theta_{nt}, \theta_{n+1\ t}, ..., \theta_{n+m\ t})$, and by the replacement of consumption x_{st} by the vector of supply tax bases $(z_{1t}, ..., z_{mt})$ in the government budget constraint. Except for the increased number of variables, the OCT problem is only modified to the extent that producer surplus becomes, instead of $(F.2)$,

$$\widetilde{PS}_t = \sum_{i=1,...,n} \widetilde{p}_{it}\widetilde{x}_{it} + \sum_{l=1,...,m} \widetilde{p}_{lt}\widetilde{z}_{lt} - \sum_{i=1,...,n} \int_0^{\widetilde{x}_{it}} S_i^{-1}(u)\, du - \sum_{l=1,...,m} \int_0^{\widetilde{z}_{lt}} \left(S_l^{-1}(u) + \widetilde{\eta}_{lt}\right) du,$$

and the resource rents become, instead of (10),

$$\widetilde{\phi}_t = \sum_{l=1,...,m} \widetilde{\eta}_{lt}\widetilde{z}_{lt}.$$

The first-order conditions for an interior solution to the choice of the taxes on resource extraction are $[D_s^{-1}(\widetilde{x}_{st}) - \theta_{lt} - S_l^{-1}(\widetilde{z}_{lt})]\frac{d\widetilde{z}_{lt}}{d\theta_{lt}} - \widetilde{z}_{lt} + \lambda(\widetilde{z}_{lt} + \theta_{lt}\frac{d\widetilde{z}_{lt}}{d\theta_{lt}}) = 0$. Since supply from deposit l is determined by condition $\widetilde{p}_{lt} = S_l^{-1}(\widetilde{z}_{lt}) + \widetilde{\eta}_{lt}$, it follows that $D_s^{-1}(\widetilde{x}_{st}) - \theta_{lt} - S_l^{-1}(\widetilde{z}_{lt}) = \widetilde{\eta}_{lt}$, $l = 1, ..., m$; the rest of the solution process is as above. If revenue needs are low, a combination of neutral taxes rising at the rate of interest is applied on the extraction of the deposits. If revenue needs are high, the analysis of the single-deposit case applies; denoting by μ_l the present-value co-state variable associated with the exhaustibility

[50] See Footnote 49.

constraint of deposit l, one obtains the optimal tax on deposit l

$$\theta_{lt}^{*} = \frac{1}{\lambda}\mu_l e^{rt} + \frac{\lambda-1}{\lambda}\left(\frac{\widetilde{p}_{lt}}{\widetilde{\epsilon}_l} - \frac{\widetilde{q}_{st}}{\widetilde{\varepsilon}_s}\right),\ l = 1, ..., m, \tag{F.5}$$

where $\epsilon_l \equiv \frac{S_l^{-1}(.)}{z_{lt}S_l^{-1\prime}(.)}$. Qualitative results are unchanged.

G OCT with Resource Substitutes or Complements

Assume that conventional goods may be substitutes or complements for the nonrenewable resource or for each other, while their marginal cost of production, as well as the marginal extraction cost, remains constant as previously. For the reasons explained in Section 2, when government revenue needs are low in the sense of Proposition 1, substitutes for, or complements to, the resource may be left untaxed, while the resource alone is taxed. However, high government revenue needs warrant that nonrenewable resource substitutes and complements be given specific tax treatments.

Assume that the demand $D_j(q_{jt}, q_{kt})$ for a conventional commodity $j \in \{1, ..., n\}$ not only depends on its own price, but also on the price of another commodity $k \in \{1, ..., j-1, j+1, ..., n, s\}$, with $\frac{\partial D_j(.)}{\partial q_j} < 0$, $\frac{\partial D_k(.)}{\partial q_k} < 0$, and $\frac{\partial D_j(.)}{\partial q_k}, \frac{\partial D_k(.)}{\partial q_j} > 0$ (< 0) if the goods are substitutes (complements). The joint consumer surplus arising from that pair of goods is given by the concave money-metric

$$\psi(\widetilde{x}_{jt}, \widetilde{x}_{kt}),\ \text{with}\ \frac{\partial\psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_j} = \widetilde{q}_j\ \text{and}\ \frac{\partial\psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_k} = \widetilde{q}_k. \tag{G.1}$$

Redefining the consumer surplus (8) accordingly, Appendix H shows that the first-order conditions for the constrained maximization of (2) take account of the effect of any tax θ_{jt} on the tax income raised in sector k.

Consider two conventional commodities $j \in \{1, ..., n\}$ and $k \in \{1, ..., n\}$ that are substitutes for or complements to each other. The optimal tax on conventional commodity j is—see Appendix H for details—

$$\theta_{jt}^{*} = \frac{\lambda-1}{\lambda}\frac{\widetilde{q}_{jt}}{-\widetilde{\varepsilon}_{jj}} + \theta_{kt}\frac{\widetilde{x}_{kt}\widetilde{\varepsilon}_{kj}}{-\widetilde{x}_{jt}\widetilde{\varepsilon}_{jj}},\ k,\ j \neq s. \tag{G.2}$$

Obviously, the optimal tax on good $k \neq s$ is given by the same expression where k and j

are interchanged. Consequently time does not enter the above tax formula either directly or through θ_{kt}. Therefore, the optimal taxes and induced tax rates for commodities that are neither nonrenewable resources, nor substitutes for (or complements to) nonrenewable resources, are constant over time.

By contrast, a similar derivation for a conventional good j when its substitute or complement is a resource ($k = s$) yields a time dependent optimal tax

$$\theta_{jt}^{*} = \frac{\lambda - 1}{\lambda} \frac{\widetilde{q}_{jt}}{-\widetilde{\varepsilon}_{jj}} + \left(\theta_{st} - \frac{1}{\lambda}\mu e^{rt}\right) \frac{\widetilde{x}_{st}\widetilde{\varepsilon}_{sj}}{-\widetilde{x}_{jt}\widetilde{\varepsilon}_{jj}},\ j \neq s,\ k = s. \tag{G.3}$$

In both formulas (G.2) and (G.3), the own-price elasticity of the demand for good j is now denoted by $\varepsilon_{jj} = \frac{q_{jt}\frac{\partial D_j(.)}{\partial q_j}}{x_{jt}}$, while $\varepsilon_{kj} = \frac{q_{jt}\frac{\partial D_k(.)}{\partial q_j}}{x_{kt}}$ is the cross-price elasticity of the demand for commodity k with respect to the price of commodity j. The former is negative as in Section 2; the latter is positive for substitutes and negative for complements.

Finally, for comparison, consider the taxation of the resource sector. When the resource admits conventional commodity j as a substitute or complement, the optimal resource tax is—see Appendix H for details—

$$\theta_{st}^{*} = \frac{1}{\lambda}\mu e^{rt} + \frac{\lambda - 1}{\lambda} \frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_{ss}} + \theta_{jt} \frac{\widetilde{x}_{jt}\widetilde{\varepsilon}_{js}}{-\widetilde{x}_{st}\widetilde{\varepsilon}_{ss}}. \tag{G.4}$$

All three tax formulas (G.2)-(G.4) are identical to their independent-demand counterparts in Section 2 (see (13) for conventional commodities and (19) for the resource), except for the last term on the right-hand side of (G.2)-(G.4). This new term reflects the change in the fiscal revenues levied on the sector indirectly affected by the tax. The tax θ_{it} can be interpreted as a producer mark-up.[51] The adjustment to the mark-up (to the tax) is positive (negative) when commodities j and k are substitutes (complements) and does not depend on time directly. This applies whether the sector indirectly affected by the tax is a conventional sector as in (G.2) or a nonrenewable resource sector as in (G.4).

When the commodity indirectly impacted by the tax is a resource, the additional term must also be interpreted as a correction accounting for monopoly power on two markets. However the correction in (G.3) is reduced by the time-dependent term $\frac{1}{\lambda}\mu e^{rt}$ that reflects

[51]When the commodity indirectly impacted is a conventional good, the new term is the same as in the formula giving the price chosen by a firm that holds monopoly power on two commodities with interrelated demands and separable costs—see, e.g., Tirole (1988, p. 70).

the scarcity of the resource. Proposition 6 summarizes the results.

Proposition 6 *(Resource substitutes and complements) Assume that the nonrenewable resource has substitutes or complements.*

1. *If $R_0 \leq \overline{R}_0$, resource substitutes and complements should be left untaxed, while the resource should be taxed positively;*

2. *If $R_0 > \overline{R}_0$,*

   (a) *Other things equal, conventional goods that are substitutes for a resource should be taxed at lower rates than substitutes for a conventional good; vice versa conventional goods that are complements to a resource should be taxed more;*

   (b) *The optimal unit tax on resource substitutes and complements depends on time.*

The rationale behind the special tax treatment of substitutes or complements is clear. On the one hand, a higher mark-up on any good positively affects the demand for substitutes, thus their tax base. On the other hand, this effect is less pronounced when the impacted substitute or complement is a nonrenewable resource—compare the right-hand sides of (G.3) and (G.2). The reason is that the tax on a resource substitute shifts demand towards a sector with an inelastic supply (the natural resource), unlike the tax on other conventional goods.

This specific treatment differs from the treatment of substitutes for, or complements of, externality-generating commodities in the absence of government financial restrictions. In Sandmo (1975), the "marginal social damage . . . does not enter the formulas for the other [non externality-creating] commodities, regardless of the pattern of complementarity and substitutability" (p. 92). In contrast, financially strapped governments should tax substitutes to fossil resources more lightly, and complements more heavily, than commodities unrelated to nonrenewable resources.

Proposition 6 further indicates that the optimal tax on the substitutes for, or complements to, a resource depends on time, unlike the optimal tax on conventional commodities without links to a resource. The link with the nonrenewable resource confers a dynamic dimension to the tax on its substitutes and complements, as can be seen by comparing

(G.3) with (G.2). For a given resource tax level, the term in μ indicates that the tax on a resource substitute should fall as the resource scarcity increases; vice versa the tax on a resource complement should rise. The tax on resource substitutes and complements is also compounded by the motion of the resource tax θ_{st} itself: The tax on resource substitutes should rise as the resource tax increases; vice versa for resource complements.

H Proof of Proposition 6

1. Shown in the main text.

2.(a) The result relies on the comparison of (G.2) with (G.3), which can be obtained as follows. In presence of substitutes and complements, the present-value Hamiltonian associated with the maximization of (2) subject to (4) and (5) takes the same form as in Section 2:

$$\mathcal{H}\left(a_t, \theta_t, \lambda_t\right) = \left(\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\Phi}_t\right) e^{-rt} + \lambda_t (r a_t + \theta_t \widetilde{x}_t).$$

However, the consumer surplus (8) is adjusted to comprise the relation (G.1):

$$\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\Phi}_t = \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt}) + \sum_{i=1,\dots,n,s \ i \neq j,k} \int_0^{\widetilde{x}_{it}} D_i^{-1}(u)\, du - \sum_{i=1,\dots,n,s} (c_i + \theta_{it}) \widetilde{x}_{it},$$

where $D_i^{-1}(\widetilde{x}_{it}) - c_i - \theta_{it} = 0$ for conventional goods $i = 1, \dots, n$, and $D_s^{-1}(\widetilde{x}_{st}) - c_s - \theta_{st} = \widetilde{\eta}_t$ for the nonrenewable resource. The maximum principle implies $\lambda_t = \lambda e^{-rt}$ as in Section 2. Furthermore, the first-order condition for the optimal choice of the tax θ_{jt}, $j \in \{1, \dots, n\}$, takes account of the effect of this tax on the tax income raised in sector k.

Consider two conventional commodities $j \in \{1, \dots, n\}$ and $k \in \{1, \dots, n\}$ that are substitutes for or complements to each other. The first-order condition for the tax on j is

$$\begin{aligned} &\left[\frac{\partial \psi(.)}{\partial x_{jt}} - \theta_{jt} - c_j\right] \frac{d\widetilde{x}_{jt}}{d\theta_{jt}} + \left[\frac{\partial \psi(.)}{\partial x_{kt}} - \theta_{kt} - c_k\right] \frac{d\widetilde{x}_{kt}}{d\theta_{jt}} - \widetilde{x}_{jt} \\ &\qquad + \lambda \left(\widetilde{x}_{jt} + \theta_{jt} \frac{d\widetilde{x}_{jt}}{d\theta_{jt}} + \theta_{kt} \frac{d\widetilde{x}_{kt}}{d\theta_{jt}}\right) = 0, \qquad k \neq s, \end{aligned} \tag{H.1}$$

where $\frac{\partial \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_{jt}} = c_j + \theta_{jt}$ and $\frac{\partial \psi(\widetilde{x}_{jt}, \widetilde{x}_{kt})}{\partial x_{kt}} = c_k + \theta_{kt}$. Moreover, $\widetilde{x}_{jt} = D_j(\widetilde{q}_{jt}, \widetilde{q}_{kt})$ and $\widetilde{x}_{kt} = D_k(\widetilde{q}_{kt}, \widetilde{q}_{jt})$ so that $\frac{d\widetilde{x}_{jt}}{d\theta_{jt}} = \frac{\partial D_j(.)}{\partial q_{jt}}$ and $\frac{d\widetilde{x}_{kt}}{d\theta_{jt}} = \frac{\partial D_k(.)}{\partial q_{jt}}$. It follows that the optimal tax on conventional commodity j when its complement or substitute k is not a resource is given by (G.2).

By contrast, a similar derivation for a conventional good j when its substitute or complement is a resource ($k = s$) yields formula (G.3), which differs from (G.2) by the intervention of the time-varying term $\frac{1}{\lambda}\mu e^{rt}$.

For comparison, we also consider the taxation of the nonrenewable resource sector. When the resource admits conventional commodity j as a substitute or complement, the first-order condition for the choice of the resource tax is the same as (H.1) except that s and j must be interchanged on the left-hand side and that the right-hand side is $\mu e^{rt}\frac{d\widetilde{x}_{st}}{d\theta_{st}}$ rather than zero. It follows that the optimal tax on a nonrenewable resource that has a substitute or complement j is given by (G.4).

The result follows from the fact that the tax in (G.3) is adjusted by a positive term when $\widetilde{\varepsilon}_{sj}, \widetilde{\varepsilon}_{js} > 0$ and by a negative term otherwise.

2.(b) Shown in the main text. ■

I Proof of Expressions (22) and (25)

The Hamiltonian (21) associated with the ex post problem is identical to (17). Hence, the application of the maximum principle also gives $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu$. The first-order condition for the choice of the tax is also (18). However, unlike in Section 2, the first term on the left-hand side is not zero since the government is subject to its ex ante commitment, which determines $\widetilde{\eta}_0$ at this stage: $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st}^* - c_s = \widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt} > 0$. Therefore, $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$. Substituting into the first-order condition and rearranging gives (22), where $\varepsilon_s \equiv \frac{q_{st}}{x_{st} D_s^{-1\prime}(.)}$.

Let us now turn to the ex ante problem. Denote by $V^*\left(\widetilde{S}_0, R; \rho\right)$ the value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt$ maximized under (24) with respect to Θ given $\widetilde{\eta}_0$; because (20) holds, this value function may be defined indifferently as a function of $\widetilde{S}_0$ or $\widetilde{\eta}_0$. Thus, by definition, $V^*\left(\widetilde{S}_0, R; \rho\right)$ is the value function for the ex post problem just analyzed, whose Hamiltonian is (21) and which requires that the optimal tax satisfies (22). The constant co-state variable μ in (21) gives the value $\frac{\partial V^*}{\partial \widetilde{S}_0}$ of a marginal unit of reserves, while $-\lambda$ gives the marginal impact $\frac{\partial V^*}{\partial R}$ of a tightening of the budget constraint. Define $\mathcal{V}\left(\widetilde{S}_0; R_0, \rho\right) \equiv V^*\left(\widetilde{S}_0, R; \rho\right)$, making use of the definition $R = R_0 + \rho\widetilde{S}_0$; we have $\frac{\partial \mathcal{V}}{\partial \widetilde{S}_0} = \frac{\partial V^*}{\partial \widetilde{S}_0} + \rho\frac{\partial V^*}{\partial R} = \mu - \rho\lambda$. Problem (23) can thus be written as that of maximizing $\mathcal{V}\left(\widetilde{S}_0; R_0, \rho\right) + \rho\widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS$ with respect to $\widetilde{S}_0$.

The first-order condition is $\frac{\partial \mathcal{V}}{\partial \widetilde{S}_0} + \frac{\partial \left(\rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS \right)}{\partial \widetilde{S}_0} = 0$, i.e., $\mu - \rho\lambda + \rho - \mathcal{S}^{-1}(\widetilde{S}_0) = 0$ so that, using (20), (25) is obtained.

J Proof of Proposition 3

1. This is a restatement of (26), which is immediately obtained by substituting (25), shown in the main text, into (22), proven in Appendix I. The rest of the proposition summarizes findings established in the text preceding it.
2. Shown in the main text. ■

K Proof of Proposition 4

Assuming that the subsidy ρ is sufficiently low to justify that the government leaves non-zero extraction rents ex post, any parametric change $\Delta\rho$ exactly compensated by a one-to-one change $\Delta\widetilde{\eta}_0 = -\Delta\rho$ and by a change $\Delta\theta^*_{st} = -\Delta\widetilde{\eta}_0 e^{rt}$ not only leaves $\widetilde{q}_{st}$ unchanged but ensures that (26) remains satisfied without any further adjustment. This is because $\widetilde{\eta}_0 + \rho$ is then unchanged so that the new combination of subsidy and after-tax rent commands the same reserve level; as $\widetilde{q}_{st}$ is unchanged it generates the same extraction path; all constraints remain satisfied. In other words, the optimal after-tax rent depends on the ex ante subsidy: $\widetilde{\eta}_0 = \widetilde{\eta}_0(\rho)$; similarly $\theta^*_{st} = \theta^*_{st}(\rho)$, with $\frac{d\widetilde{\eta}_0(\rho)}{d\rho} = -1$ and $\frac{d\theta^*_{st}(\rho)}{d\rho} = e^{rt}$. However the optimal level of reserves $\widetilde{S}_0$ and the equilibrium price profile are independent of ρ.

Having defined the functions $\widetilde{\eta}_0(\rho)$ and $\theta^*_{st}(\rho)$, the subsidy threshold $\bar{\rho}$ below which non-zero rents are left by the government can easily be established, as explained in the main text. ■

L Proof of Expression (28)

Expression (28) is established under the assumption that extraction cost is zero, $c_s = 0$, and that the demand for the resource is isoelastic, $\varepsilon_s(q_{st}) = \varepsilon_s$. As mentioned in the main text, substituting $\widetilde{q}_{st} = \widetilde{\eta}_0 e^{rt} + \theta^*_{st}$ into (19) with $\widetilde{\eta}_0 = 0$, or into (22) and into (26) with $\widetilde{\eta}_0 \geq 0$, while using the constancy of $\widetilde{\varepsilon}_s$, immediately shows that the optimal extraction unit tax then grows at the rate of interest:

$$\theta^*_{st} = \theta^*_{s0} e^{rt}, \tag{L.1}$$

where θ^*_{s0} is to be determined.

For a given ρ, the ex ante choice of θ^*_{s0} is equivalent to the choice of the unit rent $\widetilde{\eta}_0$ it induces, account being taken of (20). The first-order condition for the ex ante static maximization of (23) with respect to θ^*_{s0} subject to (24), taking the ex post solution (L.1) into account is

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt}\, dt + \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} - \mathcal{S}^{-1}(.) \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0,$$

where $\frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt} = \left(D_s^{-1}(\widetilde{x}_{st}) - \theta^*_{s0} e^{rt}\right) \frac{d\widetilde{x}_{st}}{d\theta_{s0}} e^{-rt} - \widetilde{x}_{st} = \widetilde{\eta}_0 \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st}$ and where $\mathcal{S}^{-1}(.) = \widetilde{\eta}_0 + \rho$. Substituting, one has

$$\int_0^{+\infty} \left(\widetilde{\eta}_0 \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st} \right) dt - \widetilde{\eta}_0 \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0.$$

Integrating with $\int_0^{+\infty} \widetilde{x}_{st}\, dt = \widetilde{S}_0$ and $\int_0^{+\infty} \frac{d\widetilde{x}_{st}}{d\theta_{s0}}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$ gives

$$\theta^*_{s0} = \rho - \frac{(\lambda - 1)}{\lambda} \frac{\widetilde{S}_0}{\frac{d\widetilde{S}_0}{d\theta_{s0}}}. \tag{L.2}$$

In long-run market equilibrium $\mathcal{S}^{-1}(\widetilde{S}_0) = \widetilde{\eta}_0 + \rho$ and $\int_0^{+\infty} D_s(\widetilde{\eta}_t + \theta^*_{st})\, dt = \int_0^{+\infty} D_s\big((\widetilde{\eta}_0 + \theta^*_{s0})e^{rt}\big)\, dt = \widetilde{S}_0$. It follows by differentiation with respect to θ_{s0} that $\mathcal{S}^{-1\prime}(.) \frac{d\widetilde{S}_0}{d\theta_{s0}} = \frac{d\widetilde{\eta}_0}{d\theta_{s0}}$ and $\left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}} + 1 \right) \int_0^{+\infty} D_s'(.) e^{rt}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$. Substituting in $\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$, one obtains $\frac{d\widetilde{S}_0}{d\theta_{s0}} = \frac{\int_0^{+\infty} D_s'(.) e^{rt}\, dt}{1 - \mathcal{S}^{-1\prime}(.) \int_0^{+\infty} D_s'(.) e^{rt}\, dt}$. Introducing this expression into (L.2) yields

$$\theta^*_{s0} = \rho + \frac{\lambda - 1}{\lambda} \left[\widetilde{S}_0 \mathcal{S}^{-1\prime}(.) - \frac{\widetilde{S}_0}{\int_0^{+\infty} D_s'(.) e^{rt} dt} \right], \tag{L.3}$$

from which (28) is derived after substituting the expressions for $\widetilde{\zeta}$ and $\widetilde{\xi}$ defined in the main text and using the fact that $\widetilde{q}_{st} = (\widetilde{\eta}_0 + \theta^*_0)e^{rt} = \widetilde{q}_{s0} e^{rt}$ under (L.1) so that $\frac{d\widetilde{\mathcal{D}}}{dq_{s0}} = \int_0^{+\infty} D_s'(.) e^{rt}\, dt$. Furthermore, the constancy of ε_s implies $\widetilde{\xi} = \varepsilon_s$.

M Proof of Proposition 5

The proposition summarizes findings established in the main text. ∎

N Extension to an Open Economy

OCT in an open economy raises a number of issues. In a static, closed economy, commodity taxes applied on the demand side are equivalent to taxes applied on the supply side. In the closed economy, taxation during the extraction phase can be interpreted to apply to resource demand while the reserve development subsidy can be interpreted to apply to resource supply. Proposition 4 then means that the equivalence of supply and demand taxation extends to the resource sector, despite the difference in timing between reserve development and resource extraction. In the open economy, domestic consumption generally differs from domestic production, so that OCT must be addressed by considering taxes or subsidies on both supply and demand, rather than a single tax on demand or supply indifferently. The result of Proposition 4 nonetheless allows us to simplify the taxation of domestic resource supply by focusing on the domestic reserve subsidy, rather than on the taxation of domestic extraction, while combining that subsidy with a commodity tax on resource consumption, whether from domestic or foreign origin. That way, much of the model structure used in the main text will be preserved.

In fact, the combination of a tax or subsidy on domestic demand and a tax or subsidy on domestic supply can be designed so as to be equivalent to a tariff (Mundell, 1960, p. 96). Consequently, the use of Ramsey's traditional tax instruments in an open economy could achieve the objective pursued by optimal tariffs (Friedlander and Vandendorpe, 1968; Dornbusch, 1971). Since the OCT problem and the optimal-tariff problem then differ only by the constraint to collect a minimum revenue, the latter characterizes an optimum of Pareto from the country's point of view while optimal commodity taxes are distortionary: As Boadway et al. (1973) put it "domestic commodity taxes *introduce* a distortion while optimal tariffs *eliminate* a distortion" (p. 397, their italics).

For reasons that need no explanation, tariffs will not be directly available as tax instruments in the open-economy OCT problem. However, demand and supply commodity taxes will seek the same objective as optimal tariffs and, consequently, their first-best levels (that is, unconstrained by revenue needs) will differ from zero.[52] Besides the obvious difference in domestic versus world surplus, the ability of the government to affect national surplus differs in the closed economy, where the government has the power to affect prices as a monopoly, from the open economy, where the government is competing with other countries much like an oligopolist. Nonrenewable resources are very different from conventional goods in that respect; roughly, the supply of conventional goods is elastic while the supply of the Hotelling resource is inelastic in a closed economy. In an open economy, if the country is small and trades the resource competitively, the nonrenewable resource behaves just like another commodity; its supply is infinitely elastic and optimal commodity taxes on the nonrenewable resource obey the conventional closed-economy inverse elasticity rule.

Consequently, the interesting setup to study Ramsey taxation in an open economy is strategic. The country trades the nonrenewable resource and is big enough to affect suppliers' surplus, whether supply is domestic or foreign.[53] In this section we are going to assume that the country has no influence on the prices of other commodities. Three reasons justify this restriction. First, it does not affect the generality of the results presented; second, it puts the focus on the key difference between nonrenewable resources and conventional goods and services: supply elasticity. Third, it connects with the literature on rent capture and optimal tariffs in the presence of a nonrenewable resource; more on this further below.

N.1 Analysis and Results

The government faces a problem similar to that of Section 2, i.e., of choosing linear commodity taxes to maximize domestic surpluses subject to a minimum tax revenue constraint and to a stock of endogenously supplied mineral reserves. These reserves are located either within the country, or outside, or both but have the same constant unitary extraction

[52]Since the distortion results from the failure by the country to exert market power, only "large" countries should adopt different domestic taxes when they are open to trade than when they are closed to trade. This is also true when some tariffs are set at suboptimal levels; then, as shown by Dornbusch (1971, p. 1364), domestic taxes are conferred a corrective role. Not surprisingly, if the government can freely use both tariffs and commodity taxes, it can achieve its surplus maximization objective with tariffs and satisfy its revenue collection needs using commodity taxes; then, as Boadway et al. (1973) showed, Ramsey optimal domestic commodity taxes are "the same as in the case of a closed economy" (p. 391).

[53]The literature on resource oligopolies and oligopsonies is relevant to the problem of OCT in an open economy. According to Karp and Newbery (1991) "the evidence for potential market power on the side of importers is arguably as strong as for oil exporters" (p. 305); the more so when suppliers and/or buyers act in concert as suggested by Bergstrom (1982). On market power on the demand side, see also Liski and Montero (2011).

cost.[54] The nonrenewable resource sector is now open to trade. World scarcity rents are equalized by free trade but domestic reserve supply is determined by the sum of the rent and the domestic reserve subsidy. As in our treatment of the closed economy, we simplify and sharpen the analysis by assuming that there is an ex ante step where domestic and world reserve stocks are established, followed by an ex post extraction phase.

Although the government has less power to affect the resource price than when the economy is closed, its choice of consumption taxes applied during the extraction period and the domestic reserve subsidy applied ex ante determine the scarcity rent enjoyed by both foreign producers and domestic ones, if any; they amount to a rent commitment towards the latter. This rent depends on the policies implemented in the rest of the world, which are taken as given in Nash equilibrium by the home government. Unlike the closed economy, the government is restricted to leaving its suppliers a rent at least as high as they would get if the domestic market was taxed to extinction.[55] The rent commitment occurs ex ante and is simultaneous with the choice of the reserve subsidy. Given that government market power is limited to the nonrenewable resource, because the supply of conventional goods is infinitely elastic, no tax or subsidy is applied on the supply of conventional commodities. Trade in these commodities combines with resource trade as in Bergstrom in such a way that the trade balance constraint is satisfied. For simplicity, and with no consequence on the results, it is assumed that there are only two countries.

Unless otherwise mentioned, all variables and functions are redefined so as to refer to the home country. Variables or functions pertaining to the rest of the world will be denoted by the same symbol and identified with the superscript F. Given the absence of rents or taxes on the supply side of conventional goods, surpluses on conventional goods are defined in terms of the (domestic) demands x_{it} as before. In the case of the resource, x_{st} now denotes instantaneous domestic demand while y_t denotes instantaneous domestic supply, and θ_{st} denotes the tax on demand. The resource supply tax or subsidy ρ is applied ex ante as in Section 3. Given these remarks and redefinitions, the equilibrium domestic consumer

[54] See Appendix F for generalizations.

[55] As justified above, we do not allow the government to tax domestic extraction. If it would, domestic rents would be allowed to differ from world rents; however the sum of extraction rent and support to exploration could be kept unchanged by adjusting the reserve subsidy, implying identical domestic reserves. Thus our treatment is compatible with a continuum of domestic resource taxation systems of combining extraction taxes and support to exploration as in many observed situations.

surplus $\widetilde{CS}_t$ is still given by (8), the producer surplus under competitive equilibrium is identical to (9) except that $\widetilde{y}_t$ replaces $\widetilde{x}_{st}$, and the home producers' total resource rent, formerly (10) becomes $\widetilde{\phi}_t = \widetilde{\eta}_t \widetilde{y}_t$.

The analysis replicates that of Section 3. Consider first the ex post extraction stage under the ex ante commitment to consumption taxes that induce a given unit rent $\widetilde{\eta}_0 > 0$. The choice of $\widetilde{\eta}_0$ and of the supply subsidy ρ will be discussed immediately thereafter. Given that the resource is traded and that its marginal extraction cost is the same in the rest of the world as in the home country, unit rents are equalized: $\widetilde{\eta}_0 = \widetilde{\eta}_0^F$. The relevant supply to the home country is the residual world supply, that is the supply remaining once demand from the rest of the world has been met. At each date, the remaining stock of reserves available for consumption in the home country is thus $\widetilde{S}_t^H \equiv \widetilde{S}_t + \widetilde{S}_t^F - \int_t^{+\infty} \widetilde{x}_{su}^F \, du$ where home and foreign reserves $\widetilde{S}_0$ and $\widetilde{S}_0^F$ are established ex ante so that they are given when extraction starts; and where, since $\widetilde{x}_{st}^F = D_s^F(c_s + \widetilde{\eta}_t)$, the remaining foreign demand $\int_t^{+\infty} \widetilde{x}_{su}^F \, du$ is determined by the ex ante rent commitment. The exhaustibility constraint relevant to the home government is thus

$$\dot{\widetilde{S}}_t^H = -\widetilde{x}_{st}. \tag{N.1}$$

The Hamiltonian corresponding to this open-economy problem differs from its closed-economy counterpart (21) only by the producer surplus and the resource rent:

$$\mathcal{H}(a_t, \theta_t, \lambda_t, \mu_t) = \left(\widetilde{CS}_t + \widetilde{PS}_t + \widetilde{\phi}_t\right) e^{-rt} + \lambda_t (r a_t + \theta_t \widetilde{x}_t) - \mu_t \widetilde{x}_{st}, \tag{N.2}$$

where μ_t is now associated with (N.1). From the maximum principle, as in Section 3, $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu \geq 0$, with μ again given by (25); then (see the proof below),

$$\frac{\theta_{st}^*}{\widetilde{q}_{st}} = \rho \frac{e^{rt}}{\widetilde{q}_{st}} + \frac{\lambda - 1}{\lambda} \frac{1}{-\widetilde{\varepsilon}_s} + \frac{1}{\lambda}(1 - \widetilde{\alpha}_t) \widetilde{\eta}_0 \frac{e^{rt}}{\widetilde{q}_{st}}, \tag{N.3}$$

where $\widetilde{\alpha}_t \equiv \frac{d\widetilde{y}_t / d\theta_{st}}{d\widetilde{x}_{st} / d\theta_{st}}$ is the change in domestic resource production relative to the change in domestic consumption, induced by domestic taxation.

Formula (N.3) is the open-economy counterpart of (26) and differs from it by the last term; if $\widetilde{\alpha}_t$ equalled unity, this term would vanish. By the definition of $\widetilde{\alpha}_t$, this happens if any change in domestic consumption is exclusively met by domestic supply. Clearly, this includes the limit case where the rest of the world is negligible as well as situations where the foreign country does not hold any resource. In contrast, $0 < \widetilde{\alpha}_t < 1$ whenever foreign supply to the domestic resource market adjusts to a change in the tax on domestic demand in the same direction as domestic supply does. This reinforces the closed-economy result stated in Proposition 3 that the consumption of the nonrenewable resource is taxed at a higher rate than the consumption of a conventional good or service having the same demand elasticity when $\rho \geq 0$.

N.2 Proof of Expression (N.3)

The Hamiltonian associated with the ex post open-economy problem is (N.2). Applying the maximum principle also gives $\lambda_t = \lambda e^{-rt}$ and $\mu_t = \mu$. Since the government is subject to its ex ante commitment, $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$ is determined at this stage, as well as $\widetilde{x}_{st}^F$, which depends on θ_{st} only via $\widetilde{\eta}_t$. Hence, the first-order condition for the choice of the tax is

$$\left[D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s - \widetilde{\eta}_t\right] \frac{d\widetilde{x}_{st}}{d\theta_{st}} + \widetilde{\eta}_t \frac{d\widetilde{y}_t}{d\theta_{st}} - \widetilde{x}_{st} + \lambda(\widetilde{x}_{st} + \theta_{st}\frac{d\widetilde{x}_{st}}{d\theta_{st}}) = \mu e^{rt} \frac{d\widetilde{x}_{st}}{d\theta_{st}}.$$

Since $D_s^{-1}(\widetilde{x}_{st}) - \theta_{st} - c_s = \widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$, where $\widetilde{\eta}_0$ is given, the first term on the left-hand side is zero and $\frac{d\widetilde{x}_{st}}{d\theta_{st}} = \frac{1}{D_s^{-1\prime}(.)}$. Inserting into the above condition and rearranging give

$$\theta_{st}^* = \frac{1}{\lambda}(\mu - \widetilde{\alpha}_t \widetilde{\eta}_0)e^{rt} + \frac{\lambda - 1}{\lambda} \frac{\widetilde{q}_{st}}{-\widetilde{\varepsilon}_s}, \tag{N.4}$$

where $\widetilde{\alpha}_t = \frac{d\widetilde{y}_t/d\theta_{st}}{d\widetilde{x}_{st}/d\theta_{st}}$ and $\widetilde{\varepsilon}_s \equiv \frac{\widetilde{q}_{st}}{\widetilde{x}_{st} D_s^{-1\prime}(.)}$.

In the open economy, the ex post maximized value of $\int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt$, $V^*(\widetilde{S}_0^H, R; \rho)$, is a function of the residual reserves available to the home country $\widetilde{S}_0^H \equiv \widetilde{S}_0 + \widetilde{S}_0^F - \int_0^{+\infty} \widetilde{x}_{st}^F\, dt$. The constant co-state variable μ in (N.2) should be interpreted as giving the value $\frac{\partial V^*}{\partial \widetilde{S}_0^H}$ of a marginal unit of residual reserves. By definition of $\widetilde{S}_0^H$ it must be that μ is also the value $\frac{\partial V^*}{\partial \widetilde{S}_0}$ of a marginal unit of domestic reserves. The rest of the reasoning leading to (25) in Section 3 applies.

Substituting (25) into (N.4) yields (N.3). ■

N.3 Resource Consumption Tax in Open Economy

Clearly there is an intertemporal equilibrium where $\widetilde{\alpha}_t$ is time invariant.[56] In that case the last term in (N.3) defines a component of the unit tax θ^*_{st} which is rising at the discount rate; hence, the extra taxation imposed upon resource consumption in the open economy relative to the closed economy is neutral. The second term, the distortionary Ramsey component, is the same as in the closed economy.

Consider now the ex ante open-economy problem. Given that the resource consumption taxes must satisfy (N.3) ex post, the problem of choosing $\widetilde{\eta}_0$ and ρ is

$$\max_{\widetilde{\eta}_0,\ \rho} \int_0^{+\infty} \widetilde{W}_t e^{-rt}\, dt + \rho \widetilde{S}_0 - \int_0^{\widetilde{S}_0} \mathcal{S}^{-1}(S)\, dS \tag{N.5}$$

subject to

$$\int_0^{+\infty} \theta^*_t \widetilde{x}_t e^{-rt}\, dt \geq R_0 + \rho \widetilde{S}_0 \equiv R. \tag{N.6}$$

There is an important difference between this problem and its closed-economy counterpart (23). In the closed-economy problem, the first-order condition with respect to ρ and the expression for the ex post tax (26) are linearly dependent. This is why an infinity of ex post taxes-ex ante subsidy combinations were shown to be optimal and equivalent: In the closed economy the equivalence of demand taxation and supply taxation extends from the static realm of conventional goods to the dynamic framework of resource extraction where ρ is applied prior to θ_{st}. This is not so in the open economy; the first-order condition for ρ in problem (N.5) and expression (N.3) for the optimal extraction tax, are not linearly

[56]This is because in any intertemporal equilibrium domestic and foreign resource supply flows are only determined to the extent that their sum is determined and that domestic and foreign exhaustibility constraints must be met. This can be shown as follows. For any given tax schedule, the rent must rise at the rate of interest: $\widetilde{\eta}_t = \widetilde{\eta}_0 e^{rt}$. The resource market must clear at each date so that $\widetilde{x}_{st} + \widetilde{x}^F_{st} = \widetilde{y}_t + \widetilde{y}^F_t$. On the demand side, $\widetilde{x}_{st}$ and $\widetilde{x}^F_{st}$ are demanded quantities for the current resource price, uniquely determined at each date by $\widetilde{\eta}_0$, thus giving the world equilibrium supply $\widetilde{y}^W_t = \widetilde{x}_{st} + \widetilde{x}^F_{st}$. On the supply side, however, producers are indifferent about when to extract since $\widetilde{\eta}_t$ rises at the rate of interest. Hence, equilibrium domestic and foreign supplies $\widetilde{y}_t$ and $\widetilde{y}^F_t$ are only determined to the extent that they must fulfill the exhaustibility constraints for established reserves, $\widetilde{S}_0 = \int_0^{+\infty} \widetilde{y}_t\, dt$ and $\widetilde{S}^F_0 = \int_0^{+\infty} \widetilde{y}^F_t\, dt$, as well as the clearing condition $\widetilde{y}_t + \widetilde{y}^F_t = \widetilde{y}^W_t$, where $\widetilde{y}^W_t$ is determined as above.

Clearly, there is an infinity of combined paths of domestic supply $\widetilde{y}_t$ and foreign supply $\widetilde{y}^F_t$ satisfying these two conditions. A simple and natural combination is the one along which relative instantaneous supplies remain constant, so that $\frac{\widetilde{y}_t}{\widetilde{y}^F_t} = \frac{\widetilde{S}_0}{\widetilde{S}^F_0} \equiv \sigma$. For a given rent-commitment $\widetilde{\eta}_0$, foreign consumption $\widetilde{x}^F_{st}$ is given, so that tax changes only affect $\widetilde{x}_{st}$. Hence, the above condition implies that the domestic supply reaction to a change $d\widetilde{x}_{st}$ must be $d\widetilde{y}_t = \frac{\sigma}{1+\sigma} d(\widetilde{y}_t + \widetilde{y}^F_t) = \frac{\sigma}{1+\sigma} d\widetilde{x}_{st}$, which defines $\widetilde{\alpha} \equiv \frac{\sigma}{1+\sigma}$, constant and lower than unity.

dependent; they combine to determine the optimal tax path and the optimal subsidy for any feasible rent-commitment $\widetilde{\eta}_0$ by the government.

Consequently, while Proposition 3 survives almost unscathed the extension from the closed economy to the open economy, Proposition 4, which states that an infinity of tax-subsidy mixes yield the optimal level of reserves and extraction path in a closed economy, does not hold in an open economy.

Consider the afore-mentioned equilibrium in whcih $\widetilde{\alpha}_t$ is time invariant; see Footnote 56 for a detailed description. The comparison of (N.3) with (13) and (26) yields the following proposition.

Proposition 7 *(Resource consumption tax in open economy) When the nonrenewable resource is traded, there is an equilibrium such that the Home country and the Rest of the world contribute to world resource supply in the same proportion as they share reserves. Then,*

1. *Domestic resource consumption is taxed at a strictly higher rate than the consumption of conventional goods of the same demand elasticity when supply subsidies in the resource sector are non negative ($\rho \geq 0$).*

2. *The optimal tax rate (N.3) on resource consumption is made up of non-distortionary and distortionary components. The distortionary component is the same as in the closed economy and expresses Ramsey's inverse elasticity rule.*

N.4 Rent Capture and Ramsey Taxation

In the closed economy with endogenous reserves, first-best optimal commodity taxes do not yield any fiscal revenues. In contrast, in the open economy, it is well known that optimal tariffs are not nil, so that a combination of commodity taxes mimicking optimal tariffs produces tax revenues and may meet government needs without involving any distortion. The distinction between low and high revenue needs, made in Section 2 with exogenous reserves, thus arises again when the economy is open in spite of the endogeneity of reserves. Low and high revenue needs should be defined according to whether government needs are below or above the amount $\overline{\overline{R}}_0$ raised when the resource tax is set so as to maximize welfare in the absence of tax-revenue constraint ($\overline{\overline{R}}_0$ is established in the proof of Proposition 8 below). Call this the rent-capture component of the optimal domestic consumption tax. If $R_0 > \overline{\overline{R}}_0$, the rent-capture component of the domestic resource consumption tax is not sufficient to meet revenue needs and it must be true that $\lambda > 1$; only then does the second term in (N.3), the distortionary component of the optimal consumption tax, become positive.

When the taxation of resources is distortionary, the distortion may affect both the extraction path and the amount of initial reserves. Consider the international equilibrium where $\widetilde{\alpha}_t$ is time invariant; the first and third terms in (N.3) then rise at the rate of discount while the distortionary component is identical to its counterpart in (26). Stiglitz's (1976) special case of isoelastic domestic demand and zero extraction costs then again implies that the optimal tax on resource demand is neutral and rises at the rate of interest.

An additional interest of Stiglitz's special case is that, when extraction costs are zero, a unit resource consumption tax that is rising at the rate of interest induces the final price $\widetilde{q}_{st}$ to rise at the same rate. Hence, such a tax is tantamount to the constant *ad valorem* tax in Bergstrom (1982). Our open-economy model then differs from Bergstrom's only in the treatment of reserves, exogenous in his paper, endogenous here. Bergstrom's inverse elasticity rule maximizes the country's surplus without any constraint on tax revenues, so that it is equivalent to an optimal tariff. Stiglitz's special case then enables us to investigate how the optimal resource tax of the Ramsey government differs from a commodity tax that would pursue the objective of an optimal tariff.

With θ^*_{st} now equal to $\theta^*_{s0}e^{rt}$, expressions (N.3) are determined at all dates by the initial level of the optimal resource tax. The maximization of (N.5) with respect to θ_{s0} is equivalent to its maximization with respect to the rent $\widetilde{\eta}_0$ induced by θ_{s0}. The resulting optimal tax rate, the open-economy counterpart of (28) is:

$$\frac{\theta^*_{s0}}{\widetilde{q}_{s0}} = \frac{\rho}{\widetilde{q}_{s0}} \frac{\widetilde{S}_0 \widetilde{\zeta}}{\widetilde{S}_0^H \widetilde{\zeta}^H} + \frac{\lambda - 1}{\lambda} \left[\frac{1 - \frac{\theta^*_{s0}}{\widetilde{q}_{s0}}}{\widetilde{\zeta}^H} + \frac{1}{-\widetilde{\xi}} \right] + \frac{1}{\lambda} \frac{1 - \frac{\theta^*_{s0}}{\widetilde{q}_{s0}}}{\widetilde{S}^H \widetilde{\zeta}^H} \left[\widetilde{\mathcal{D}} - \widetilde{S}_0 \right], \tag{N.7}$$

where $\widetilde{S}_0^H \equiv \widetilde{S}_0 + \widetilde{S}_0^F - \widetilde{\mathcal{D}}^F$ is the residual supply of reserves available for home country consumption, whose elasticity is defined as $\widetilde{\zeta}^H \equiv \left(\frac{d\widetilde{S}_0^H}{d\eta_0} \right) \frac{\widetilde{\eta}_0}{\widetilde{S}_0^H}$. The proof of (N.7) is presented below, in the next subsection.

Expression (N.7) simplifies to (28) when the totality of domestic consumption is met by domestic production.[57] Although complex, it brings up simple and important insights. First it shows the role of resource supply and its elasticity explicitly. It stresses the distinction between domestic production $\widetilde{S}_0$, which may be consumed locally or exported and can be taxed or subsidized in both cases, and foreign supply to the domestic market, which cannot be taxed or subsidized; $\widetilde{S}_0^H$ combines both. For a resource importer $\left(\widetilde{\mathcal{D}} - \widetilde{S}_0 > 0\right)$ that does not tax reserve production ($\rho \geq 0$), the optimal tax rate decreases when the elasticity of residual reserve supply $\widetilde{\zeta}^H$ increases. Indeed, Pigou (1947, p. 113) attempted to extend Ramsey's principles to trading economies. Since the residual supply of internationally-traded commodities presumably has a greater elasticity than total supply, he conjectured that Ramsey's analysis would imply imposing lower tax rates on those commodities.

Second, (N.7) connects neatly with the literature on the capture of resource rents initiated by Bergstrom (1982) and with the question of optimal tariffs in the presence of nonrenewable resources. Bergstrom treats reserves as given so he does not envisage a subsidy: $\rho = 0$. Bergstrom does not consider that the government faces any revenue constraint: $\lambda = 1$. Consequently the first and second terms disappear under his setup. Multiplying by $\widetilde{q}_{s0}$, substituting $\widetilde{\eta}_0 = \widetilde{q}_{s0} - \theta^*_{s0}$, we obtain

$$\frac{\theta^*_{s0}}{\widetilde{\eta}_0} = \frac{1}{\widetilde{S}_0^H \widetilde{\zeta}^H} \left(\widetilde{\mathcal{D}} - \widetilde{S}_0\right). \tag{N.8}$$

Since extraction costs are assumed nil, $\frac{\theta^*_{s0}}{\widetilde{\eta}_0}$ is the optimal, constant *ad valorem* tax given by Bergstrom in Expression (32), p. 198. One may wonder why Bergstrom's formula involves countries' demand elasticities and no supply elasticity. The reason is the assumption of exogenous world reserves. A country's residual supply then only depends on other countries' demands and not on the technology of reserve discovery as in this paper. Once $\widetilde{S}_0^H$ and its elasticity are written in terms of resource demands using $\widetilde{S}_0^H = \widetilde{S}_0 + \widetilde{S}_0^F - \widetilde{\mathcal{D}}^F$, we obtain Bergstrom's Expression (32).[58]

This formula is famous because it implies that a net importer should tax the resource, at least to the extent that it holds market power. This is Pareto optimal from that country's

[57] The last term vanishes when $\widetilde{\mathcal{D}} - \widetilde{S}_0 = 0$, and it must then also be the case that $\widetilde{S}_0^F - \widetilde{\mathcal{D}}^F = 0$ so that $\widetilde{S}_0^H = \widetilde{S}_0$, $\widetilde{\zeta}^H = \widetilde{\zeta}$, and the first term reduces to $\frac{\rho}{\widetilde{q}_{s0}}$ as in (28).

[58] This being the two-country case, the summation symbol in Bergstrom disappears, so that, in our notations—we also corrected a typo in Bergstrom—the formula reads $\frac{\theta^*_{s0}}{\widetilde{\eta}_0} = \frac{\widetilde{\mathcal{D}} - \widetilde{S}_0}{-\widetilde{\mathcal{D}}^F \widetilde{\xi}^F}$.

point of view and allows it to capture some of the rents otherwise falling into the hands of exporters. When reserves are endogenous this power to capture rents is attenuated: $\widetilde{S}_0^H\widetilde{\zeta}^H$ being higher than its exogenous-reserve counterpart $-\widetilde{\mathcal{D}}^F\widetilde{\xi}^F$, the importer must not tax resource consumption as much: Depriving foreign suppliers of resource rents would reduce their supply of reserves.

Third, the first term in (N.7) shows the arbitrage between ex ante reserve subsidization and ex post taxation of resource consumption: The consumption tax increases with reserve subsidization by a factor of proportionality equal to the ratio of local production over residual supply to the home country, both weighted by their respective elasticities. This ratio is unity in the closed economy, so that the trade-off between taxing extraction or subsidizing reserves is financially neutral. The trade-off would be financially neutral in a competitive open economy if the coefficient of ρ were $\frac{\widetilde{S}_0}{\widetilde{S}_0^H}$, reflecting the fact that the tax base of domestic production is smaller than the tax base of domestic consumption; the presence of elasticities in the coefficient of ρ makes it plain that the optimal tax-subsidy combination further reflects the ability of the country to manipulate prices by its choice of the tax instruments.

The main results are gathered in the following proposition; see the proof below.

Proposition 8 *(Rent capture and Ramsey taxation) When further to the conditions of Proposition 7, domestic demand is isoelastic, and extraction is costless, the maximum revenue need $\overline{\overline{R}}_0$ compatible with neutral resource taxation is given by (N.13) and the optimal taxes or subsidies on resource consumption and reserve supply are jointly determined by (N.7) and (N.12). More precisely,*

1. *When $R_0 \le \overline{\overline{R}}_0$, so that (N.7) and (N.12) hold with $\lambda = 1$, OCT is Pareto optimum and fulfills a resource-rent-capture objective. For an importing country, this involves taxing resource consumption while subsidizing domestic production, and vice versa for an exporter.*

2. *When government revenue needs are high, (N.7) and (N.12) apply with $\lambda > 1$. Optimal resource taxes are then higher than when $R_0 \le \overline{\overline{R}}_0$ (reserve subsidies are lower) by an amount that reflects both domestic and foreign demand elasticities, as well as domestic and foreign supply elasticities.*

The formula giving the optimal level of ρ is (N.12); being the sister of Formula (N.7), it can be read and interpreted in much the same way. When revenue needs are low, ρ is always strictly positive for importing countries, as is well understood from the optimal tariff literature. Sufficiently high revenue needs, however, may reverse the result, implying that it may be optimal to tax reserve production, even in importing countries. Similarly, under sufficiently high revenue needs, exporters may tax consumption according to (N.7).

N.5 Proof of Proposition 8, and of Expressions (N.7) and (N.12)

1. Most of the first part of Proposition 8 is shown in the main text and in the proof of (N.3). Expressions (N.7) and (N.12) remain to be proven. They can be obtained as follows.

Throughout this proof, Stiglitz (1976)'s conditions hold: The elasticity of domestic demand $\varepsilon_s(q_{st})$ is a constant ε_s and marginal extraction cost c_s is zero. Without any further loss of generality, we may then restrict attention to the equilibrium in which $\widetilde{\alpha}_t = \widetilde{\alpha}$ is time invariant.

In this case the optimal extraction unit tax is given by (N.3) multiplied by $\widetilde{q}_{st}$; it rises at the rate of interest. This formula only differs from (26) by its last term, which is, after multiplying by $\widetilde{q}_{st}$, $\frac{1}{\lambda}(1-\widetilde{\alpha})\widetilde{\eta}_0 e^{rt}$. Recalling that the unit tax given by (26) has been shown to rise at the rate of interest in Appendix L, it remains to show that the new term does so, which is immediate since $\widetilde{\alpha}$ is constant. Hence, (L.1) is valid, where θ^*_{s0} is to be determined as follows.

The first-order condition for the ex ante static maximization of (N.5) with respect to θ^*_{s0} subject to (N.6), taking the ex post solution (L.1) into account, is, as in Appendix L,

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt}\, dt + \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} - \mathcal{S}^{-1}(.) \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0.$$

Furthermore, $\frac{d\widetilde{W}_t}{d\theta_{s0}} e^{-rt} = (D_s^{-1}(\widetilde{x}_{st}) - \widetilde{q}_{st}) \frac{d\widetilde{x}_{st}}{d\theta_{s0}} - \widetilde{x}_{st} + \frac{d\widetilde{\eta}_0}{d\theta_{s0}} (\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0 \frac{d\widetilde{y}_t}{d\theta_{s0}} = \frac{d\widetilde{\eta}_0}{d\theta_{s0}} (\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0 \frac{d\widetilde{y}_t}{d\theta_{s0}} - \widetilde{x}_{st}$ since $D_s^{-1}(\widetilde{x}_{st}) = \widetilde{q}_{st}$, and $\mathcal{S}^{-1}(.) = \widetilde{\eta}_0 + \rho$. Substituting, one has

$$\int_0^{+\infty} \left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}} (\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0 \frac{d\widetilde{y}_t}{d\theta_{s0}} - \widetilde{x}_{st} \right) dt - \widetilde{\eta}_0 \frac{d\widetilde{S}_0}{d\theta_{s0}} + \lambda \left(\int_0^{+\infty} \left(\widetilde{x}_{st} + \theta^*_{s0} \frac{d\widetilde{x}_{st}}{d\theta_{s0}} \right) dt - \rho \frac{d\widetilde{S}_0}{d\theta_{s0}} \right) = 0.$$

Integrating with $\int_0^{+\infty} \widetilde{x}_{st}\, dt = \widetilde{\mathcal{D}}$, $\int_0^{+\infty} \widetilde{y}_t\, dt = \widetilde{S}_0$, $\int_0^{+\infty} \frac{d\widetilde{x}_{st}}{d\theta_{s0}}\, dt = \frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}$ and $\int_0^{+\infty} \frac{d\widetilde{y}_t}{d\theta_{s0}}\, dt = \frac{d\widetilde{S}_0}{d\theta_{s0}}$, and rearranging give

$$\theta_{s0}^{*} = \rho \frac{\frac{d\widetilde{S}_0}{d\theta_{s0}}}{\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}} - \frac{(\lambda-1)}{\lambda} \frac{\widetilde{\mathcal{D}}}{\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}}} + \frac{1}{\lambda} \frac{\frac{d\widetilde{\eta}_0}{d\theta_{s0}}}{\frac{d\widetilde{\mathcal{D}}}{\theta_{s0}}} \left[\widetilde{\mathcal{D}} - \widetilde{S}_0\right]. \tag{N.9}$$

In long-run market equilibrium, $\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho)$ and $\widetilde{\mathcal{D}} = \int_0^{+\infty} D_s\left((\widetilde{\eta}_0 + \rho)e^{rt}\right) dt = \widetilde{S}_0^H$, where $\widetilde{S}_0^H$ is the residual supply as defined in the main text. It follows by differentiation with respect to θ_{s0} that $\frac{d\widetilde{S}_0}{d\theta_{s0}} = \mathcal{S}'(.)\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$ and that $\frac{d\widetilde{\mathcal{D}}}{d\theta_{s0}} = \left(\frac{d\widetilde{\eta}_0}{d\theta_{s0}} + 1\right) \int_0^{+\infty} D_s'(.)e^{rt}\, dt = \frac{d\widetilde{S}_0^H}{d\eta_0}\frac{d\widetilde{\eta}_0}{d\theta_{s0}}$. From that equality, we obtain $\frac{d\widetilde{\eta}_0}{d\theta_{s0}} = \frac{\int_0^{+\infty} D_s'(.)e^{rt}\, dt}{\frac{d\widetilde{S}_0^H}{d\eta_0} - \int_0^{+\infty} D_s'(.)e^{rt}\, dt}$. Introducing these expressions in (N.9) yields

$$\theta_{s0}^{*} = \rho \frac{\mathcal{S}'(.)}{\frac{d\widetilde{S}_0^H}{d\eta_0}} + \frac{\lambda-1}{\lambda}\left[\frac{\widetilde{S}_0^H}{\frac{d\widetilde{S}_0^H}{d\eta_0}} - \frac{\widetilde{\mathcal{D}}}{\int_0^{+\infty} D_s'(.)e^{rt}\, dt}\right] + \frac{1}{\lambda}\frac{1}{\frac{d\widetilde{S}_0^H}{d\eta_0}}\left[\widetilde{\mathcal{D}} - \widetilde{S}_0\right], \tag{N.10}$$

from which (N.7) is obtained after substituting $\widetilde{\zeta}$, $\widetilde{\zeta}^H$, $\widetilde{\xi}$. For the latter, we proceed in the same way as described in Appendix L.

The first-order condition for the ex ante static maximization of (N.5) with respect to ρ subject to (N.6), taking the ex post solution (L.1) into account is

$$\int_0^{+\infty} \frac{d\widetilde{W}_t}{d\rho} e^{-rt}\, dt + \widetilde{S}_0 + \rho\frac{d\widetilde{S}_0}{d\rho} - \mathcal{S}^{-1}(.)\frac{d\widetilde{S}_0}{d\rho} + \lambda\left(\int_0^{+\infty} \theta_{s0}\frac{d\widetilde{x}_{st}}{d\rho}\, dt - \widetilde{S}_0 - \rho\frac{d\widetilde{S}_0}{d\rho}\right) = 0,$$

where $\frac{d\widetilde{W}_t}{d\rho}e^{-rt} = \left(D_s^{-1}(\widetilde{x}_{st}) - \widetilde{q}_{st}\right)\frac{d\widetilde{x}_{st}}{d\rho} + \frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho} = \frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho}$ since $D_s^{-1}(\widetilde{x}_{st}) = \widetilde{q}_{st}$. Substituting and using $S^{-1}(.) = \widetilde{\eta}_0 + \rho$, one has

$$\int_0^{+\infty}\left(\frac{d\widetilde{\eta}_0}{d\rho}(\widetilde{y}_t - \widetilde{x}_{st}) + \widetilde{\eta}_0\frac{d\widetilde{y}_t}{d\rho}\right) dt + \widetilde{S}_0 - \widetilde{\eta}_0\frac{d\widetilde{S}_0}{d\rho} + \lambda\left(\int_0^{+\infty} \theta_{s0}\frac{d\widetilde{x}_{st}}{d\rho}\, dt - \widetilde{S}_0 - \rho\frac{d\widetilde{S}_0}{d\rho}\right) = 0.$$

Integrating as above and rearranging give

$$\rho^{*} = \theta_{s0}\frac{\frac{d\widetilde{\mathcal{D}}}{d\rho}}{\frac{d\widetilde{S}_0}{d\rho}} - \frac{(\lambda-1)}{\lambda}\frac{\widetilde{S}_0}{\frac{d\widetilde{S}_0}{d\rho}} + \frac{1}{\lambda}\frac{\frac{d\widetilde{\eta}_0}{d\rho}}{\frac{d\widetilde{S}_0}{\rho}}\left[\widetilde{S}_0 - \widetilde{\mathcal{D}}\right]. \tag{N.11}$$

In long-run market equilibrium, $\widetilde{\mathcal{D}} = \int_0^{+\infty} D_s\left((\widetilde{\eta}_0 + \theta_{s0})e^{rt}\right) dt$ and $\widetilde{S}_0 = \mathcal{S}(\widetilde{\eta}_0 + \rho) = \widetilde{\mathcal{D}}^H$, where $\widetilde{\mathcal{D}}^H \equiv \widetilde{\mathcal{D}} + \widetilde{\mathcal{D}}^F - \widetilde{S}_0^F$, is the residual cumulative demand of the rest of the world, which has to be met by the supply of domestic reserves. It follows by differentiation with respect to ρ that $\frac{d\widetilde{\mathcal{D}}}{d\rho} = \frac{d\widetilde{\eta}_0}{d\rho}\int_0^{+\infty} D_s'(.)e^{rt}\,dt$ and $\frac{d\widetilde{S}_0}{d\rho} = \mathcal{S}'(.)\left(\frac{d\widetilde{\eta}_0}{d\rho}+1\right) = \frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}\frac{d\widetilde{\eta}_0}{d\rho}$. From that equality, we obtain $\frac{d\widetilde{\eta}_0}{d\rho} = \frac{-\mathcal{S}'(.)}{\mathcal{S}'(.)-\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}}$. Introducing these expressions into (N.11) yields

$$\rho^* = \theta_{s0}\frac{\int_0^{+\infty} D_s'(.)e^{rt}\,dt}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}} - \frac{\lambda-1}{\lambda}\left[\frac{\widetilde{S}_0}{\mathcal{S}'(.)} - \frac{\widetilde{\mathcal{D}}^H}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}}\right] + \frac{1}{\lambda}\frac{1}{\frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}}\left[\widetilde{S}_0 - \widetilde{\mathcal{D}}\right].$$

Using the definition $\widetilde{\xi}^H \equiv \frac{d\widetilde{\mathcal{D}}^H}{d\eta_0}\frac{\widetilde{\eta}_0}{\widetilde{\mathcal{D}}^H} < 0$ and redefining $\widetilde{\xi} \equiv \frac{d\widetilde{\mathcal{D}}}{d\eta_0}\frac{\widetilde{\eta}_0}{\widetilde{\mathcal{D}}}$ as well as $\widetilde{\zeta} \equiv \frac{(\widetilde{\eta}_0+\rho)\mathcal{S}'(.)}{\widetilde{S}_0}$, we obtain

$$\frac{\rho^*}{\widetilde{\eta}_0+\rho^*} = \frac{\theta_{s0}}{\widetilde{\eta}_0+\rho^*}\frac{\widetilde{\mathcal{D}}\widetilde{\xi}}{\widetilde{\mathcal{D}}^H\widetilde{\xi}^H} - \frac{\lambda-1}{\lambda}\left[\frac{1}{\widetilde{\zeta}} + \frac{1-\frac{\rho^*}{(\widetilde{\eta}_0+\rho^*)}}{-\widetilde{\xi}^H}\right] + \frac{1}{\lambda}\frac{1-\frac{\rho^*}{(\widetilde{\eta}_0+\rho^*)}}{\widetilde{\mathcal{D}}^H\widetilde{\xi}^H}\left[\widetilde{S}_0 - \widetilde{\mathcal{D}}\right]. \quad \text{(N.12)}$$

When $\lambda = 1$, the second term on the right-hand side, the distortionary Ramsey component of the subsidy, vanishes. If $\theta_{s0} > 0$ and the home country is importing the resource, i.e., $\widetilde{S}_0 - \widetilde{\mathcal{D}} < 0$, ρ^* is non-ambiguously positive. Since $\frac{\widetilde{S}_0\widetilde{\zeta}}{\widetilde{S}_0^H\widetilde{\zeta}^H} < 1$ and $\frac{\widetilde{\mathcal{D}}\widetilde{\xi}}{\widetilde{\mathcal{D}}^H\widetilde{\xi}^H} < 1$ by the definitions of S_0^H and $\mathcal{D}^H$, combining (N.12) with (N.7), computed for $\lambda = 1$, yields a strictly positive tax $\theta_{s0}^* > 0$ and a strictly positive subsidy $\rho^* > 0$. The second term on the right-hand side of (N.12) is negative. Therefore, for sufficiently high revenue needs, ρ^* may turn negative, i.e., may become a tax on reserve development.

Symmetrically, if the home country is exporting the resource, i.e., $\widetilde{S}_0 - \widetilde{\mathcal{D}} > 0$, then θ_{s0}^* and ρ^* are strictly negative when $\lambda = 1$; the second term on the right-hand side of (N.7) being positive, θ_{s0}^* may turn positive for sufficiently high revenue needs, i.e., may become a tax on domestic resource consumption.

2. The proof is similar to the Proof of Proposition 1. We know that when $\lambda = 1$, $\theta_i^* = 0$, $i = 1, ..., n$, so that the totality of fiscal revenues is raised from the resource sector. In the context of Proposition 8, $\theta_{st}^* = \theta_{s0}^* e^{rt}$, where θ_{s0}^*, given by (N.7), is jointly determined with ρ^*, given by (N.12). Combining both expressions for $\lambda = 1$ and substituting into

$$\overline{\overline{R}}_0 \equiv \theta_{s0}^*\widetilde{\mathcal{D}} - \rho^*\widetilde{S}_0 \quad \text{(N.13)}$$

defines the net amount raised by the resource sector. Hence, when $\lambda = 1$ it must be the case that $R_0 \leq \overline{\overline{R}}_0$. The contrapositive is that any $R_0 > \overline{\overline{R}}_0$ implies $\lambda > 1$. Moreover, following the reasoning of the Proof of Proposition 1, any $R_0 \leq \overline{\overline{R}}_0$ will be raised without imposing distortion, implying $\lambda = 1$. ■

References

Alberta Royalty Review (2007), "Final Report to the Finance Minister," Government of Alberta, Edmonton
http://www.albertaroyaltyreview.ca/panel/final_report.pdf.

Atkinson, A.B., and J.E. Stiglitz (1976), "The Design of Tax Structure: Direct versus Indirect Taxation," *Journal of Public Economics*, 6: 55-75.

Auerbach, A.J., and K. Hassett (2015), "Capital Taxation in the Twenty-First Century," *American Economic Review*, 105: 38-42.

d'Autume, A., K. Schubert and C.A.A.M. Withagen (2016), "Should the Carbon Price be the Same in All Countries?," *Journal of Public Economic Theory*, forthcoming.

Barrage, L. (2017), "Optimal Dynamic Carbon Taxes in a Climate-Economy Model with Distortionary Fiscal Policy," *Review of Economic Studies*, forthcoming.

Baumol, W.J., and D.F. Bradford (1970), "Optimal Departures From Marginal Cost Pricing," *American Economic Review*, 60: 265-283.

Belan, P., S. Gauthier and G. Laroque (2008), "Optimal Grouping of Commodities for Indirect Taxation," *Journal of Public Economics*, 92: 1738-1750.

Bergstrom, T.C. (1982), "On Capturing Oil Rents with a National Excise Tax," *American Economic Review*, 72: 194-201.

Boadway, R.W., and F. Flatters (1993), "The Taxation of Natural Resources: Principles and Policy Issues," World Bank Policy Research Department Working Papers, 1210.

Boadway, R.W., and M. Keen (2010), "Theoretical Perspectives on Resource Tax Design," in: Daniel, P., M. Keen and C.P. McPherson (Eds.), *The Taxation of Petroleum and Minerals: Principles, Problems and Practice*, 13-74, Routledge.

Boadway, R.W., and M. Keen (2014), "Rent Taxes and Royalties in Designing Fiscal Regimes for Non-Renewable Resources," CESifo Working Papers, 4568.

Boadway, R.W., S. Maital and M. Prachowny (1973), "Optimal Tariffs, Optimal Taxes and Public Goods," *Journal of Public Economics*, 2: 391-403.

Chamley, C. (1986), "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica*, 54: 607-622.

Conrad, R.F., and B. Hool (1981), "Resource Taxation with Heterogeneous Quality and Endogenous Reserves," *Journal of Public Economics*, 16: 17-33.

Corlett, W.J., and D.C. Hague (1953), "Complementarity and the Excess Burden of Taxation," *Review of Economic Studies*, 21: 21-30.

Daniel, P., M. Keen and C.P. McPherson (2010), *The Taxation of Petroleum and Minerals: Principles, Problems and Practice*, Routledge.

Dasgupta, P.S., and G.M. Heal (1979), *Economic Theory and Exhaustible Resources*, Cambridge University Press.

Diamond, P.A., and J.A. Mirrlees (1971), "Optimal Taxation and Public Production I: Production Efficiency," *American Economic Review*, 61: 8-27.

Dornbusch, R. (1971), "Optimal Commodity and Trade Taxes," *Journal of Political Economy*, 79: 1360-1368.

Ernst and Young (2015) "Indirect Tax in 2015: A Review of Global Indirect Tax Developments and Issues." `http://www.ey.com/Publication/vwLUAssets/ey-indirect-tax-developments-in-2015/\$FILE/ey-indirect-tax-developments-in-2015.pdf`

Friedlander, A.F., and A.L. Vandendorpe (1968), "Excise Taxes and the Gains from Trade," *Journal of Political Economy*, 76: 1058-1068.

Garnaut, R.G. (2010), "Principles and Practice of Resource Rent Taxation," *Australian Economic Review*, 43: 347-356.

Gaudet, G. (2007), "Natural Resource Economics under the Rule of Hotelling," *Canadian Journal of Economics*, 40: 1033-1059.

Gaudet, G., and P. Lasserre (1986), "Capital Income Taxation, Depletion Allowances, and Nonrenewable Resource Extraction," *Journal of Public Economics*, 29: 241-253.

Hotelling, H. (1931), "The Economics of Exhaustible Resources," *Journal of Political Economy*, 39: 137-175.

Karp, L., and D.M. Newbery (1991), "OPEC and the U.S. Oil Import Tariff," *Economic Journal*, 101: 303-313.

Kopczuk, W. (2003), "A Note on Optimal Taxation in the Presence of Externalities," *Economics Letters*, 80: 81-86.

Levhari, D., and N. Liviatan (1977), "Notes on Hotelling's Economics of Exhaustible Resources," *Canadian Journal of Economics*, 10: 177-192.

Ley, E. (1992), "A Note on Ramsey and Corlett-Hague Rules," Universidad Carlos III – Departamento de Economía Working Papers, 1992-14.

Liski, M., and J.-P. Montero (2011), "On the Exhaustible-Resource Monopsony," mimeo, Aalto University.

Lucas, R.E. (1990), "Supply-Side Economics: An Analytical Review," *Oxford Economic Papers*, 42: 293-316.

Mundell, R.A. (1960), "The Pure Theory of International Trade," *American Economic Review*, 50: 67-110.

Pigou, A.C. (1947), *A Study in Public Finance*, Macmillan.

Piketty, T. (2015), "About Capital in the Twenty-First Century," *American Economic Review*, 105: 1-6.

Piketty, T., and E. Saez (2013), "A Theory of Optimal Inheritance Taxation," *Econometrica*, 81: 1851-1886.

Pindyck, R.S. (1978), "The Optimal Exploration and Production of Nonrenewable Resources," *Journal of Political Economy*, 86: 841-861.

Ramsey, F.P. (1927), "A Contribution to the Theory of Taxation," *Economic Journal*, 37: 47-61.

Sandmo, A. (1975), "Optimal Taxation in the Presence of Externalities," *Swedish Journal of Economics*, 77: 86-98.

Sandmo, A. (1976), "Optimal Taxation – An Introduction to the Literature," *Journal of Public Economics*, 6: 37-54.

Sandmo, A. (2011), "Atmospheric Externalities and Environmental Taxation," *Energy Economics*, 33: S4-S12.

Slade, M.E. (1988), "Grade Selection under Uncertainty: Least Cost Last and Other Anomalies," *Journal of Environmental Economics and Management*, 15: 189-205.

Stiglitz, J.E. (1976), "Monopoly and the Rate of Extraction of Exhaustible Resources," *American Economic Review*, 66: 655-661.

Stiglitz, J.E. (2015), "In Praise of Frank Ramsey's Contribution to the Theory of Taxation," *Economic Journal*, 125: 235-268.

Stiglitz, J.E., and P.S. Dasgupta (1971), "Differential Taxation, Public Goods, and Economic Efficiency," *Review of Economic Studies*, 38: 151-174.

Withagen, C.A.A.M. (1994), "Pollution and Exhaustibility of Fossil Fuels," *Resource and Energy Economics*, 16: 235-242.

MIT CEEPR

MIT Center for Energy and Environmental Policy Research

MIT Center for Energy and Environmental Policy Research
77 Massachusetts Avenue, E19-411
Cambridge, MA 02139
USA

Website: ceepr.mit.edu

MIT CEEPR Working Paper Series is published by the MIT Center for Energy and Environmental Policy Research from submissions by affiliated researchers.

For inquiries and/or for permission to reproduce material in this working paper, please contact:

Email ceepr@mit.edu
Phone (617) 253-3551
Fax (617) 253-9845